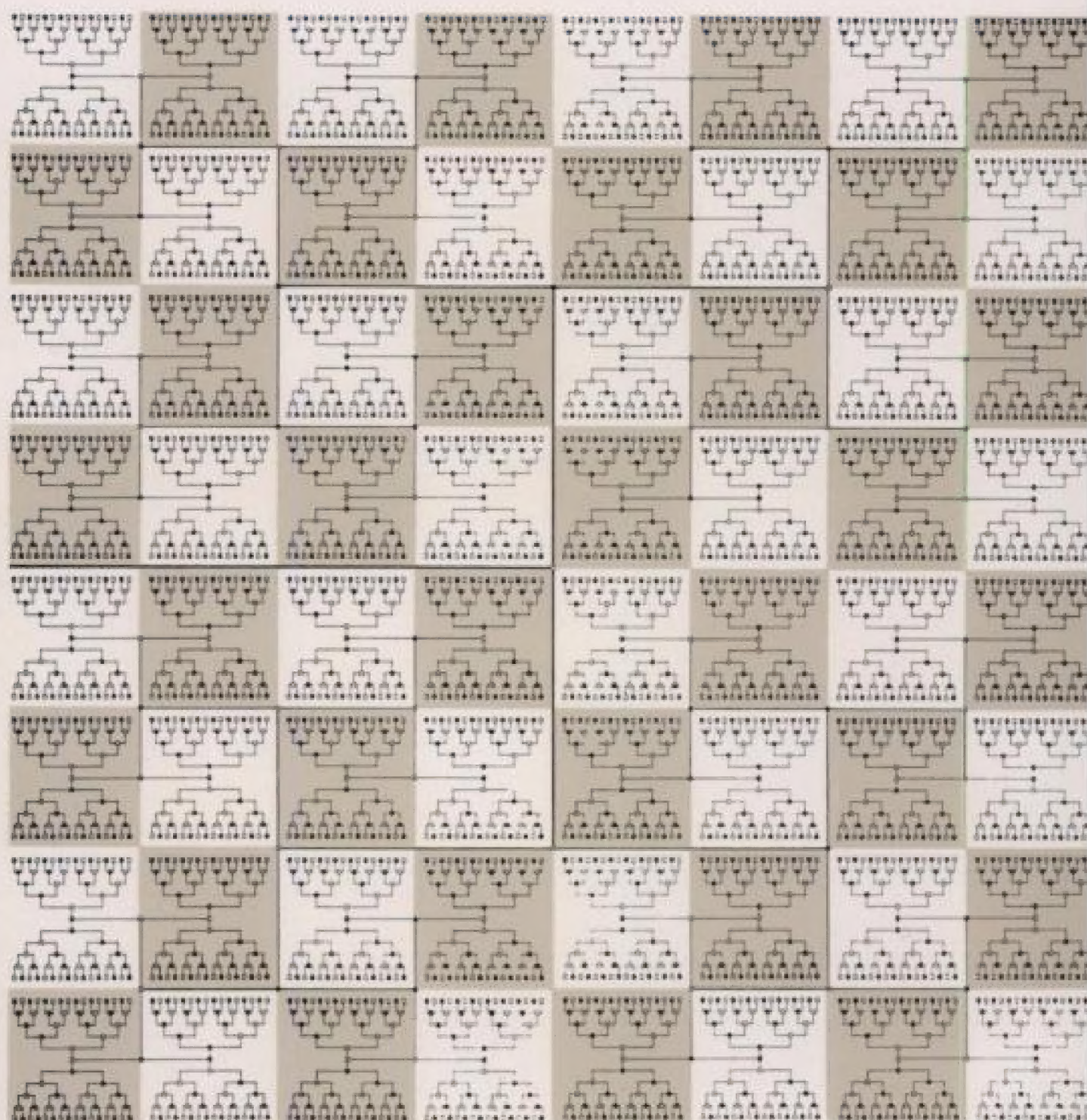


thought and choice in chess



a.d. de groot

PSYCHOLOGICAL STUDIES 4

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THOUGHT AND CHOICE IN CHESS

by

ADRIAAN D. DE GROOT

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PREFACE

What is so special about the thought processes underlying the skilled chess player's choice of a move? Why do masters find the good moves that patzers overlook? This book is an attempt at answering these questions through systematic description and interpretative analysis of a collection of 'thinking aloud protocols.' The protocols, products of experimental sessions held in the years 1938 to 1943, are verbal reports rendered by subjects performing a simple experimental task: a subject was presented with an unfamiliar position taken from an actual tournament or match game and asked to find and play a move as though he were engaged in a tournament game of his own. The verbal report was to be as full and explicit a rendering of the subject's thoughts as possible, to include his plans, calculations, and other considerations leading to the move decision. The author – at the time a psychology student at the University of Amsterdam and an active international chess player to boot – served as the principal experimenter while the subjects were players of varying strengths, up to masters, grandmasters, and even two world champions.

For the interested reader protocols of the thought processes of such titans as Alexander Alekhine, Max Euwe, Paul Keres, Reuben Fine, Salo Flohr and Savielly Tartakower are appended. Quite apart from the insight the analysis of these protocols has given us into the dynamics of thinking and choosing in chess, these protocols would seem to be of some historic and curiosity value in themselves.

Since the experiments were not in the rigorous laboratory tradition but rather 'free' occasions for goal-directed, productive thinking – so free, indeed, that some might prefer not to call them 'experiments' at all – the analysis of the resulting protocols had to remain largely descriptive and interpretative. Except for a few general expectations about masters doing better – they did – and calculating deeper – they did not – there were no present hypotheses being tested. The idea

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was rather to arrive at a generalized description of the structure and the dynamics of the thought process as a basis for theory formation; this to be accomplished through a painstakingly thorough and systematic analysis of the protocols. In other words: The main goal was to describe in general terms what 'happens' (structure) and to infer why it happens as it does (dynamics) in the mental processes preparatory to making a move decision.

As a working basis for this analysis, Otto Selz's conception of thinking as a hierarchically organized linear series of operations was adopted. In a way Selz's theory was tested too – a crucial question being the extent to which the above general conception would suffice for a gapless description and interpretation of the process of chess thinking.

So much for the central theme of the investigations – and for the body of the book. All other problems treated in the text – discussions of theory, of method, of an additional series of experiments in chess perception, the concept of mastership, the factors and the development of chess talent, etc. – can be viewed as variations, extensions, or generalizations of the main theme. But at this point we had better refer the reader to the Table of Contents or to Section 5 for a more complete description of the organization of the book.

The present work appeared originally in the Dutch language under the title of *Het Denken van den Schaker* (Amsterdam: North-Holland Publishing Co., Ltd., 1946) as a voluminous doctoral dissertation at the University of Amsterdam. The English edition, although revised, is for the most part a faithful translation of the original text. It is true that terminology was modernized, sections were rearranged, rewritten, or abbreviated, and references and new footnotes added; but in general both the content and composition of the book have remained the same – down to the numbering of sections.

Some changes and revisions should be made mention of, however. First, the English edition contains new material. Tables 8, 9, and 10 were not previously published; nor were the games from which experimental positions A, B, and C were taken (see Appendix I); nor was the collection of protocols (see Appendix II). Second, after twenty-five years the incognito of the grandmaster and master subjects has been lifted; the dead offered no objections while the quick readily consented to having their identity revealed. Third, the list of biographical information of famous masters (Table 15) has been brought up to date. Fourth, along with some other rearrangements in Chapter

II, a brief sketch of the life and work of Otto Selz has been newly written and added to the text (see Section 17).

Finally, the most important changes are in Chapter IX. Although similar problems were treated in the 1946 edition, the text of the original chapter was practically discarded: the whole chapter is a freshly written epilogue, anno 1963. In this ninth chapter too, the old findings of *Het Denken van den Schaker* have been related to new ideas and findings, in particular in the field of machine simulation of cognitive processes.

Apart from the rendering of the translation proper it is hoped that the various revisions and additions have increased the value of the book in the eyes of its prospective audience: psychologists, chess players, computer workers, and librarians.

ADRIAAN D. DE GROOT, *author*
GEORGE W. BAYLOR, *editor*

PREFACE TO THE SECOND EDITION

To the author, the task of writing a few lines by way of introduction to the second edition of his old book – published in Dutch 32 years ago and in English translation more than 12 years ago – is of course most gratifying. To him, the publisher's decision to have the text reissued spells confirmation of his hope that the book has some qualities which enable it to stand up to the test of time. In fact, there appears to be some continuing demand for it in the various quarters for which it was intended: among cognitive psychologists and computer specialists as well as among 'plain' chessplayers of all ranks. Lately, *Thought and Choice in Chess* has been quoted particularly often in publications of artificial intelligence specialists.

The text of this second edition is identical to that of the first apart from a good many relatively minor textual emendations. First, the number of printing errors has been greatly reduced, it is hoped, by correcting whatever mistakes were brought to our attention by the editor of the first edition, Dr. George W. Baylor (Université de Montréal), and by various readers. Second, and more important, in cooperation with the author the whole text has been newly edited, or rather given a linguistic facelift, by Mr. Jop Spiekerman. Awkward translations – a few on almost every page – have been carefully rephrased and closely scrutinized so as to leave meanings intact. Technical terms, as listed in the Index of Subjects, have not been changed, with only one exception: 'considerable moves' has been replaced throughout by 'pertinent moves'.

Thanks are due to all who have cooperated in preparing this second edition. Next to the printer – who did an excellent job – I feel particularly indebted to Jop Spiekerman, who again proved his great specialist competence and fine linguistic acumen by the way he led the refurbishing operation. To the extent that improvements in readability

and style were possible – given the original text and the constraints of time and format – the second edition has become substantially superior to the first.

Schiermonnikoog-Amsterdam, April 1978 ADRIAAN D. DE GROOT

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CHAPTER I

STATEMENT OF THE PROBLEM

A. INTRODUCTION

Section 1: The literature of chess

Although a good many more books have been written about chess than the uninitiated generally realize,¹ the psychological side of the game has remained a largely virgin territory.

The chess literature is for the most part of a purely technical nature. It deals with the play and not with the player and his way of thinking; it treats the problem and not the problem solver. Other than their games, the 'biographies' of famous players contain little more than a record of tournament results, prizes won, and a sketch of their chess careers. Concerning their inner development one finds only scanty accounts; their personalities are scarcely described, if at all; and with questions such as how their powers evolve and what are the bases of their skill, as yet no one has really concerned himself seriously.

Nevertheless, here and there, scattered among books and journals one can find descriptions and discussions which have some value for the psychologist. In some books and articles one finds comparative discussions of the styles of different masters,² in others analyses of a partly psychological kind about the game itself;³ while elsewhere one finds more or less extensive descriptions of individuals.⁴

Further, the psychology of blind simultaneous play has sometimes attracted the attention of chessmasters themselves.⁵ Finally, newspaper and magazine accounts of important tournaments and matches sometimes contain psychologically interesting particulars or anecdotes

¹ The old chess library of Dr. M. Niemeyer, now part of the *Koninklijke Bibliotheek* in The Hague, one of the most complete in the world, contains about 10,000 items, written in 41 different languages; the oldest books date from the 15th century.

² E.g. LASKER 1895; TARTAKOWER 1921; RETI 1923 and 1933; BRINCKMANN 1932; EUWE 1938.

³ WALKER 1850; JUNK 1918; VON SCHEVE 1919; TARTAKOWER 1921; KLEIN and PALITZSCH 1924; SPIELMANN 1929; LASKER 1932;

⁴ ALLEN 1858; MACDONNELL 1894; BUCK 1902; BACHMANN 1910-21; VON GOTTSCHALL 1912; TARRASCH 1912 and 1925; EUWE 1932; VAN HOORN and LIKET 1937.

⁵ MIESES 1918; ALEKHINE 1932.

on events and reactions during play; sometimes keen character sketches as well.

These references sound more impressive than they are, however. All too often the harvest of psychologically interesting material from the winnowing of thick books is extremely lean; moreover, *thinking* almost never receives attention. Nevertheless, there will be occasional references to the contents of some of these writings in the following pages.

Section 2: Binet's study

Just as the chess literature, written specifically for and by chess players, supplies few connections with psychology; likewise the psychological literature has little bearing on the psychology of chess.

The eminent French pioneer in intelligence testing, Alfred Binet, wrote the first work that treated problems in this field: *Psychologie des Grands Calculateurs et des Joueurs d'Echecs* (BINET 1894). It contains the findings from a study on the memory and imaginative faculty of blindfolded players.

At first Binet considered the fact that many chessmasters are able to play simultaneously a considerable number of games blindfolded, that is, without looking at the chessboard, as an achievement in the realm of *visual memory*.⁶ In the older psychological literature this had never been doubted. Hippolyte Taine, misled by the completely atypical case of a blindfold player who was surely no chessmaster, expressly defended this point of view (TAINÉ 1870, p. 80). It was only natural that Binet, who himself knew the game only superficially, should use this as his starting point. The study of the blindfold player formed for him just another part of a general investigation into *les grandes mémoires*.

In order to collect the opinions of as many experts as possible, Binet first of all conducted a survey. He drew up a list of 14 rather

6 Any player of master strength can play at least four blindfold games simultaneously, if he wants to. Some have specialized in it and have achieved larger numbers. Well known in this area are Morphy (8 games), Zukertort (16), Pillsbury (22), Reti (28), Alekhine (32), Najdorf (40), and Koltanowski (50). A blind simultaneous performance can only be considered a success if the opponents are not too weak (at least Class A or Class B players) and the percentage of wins for the simultaneous player is respectable (say, 75 per cent). Naturally the larger the number of opponents, the longer the duration of the game. Record lengths of time for Alekhine and Koltanowski were more than 12 hours. Needless to say, such performances make enormous demands on one's endurance.

detailed questions concerning the ways and means by which the blindfolded player calls the positions during a game back to mind, concerning the character of the representation, the role of auditory and kinesthetic experiences, etc. Through the assistance of chess organizations in France, England, Germany, and Spain answers were received from many prominent chess players. Furthermore, several Parisian masters (among whom were Rosenthal, Goetz, Arnous de Rivière, Janowski, and Taubenhaus) made themselves available for direct study in the psychological laboratory at the Sorbonne.

In the course of the investigations it became increasingly clear to Binet that his original plan had been too narrow. In the written and verbal communications of the chessmasters a number of points emerged that he had not foreseen at the outset and which consequently led him to revise his concepts about the factors that govern blind play. He came to the conclusion that the ability to play blindfolded rests on three fundamental conditions (BINET 1894, p. 262):

- (1) Knowledge and experience in the field of chess (*l'érudition*);
- (2) Imagination (*l'imagination*);
- (3) Memory (*la mémoire*).

Condition 1 (*l'érudition*):

It is only because the position is meaningful to the master, is a unity, a well-structured scene of battle, that he is able to keep it in mind. Each position has a character of its own. One of the correspondents wrote: 'Each position that I create or that I see develop speaks to my senses, beyond my reason, it makes on me an impression *sui generis*.'⁷ (*Ibid.*, p. 265) This unity of character naturally exists only for the initiated; as a result of his knowledge and experience, as a result of his mastery, the configuration of the position 'integrates itself.'

The same holds for the course of an entire game: to the master a game is no mere sequence of independent moves, but a development of a struggle that can be typified by a few characteristic maneuvers and ideas. The feat of memory, which Binet regarded as the most essential, is thus only possible on the basis of the chess mastery of the blindfolded player. Because of the import of the logical, dynamic, and genetic relationships between the elements in this feat of memory, Binet spoke of a *mémoire des idées* that he contrasted with the more mechanically operating memory, *la mémoire des sensations*.

7 'Chaque position que je crée ou que je vois se former devant moi parle au delà de mon raisonnement, à ma sensibilité, elle me fait une impression *sui generis*.'

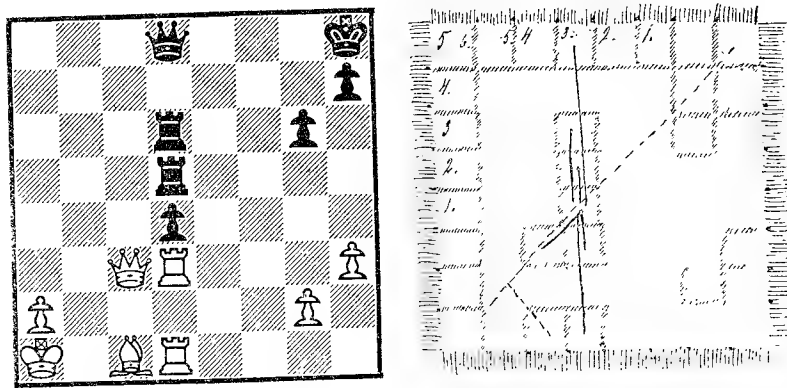
Condition 2 (l'imagination):

The reports differ about the way in which the blindfolded player pictures the board and the actual situation; in particular, about the localization of the image in space. The descriptions were not all equally clear on this point, probably as a result of a certain vagueness and ambiguity of the localization itself. But this is easily understandable in the light of the highly abstract character of the mental images (see under condition 3). Moreover, Binet concluded from his material that the blind player does not, in general, have a fully *complete* picture of the position before him, but rather only a rough Gestalt which he searches, step by step, or rather: He continually reconstructs (the details of) the position.

Condition 3 (la mémoire):

On the basis of the fact that virtually all of the correspondents laid emphasis on the lack of visual details of color and form, both of the pieces and of the board, Binet came to the conclusion that one must speak here of an *abstract* visual memory that he contrasted with the better known *concrete* visual memory. One of the correspondents wrote very characteristically: 'We know only that it is a Knight or a Pawn without bothering about anything else.' (*Ibid.*, p. 295) Another tried to make a drawing (Figure 1) which represented in a schematic way the nature of the representation of a specific position. Although one cannot show specifically that this drawing is a 'likeness,' it is still very instructive insofar as it shows that indeed everything concrete is lacking. The squares of the board have no color, have only vaguely defined boundaries, and are only incompletely present. Only the

FIGURE 1



numbers resulting from the King's trying to catch the onrushing Pawn are given. The pieces themselves do not appear in the drawing: rather the lines of force that go out from them and that schematically represent their dynamic possibilities.

Apparently one cannot properly speak of a visualization⁸ of the position. At the request of the investigator, one of the correspondents attempted to visualize the position occurring in a blind game at the laboratory in a more concrete way. He succeeded quite well but added that he would certainly abandon this method when playing six or eight games simultaneously, since in that case such a procedure 'is not of any help to him but would have no other effect than to tire him.' (*Ibid.*, p. 302)

Finally, in agreement with the remarks of the experts, Binet distinguished in his summary two forms of memory activity in blindfold play: the retention of positions (*mémoire visuelle abstraite*) and the recollection of the course of a game (*mémoire de récapitulation*), which he did not discuss further.

In studying the book by Binet one cannot escape the impression that the author did not completely overcome the mistake of his starting point. To be sure, he did retract his original opinion concerning the concrete visual character of the chess player's memory, and it is true that he added other factors which are of importance; but with respect to the dynamics of the processes of remembering and recollecting there are but a few negative conclusions and vague words. No psychological analysis of the functioning of the 'abstract visual' memory, *la mémoire des idées*, was offered by his study. Moreover, the difference between the experience (*l'érudition*) of the player and his skill in blind play was not sufficiently recognized, let alone analyzed.

In this connection the few pages that Henri Bergson devotes to Binet's results, in a paper that appeared in 1902, are much more instructive (BERGSON 1902). The author cites the professional memory of the chess player as an example of the activity of a *schéma dynamique*.⁹

⁸ Binet himself spoke of 'visualization' but he added qualifications like: 'It is most often abstract, that is to say that it abstracts, detaches, and pulls out of the visualized object exclusively those qualities that are necessary for the combinations of the game.' (*Ibid.*, p. 338) ('Elle est le plus souvent abstraite, c'est-à-dire qu'elle abstrait, qu'elle détache, qu'elle arrache de l'objet visualisé les seules qualités nécessaires aux combinaisons du jeu.' We prefer to entirely omit the term 'visualize' (see p. 7).

⁹ This is the forerunner of the 'schematic anticipation' (anticipatory schema) in the *Denkpsychologie* of Otto Selz (cf. Sections 19 and 44).

By this he means a schematic representation of a complex structure of images and/or ideas that cannot be visualized itself, but can be developed into detailed visual or other sensory representations: 'The schematic representation does not so much contain the images themselves but rather indicates the directions to follow and the operations to perform in order to reconstruct them.'¹⁰ (*Ibid.*, pp. 6-7)

According to Bergson, every mental operation that demands efforts of concentration begins with this kind of dynamic schema of the whole and/or the goal. The experimental work in the later school of *Denkpsychologie* almost completely confirms this notion. The reproduction of the image of a position in chess is, indeed, a telling example. The schema here is approximately that which the chessmasters in Binet's survey called the 'character' of the position. In particular, Dr. Tarrasch's description of the reproductive process is illustrative in this respect. First, the position is identified by its 'character,' that is, the subject must recognize 'which position it is.' In this process some characteristic move or maneuver from that game is often instrumental. Second, from here a more detailed representation of the actual situation on the board is developed; the schema is worked out into actual ('abstract visual') images. Thus according to the reports of the players, 'imagining a position' does not ordinarily occur all at once, but progressively, in successive steps. It is not a matter of simply 'seeing' the position mentally but rather of continual reconstruction which requires some effort: 'the image of the board with its pieces is not present in the memory of the player as it is in a mirror, but at every moment it demands of the player an effort of reconstruction.'¹¹ (*Ibid.*, pp. 7-8)

Binet never actually arrived at the point where he recognized the process of reproduction as a reconstruction 'by parts' of the position starting out from a dynamic total schema. Nor did he recognize the importance of the distinction between *knowing* 'which position it is' (feeling able to reconstruct it) and *imagining* a position (having a workably complete picture in mind) even though it was expressed rather clearly by several of the correspondents (among whom was Dr. Tarrasch). This would also explain why the distinction

¹⁰ 'Nous entendons par là que cette représentation contient moins les images elles-mêmes que l'indication des directions à suivre et des opérations à faire pour les réconstituer.'

¹¹ 'l'image de l'échiquier avec ses pièces n'est pas présentée à la mémoire du joueur telle quelle, 'comme dans un miroir,' mais... elle exige à tout instant, de la part du joueur, un effort de reconstitution.'

Binet makes between the function of memory and that of imagination is not completely satisfactory. He does not take into account that the development from a schematic to a complete representation of the position is not a purely reproductive process but is in part an inferential construction process and therefore thinking. Where he speaks of 'abstract visual' memory, we are now able to discern two different levels of functioning: that of memory in its narrower sense and that of imaginative inferential reconstruction.

In the above exposition the use of the term 'visual' when speaking of either memory or imagination has purposely been avoided. As to memory, much information concerning positions and games is present in the blindfolded player's mind in the form of a 'knowledge that...' which has hardly any spatial and certainly no visual character. The abstract nature of the imaginative reproduction process, too, makes the term 'visual' somewhat suspect. In the mental representations there are probably many visual as well as tactile and kinesthetic elements that correspond to the observation, picking up, and moving of the pieces, respectively. However, the abstract images with which the player really works can hardly be considered either tactile or visual; rather they are *spatial*, which is more than either tactile or visual.

If the imagery of the chess player were visual, one would have to invent an independent theory for the game without a board for a player such as the late P. A. Koetsheid (class A to B strength) who became (really) blind at the age of seven but did not learn to play chess until he was twelve (compare, too, Géza Révész's exposition on blind mathematicians; Révész 1938, p. 155 ff.).

A precise study of this matter is, of course, outside the scope of the present work. The immediate purpose of the above discussion was merely to draw attention to the problems connected with the use of such terms as 'visual' and 'visualization' in the case of the chess player.

Finally one may note that Binet does not do full justice to blindfolded playing when he places such great emphasis on memory. Alexander Alekhine has rightly remarked that one forgets too easily that the blindfold player has more to do than just remember the positions; he has a second and much greater difficulty to cope with 'namely, to fight blindly, to find in every position, blindly, the best or almost the best move.'¹² (ALEKHINE 1932, p. 19) Because the actual position is missing perceptually, playing is made that much more

¹² '...nämlich, blindlings zu kämpfen, blindlings in jeder Position ungefähr den besten Zug zu finden.'

difficult than in front of a board; the position must be continually reconstructed in the imagination.

There are other points in Binet's study that could be criticized. One could on logical grounds make the objection that his three factors are not conceptually on a par. 'Imagination' and 'memory' can best be considered capacities, faculties of the mind, but the player's 'experience' (*l'érudition*) is a concept of a different order. Binet hardly elaborates this point at all. Furthermore, his lack of knowledge of the game sometimes manifests itself in faulty interpretations of passages in the reports made by the chess players as well as in some erroneous inferences of lesser importance. We shall not, however, go into these minor matters now.

All in all, we see that there is quite a bit of criticism that could be made of this book by the French psychologist. However, if one takes into consideration that Binet began his investigation with an erroneous hypothesis (concrete visual memory as a basic factor in chess mastership), carried it out with an insufficient knowledge of the game of chess, and had to draw conclusions that found no support whatever in contemporary psychology, then one can have nothing but admiration for the results he achieved. Psychology in his day knew nothing of 'wordless thought'; it was still stumbling with the theory of sensations and images and their divisions into auditory, visual, and kinesthetic categories, and with other concepts and distinctions that were in no way helpful in understanding the thought process. For that reason Binet had to label many of the reports he received about blindfolded playing as being *fort obscurs*. In fact, they were obscure only insofar as they did not concur with the prevalent conception of thought as a series of concrete images.

The conclusion of Binet's book shows, for that matter, that he himself painfully felt the inadequacies of the academic psychology of his time. In the last paragraphs which radiate the same wholesome common sense which assured his success as a pioneer of mental measurement, he pokes fun at the lifeless, all too abstract way psychologists had so far tried to investigate a function such as memory. Later, when he talks of the role of experience in blindfolded playing he attests to an important gap in theoretical psychology: 'If I am not mistaken, modern psychology has not paid sufficient attention to the role played by old memories in the acquisition of new memories.' (BINET 1894, p. 339) Here he mentions a subject which was not worked out systematically until many years later. With regard to its im-

plications for thought processes Otto Selz was the first to incorporate it into a theoretical framework, namely, in his conception of acquiring 'experience' in some field. This process is described by Selz as a gradual construction and differentiation of a system of both general and more specific reaction dispositions (*Verhaltensweisen*) or, in the case of intellectual skills, a system of immediately available mental operations (solving methods) such as that on which the chessmaster's skill is based.¹³

The problems that were touched upon in this section will be treated later in another context, in Chapter VIII in particular.

Section 3: The question of chess talent

As well as the problem posed by the conspicuous phenomenon of blindfolded playing, the question of chess talent has also, here and there, inspired psychologists to empirical investigations. We shall briefly discuss a few of these investigations.

1. In the year 1925 three Russian professors, Djakow, Rudik, and Petrovsky invited eight participants from the large grandmaster tournament at Moscow to come to the Laboratory of Experimental Psychology and Psychotechnics of the University of Moscow to participate as subjects in a few experiments. The general purpose was to try to unearth the factors that underlie chess talent. The experimental program included, according to the German translation of the booklet the Russians published, the investigation of 'all the essential aspects of the mental activity of a chess player, as they have been sufficiently described in the literature.'¹⁴ (DJAKOW, RUDIK and PETROVSKY 1927, p. 26) The somewhat over-optimistic statement in modern eyes is then followed by a specification of the 'faculties of the mind' that were examined: memory, attention (concentration), higher intellectual processes (combination power, finding of logical regularities, speed of reaction with regard to simple intellectual

¹³ New in Selz's approach was, as we shall see in some more detail in Chapter II, the emphasis on the operational side of thought and memory. Old memories exert influence on the acquisition of new ones since it is not only content that is stored but also – in computer language – 'programs' for information retrieval and for operational thought which makes use of stored information. Selz's *System von Verhaltensweisen* is essentially a mental program for handling data or solving problems in some particular area.

¹⁴ '...alle wesentlichen Seiten der psychischen Arbeit der Schachspieler die ja in der einschlägigen Literatur zur Genüge beschrieben.'

tasks such as checking calculations, and the like), as well as imagination, will power, and psychological type.

Most of the 'faculties' were investigated – nowadays we would say: operationally defined – by means of simple experiments and tests from the early psychotechnical tradition. Judged from a more modern point of view and taking into account what we currently know about 'factors of the mind,' one could say in advance that it is highly improbable that tests of this type – tests for visual memory and combination power, for example – have anything in common with what these terms mean in chess, other than the same name. It is hardly surprising, therefore, that most of the tests did not differentiate between chessmasters and control subjects of comparable intellectual stature. The Russian investigators interpreted their findings, however, as a refutation of commonly held beliefs, namely, that high achievements in chess are based on exceptional visual memory, combinative power, speed of calculation, power of concentration, and logical thinking. They were not aware of the 'faculty fallacy' in their reasoning, which is most obvious with regard to *combination* power and speed of *calculation*: two terms that have a very specific meaning in chess.

Most worthwhile, however, were the results of their experiments in visual memory. The experimenters used different types of stimulus materials in such a way that the resemblance with the customary materials of the chessmaster – board and moving pieces – was experimentally varied. There appeared to be no superiority in the master group where the resemblance to chess was lacking, some superiority where an 8×8 board with moving spots was used, and a definite superiority where actual positions were used. Again, this is hardly surprising. Here we can probably agree with the Russians that a commonly held belief – superiority in visual memory, regardless of the stimulus materials – was refuted. The results appear to be in agreement with our reluctance to speak of 'visual' (memory) in our discussion of Binet's findings.

The faculty of imagination and the two personality traits, will power and psychological type, were operationalized in a rather primitive and highly disputable fashion: by means of Rorschach indices. The number of responses, R, was used as an index for 'imagination' (power), the number of kinesthetic answers, M, as a negative index for 'will power' (*sic*). The 'psychological type' of a subject was determined by a somewhat *ad hoc* derived combination of scores. Results indicated that chessmasters had rather high scores on 'will power' and differed very much from one another in 'imagination.'

The latter result was one of the main factors on the basis of which the investigators concluded that two types of chessmasters could be distinguished: the *pedantic* and the *imaginative* types (*Pedanten* and *Phantasten* in the German translation). The latter finding appears to be roughly in agreement with commonly held conceptions on typological differences. From the exposition in the book, however, it is not quite clear to what extent the typological findings were contaminated by *a priori* ideas of the investigators.

In summary, the methods used were rather primitive and, as a result, the outcomes far from unambiguous. Nonetheless, some of the findings and observations in the book, concerning both the character of the game of chess and chess talent, are of enough interest to be taken up again in Chapter VIII of this study, where the factors underlying chess mastership are discussed.

2. In the year 1920 Franziska Baumgarten had the opportunity to examine psychologically the infant prodigy in chess, the eight year old Samuel Rzeschewski – now U.S. Grandmaster Reshevsky. At that early age he was already quite famous for his simultaneous play against some 20 opponents. Miss Baumgarten published her results, together with findings on other prodigies, in an entertaining but superficial little book, *Wunderkinder* (BAUMGARTEN 1930).

In itself the case of Reshevsky is extremely interesting since in the history of chess there is no other example of a talent that matured so early. Révész is right, it seems, in his statement that a chess talent in general is apt to attain a very high, if not already maximal, level of ability before the 21st year of life (Révész 1921), but to arrive at the master level before the age of 10 is highly exceptional, to say the least.

It was unfortunate that Baumgarten could not examine the child wonder in a more thorough and adequate way than she did. The investigator was careful not to draw any general conclusions on chess talent. She confined herself to relating a number of outcomes from tests and observations and interpreting the case of this particular prodigy – which interpretation is, again, open to much criticism. She found in the little boy Samuel an extremely one-sided mental development. His verbal intelligence was below the average level of five year old Berlin boys. There was only one test where his achievement was exceptional, namely in a memory experiment, where the stimulus material consisted of a matrix of one digit figures: he succeeded in retaining a 4 by 7 schema after a learning period of three

minutes and a 5 by 8 schema in four minutes. On the other hand, his visual memory for less abstract data was rather poor.

Baumgarten concluded that there existed a cleavage between his inborn talent and his actual knowledge due to the rather exceptional environment in which the child grew up. According to her information, Samuel – at his eighth year – had never seen a picture book, had never made a drawing or seen anybody else draw, had not yet been to school, and had so far learned only Talmud and some Hebrew. By contrast, it is quite probable that at that tender age he had already played more games of chess than, for instance, Euwe or many other masters, at the age of fifteen.

We can agree with Baumgarten's statement that the mental development of the young Reshevsky was 'one-sided' in comparison with other children of the same age. We can also agree with her opinion that this one-sidedness can in part be ascribed to the consequences of his position as a child prodigy. As such he had to be 'spoiled' on the one hand and 'exploited' on the other to the detriment of what for normal children would be called a 'more balanced development.' But Baumgarten's interpretation and evaluation of the facts are biased. First, she misinterpreted some of the facts by not sufficiently taking into account the orthodox Jewish educational traditions. Second, the young Reshevsky happened to be extremely – albeit 'one-sidedly' – gifted. Both facts make her comparison with 'normal,' i.e., with non-Jewish, not extremely gifted children of his age, rather futile. She was certainly wrong in blaming the parents for his one-sided development as strongly as she did.

The question of what might have become of the little Samuel if he had been given a more general intellectual and cultural education in his early youth is, of course, unanswerable, but it would seem that Baumgarten's supposition that he could have become an important scientist (*Ibid.*, p. 58) is, again, ill-founded.

3. Besides Reshevsky some other 'cases' have here and there attracted the attention of psychologists and psychiatrists, in particular where there were personality deviations or symptoms of mental illness. One result of such a psychopathologically oriented interest is Ernest Jones's psychoanalytic study of Paul Morphy (JONES 1931). In this work, apart from the case study materials, the author makes a number of general remarks on the nature of the game of chess that are of importance for the analysis of chess talent and chess affinity. Jones's article will be discussed in some detail in Chapter VIII.

4. Finally, there exist a number of psychological theories of a more general scope that in some respects touch upon the problem of chess talent. Révész tackled the problems of chess talent in one article on general and specific forms of giftedness (RÉVÉSZ 1938) and in another on the phenomenon of early maturation of talents in different fields (RÉVÉSZ 1921). Also, in 1938, the present writer published a brief essay on the factors of talent in chess that was, however, not yet based on an experimental investigation (DE GROOT 1938). In the present work a further analysis of the problems of chess talent will be delayed until after the discussion of the experimental analysis of the thought process. In Chapter VIII a more critical review of some of the above mentioned publications will be found.

B. THE OBJECT OF THIS STUDY

Section 4: The analysis of chess thinking

The purpose of the investigations described in this study is first of all to carry out an experimentally based psychological analysis of chess thinking.

Until now there have been no studies that have concerned themselves with a systematic description of the chess player's characteristic attitudes and methods of thinking. Herein, however, lies the heart of the psychological problems involved in chess. Only from a knowledge of the normal thinking of the chessmaster can one understand special arts, such as 'blind' and/or simultaneous play. Only by analyzing the thought process can one arrive at a thorough insight into the demands the game makes on its practitioners. Only along this path can the question of chess aptitude be fully handled. A systematic empirical analysis of the chess player's thinking therefore forms a sound basis for practically every psychological study in the field of chess.

To arrive at such a systematic analysis, a theoretical frame of reference had to be used. The only available conceptual system, aside from obsolete association theory, was the *Denkpsychologie* of Otto Selz. In the second part of his voluminous work, *Zur Psychologie des produktiven Denkens und des Irrtums*, he wrote at the conclusion of a discussion on general methods of problem solving in productive thinking: 'Only through an analysis, as undertaken in the present investigation, of those most general operations that correspond to (mentally set) aims as such ... does a complete (literally: 'gapless') description of the causal connections that govern the total course of

intellectual and/or motor processes come within reach.¹⁵ (SELZ 1922)

Thus, the task we set ourselves was to attempt such a complete description while using the terminology and applying the laws of Selz's *Denkpsychologie*. In other words, we tried to analyze the structure of the thought process in chess and to search for and pin down the characteristic moments in the chess player's thought dynamics: organization, methods, operations, etc.

The analysis was restricted to thought processes as they occur in playing the game. The field of problem composition or endgame studies was not taken into consideration. As is well known, the field of chess problems is in many respects a domain of its own, quite apart from the playing arena. Interest in playing the game and interest in composing and solving problems do not necessarily go together, nor does skill in one field guarantee skill in the other. Generally, the player's personality differs from that of the problemist, in that the player is primarily prepared to fight and to risk and to play. For that reason we do not come across many names of grandmasters in the problemist's world even though the purely intellectual basis is largely the same in both fields.

The investigation into the player's thinking during the game had to be restricted too. A chess game is a unit; playing it is a single feat, one activity; the concentration span runs from the first to the last move. But this unit is too large for an experimental psychological investigation, as the average duration of a serious competitive game is about four hours. The abundance of material from a protocol of the player's thinking during that entire time would be practically unmanageable. Moreover, it is impossible to have different subjects play the same game.

However, the alternation of moves required by the rules of chess ensures a natural segmentation of the game. During those four hours

(two hours per person) approximately forty moves are made¹⁶ so that each player thinks an average of three minutes about his next move. Wide fluctuations occur around the three-minute-per-move mean: from part of a second to three quarters of an hour, or more. But in any case, thought processes leading to one move provide more workable units than entire games.

To create the conditions necessary for studying these thought processes a number of positions from actual games were selected and presented to a group of subjects, consisting of grandmasters, masters, experts and less skilled class players. They were not familiar with the positions presented. Each subject was requested to make a move only after proper deliberation, as if he were engaged in his own game, and, in addition, to think out loud as fully as possible so that the protocol would contain a picture of the way by which he had reached his choice of move. This means that the object of the present investigation is not the analysis of the way in which a chess game is built up in its entirety, but rather the analysis of how the chess player (subject) solves the *choice-of-move-problem*.

Section 5: Organization of the book

This study principally consists thus of an *experimentally based, psychological analysis of the chess player's thought processes preceding a move in a serious game*.

Parts C and D of Chapter I contain a closer analysis of the objective nature of the choice-of-move-problem. Chapter II is devoted to the theory of thinking, especially the *Denkpsychologie* of Otto Selz, while Chapter III describes and attempts to justify the experimental method used.

These preparatory chapters lead to the actual analysis contained in Chapters IV, V, and VI. The striking *external phase structure* which appears in chess thinking was made the starting point in the analysis of the protocol material.

Chapter IV, then, contains a detailed description based on one of the protocols as well as a statistical elaboration of a number of characteristics of the external structure. Chapter V deals with the *general problem structure* of chess thought which should be carefully distinguished from the external phase structure. The treatment here is in terms of the hierarchical structure of 'main problem' and 'sub-problems' and of 'main goal' and 'subgoals,' respectively. In Chapter VI we leave the description of the external structure still further

15 'Erst durch die in diesen Untersuchungen in Angriff genommene Analyse jener allgemeinsten Operationen, die Zielsetzungen als solchen zugeordnet sind ... wird eine lückenlose Beschreibung des Kausalzusammenhangs determinierter intellektueller und intellektuell-motorischer Gesamtverläufe möglich.' (P. 609).

16 From a sample of one thousand games that were played in master tournaments and matches in the latter half of the last century the mean number was 42 moves (actually 41.9). Of the 422 wins by White the mean was 41.0 moves; of the 322 wins by Black the mean was 42.0 moves; and of the 256 draws the mean was 43.3. The distribution over the 11-15 moves, 16-20 moves, 21-25 moves, etc., appeared to be clearly skewed as evidenced by the values of the three measures of central tendency:

mean	median	mode
41.9	39.4	34.7

behind and proceed to an explanation of the *dynamics of the thought process*. Here the main theme is the psychological development of the thinker's problem. In Chapter VII the most important results from the previous analyses are assembled and worked out theoretically with respect to the general *organization and methodology of the thought process*. With that the systematic analysis may be considered complete.

Having come so far, however, it seemed quite natural to keep going. The results obtained made it possible to reconsider, from a new point of view, other psychological questions with regard to chess and to compare thought in chess to that in other fields. Questions concerning the characteristics of the game and the player, his attitudes in contrast to his aptitudes, could be examined; the factors of talent and its development into mastery of the game could be looked into. It was, of course, impossible to consider these problems as thoroughly as the analysis of chess thought itself. The author had to limit himself to collecting some supplementary data from the literature and to extrapolating from his own experience and some additional experimentation. For that reason Chapter VIII has been written with perhaps less scientific rigor than the preceding chapters. By virtue of this, however, it will probably make the chapter that much more readable, stimulating, and accessible, in particular to a reader not directly interested in the complexities of thought psychology.

Finally, Chapter IX contains a brief, critical discussion of the *Denkpsychologie* of Selz and its further developmental possibilities. Since the attempt to base the analysis of the thought process of the chess player on Selz's ideas can be considered a test of the applicability of his theories and terminology, this subject, too, is a natural outgrowth of the main object of the present study.

In the revised English edition Chapter IX has been enlarged by some methodological remarks and by a discussion of some of the consequences of the findings for the machine simulation of chess thinking and for theories on human thought.

C. THE CHOICE-OF-MOVE-PROBLEM

Section 6: The variability of the problem

It is somewhat misleading to speak of *the* choice-of-move-problem. The character of the problem which confronts the player on move is

extremely variable, both from a psychological and a chess-technical standpoint. It is, of course, highly dependent on the position on the chessboard. It makes a great difference whether the player on move must construct a plan or execute one; whether he must conceive a combination or parry one; whether he must simply exchange pieces, get out of check, look for a safe continuation in a threatening position, or 'lie in wait' for the right opportunity. It makes a difference whether he can follow well-known lines of play – in the opening, the endgame, or a stereotyped middle game position – or must discover and work out new possibilities for himself.

Most 'interesting,' of course, are the turning points in the development of the battle: the moments in which the player discovers and starts to carry out a deep combination or works out a long range plan: those moments that would ordinarily appear in a diagram from the game. It would be most fascinating, for example, to gain some insight into the thought processes of Anderssen during his 'immortal game' against Kiezeritsky, when he decided to sacrifice both of his Rooks for the furthering of his attack.

Mainly, the layman wonders about two things: how the player 'hits upon the idea' and how far he foresees the coming development of the game. How did Euwe hit upon the idea of sacrificing a piece for three Pawns against Alekhine in the famous 26th (Zandvoort) game of his first match for the world championship? And how many of the combinatorial possibilities did Alekhine weigh in his brilliant game against Réti at Baden-Baden in 1925, when he put his Rook *en prise* at K6?

Such questions concerning the tense high points of competitive chess are of most interest to the chess devotee and amateur – next to the psychological explanation of special skills such as simultaneous blind-folded play. It is self-evident, however, that such sensational moments rarely occur in chess and surely are not generally representative of the choice-of-move-problem. Ordinarily positions are not so colorful, and it would certainly have been a serious mistake if brilliant combinations had been at hand in all of the experimental positions. The attempt was to avoid such one-sidedness by selecting a series of experimental positions of a wide and varied character. Only trivial choice-of-move-problems were purposely omitted.

For our purposes one of the important variable characteristics of a chess position is the number of possibilities available, i.e., the player's

freedom of choice. The choice of considerable moves can be large or small, up to the extreme case where there is no choice at all.

The question of the number of possibilities can be examined from three angles: the formal, the chess objective, and the psychological. That is, one can ask:

- for the number of *possible* moves according to the rules of the game (the *legal* freedom of choice);
- for the number of *good* moves; and
- for the number of moves from which the player or subject *really* makes his choice – if there is a question of choice.

To do full justice to point *c*, the most important one, it is first necessary to discuss briefly *a* and *b*.

Section 7: The legal freedom of choice (*K*)

According to the rules of the game the number of legally possible moves in a given chess position ($= K$) ranges from zero (in a stalemate or checkmate position) to over a hundred. Practically speaking, however, the boundaries lie closer to each other: positions with more than seventy legal possibilities can be constructed but rarely occur in actual play, whereas the lower numbers – under eight – almost only occur when the King is in check. In the latter case the King must be removed from check which can only be done in a limited number of ways.

How great is the legal freedom of choice on the average? If we examine the universe of all positions which have occurred in past tournament and match games, what distribution for *K* would result? In order to get at least an approximate answer to these questions, the following variables have been tabulated from the 'diagramed positions' of a richly illustrated game collection, TARRASCH's *Dreihundert Schachpartien*:¹⁷

- the number of the move to be played at that certain moment;
- the legal number of possible moves in the given position;
- the legal number of possible moves for the opponent, after the execution of the actually chosen move.

Variables 2 and 3 were classified according to intervals for variable 1: the 11th to the 15th move, 16th to 20th, 21st to 25th, 26th to

¹⁷ The sample is, of course, not random. Both games to be published and positions to be diagramed are chosen for the interest they may arouse in the reader. It does not seem likely, however, that this selective factor and the purely formal variable *K* are correlated, especially not if the number of legal possibilities for both White and Black are considered in every position.

30th, 31st to 40th, 41st to 50th, 51st and higher. Positions in which check must be answered (check-positions) were excluded from these tabulations. The collection of data was continued until each of the first five classes (starting with the 11th to the 15th) contained 20 positions with White on move and 20 positions with Black on move. Thus, for example, the 21st to the 25th move class yielded for 40 positions:

mean move number: 23.4		
mean of <i>K</i> (number of legal possibilities)		
for White:	38.1	} total mean K = 38.0
mean of <i>K</i> (number of legal possibilities)		
for Black:	37.9	

For the same move interval the following results were obtained for the extreme *K*-values (lo and hi), the median (Mdn), and the quartiles (*Q*₁ and *Q*₃):

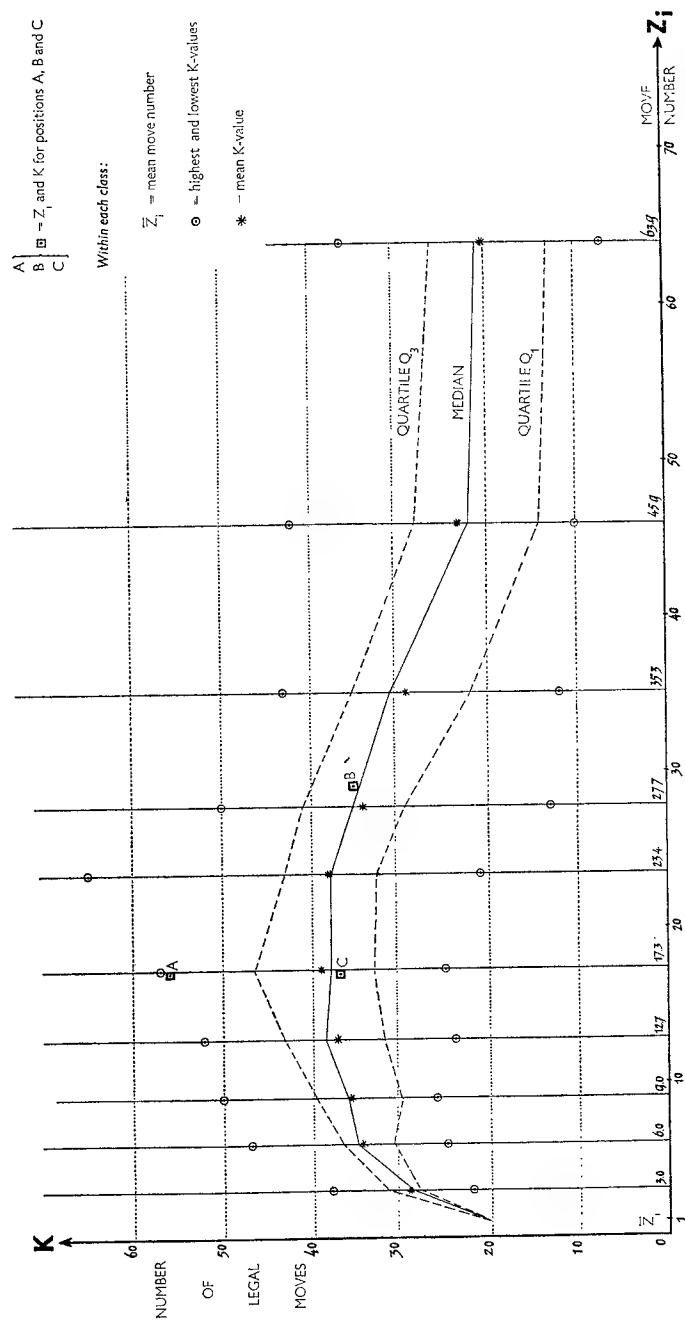
lo	<i>Q</i> ₁	Mdn	<i>Q</i> ₃	hi
21	32.5	38.0	42.5	65

Apparently the median and the total mean coincide in this class. Assuming that the sample is representative, we can now offer the following generalization of the results:

Around the 23rd and 24th move (mean 23.4) the legal freedom of choice (*K*) lies between 32 and 43 move possibilities in about 50% of all the non-check-positions from master (tournament and match) games; *K*-values under 21 and above 65 are rare.

The results of all the move intervals are summarized in the graph, Figure 2. The three curves were formed by connecting the values of *K* of each interval. Approximate data with regard to the value of *K* for move number *i* can be found by interpolation. Thus, for instance, the quartiles for *K* at the *i*th move, for Black and White averaged together, can be estimated by intersecting the vertical at *i* with the dotted quartile lines. The extreme values as found in each sample have been indicated by circles on the corresponding vertical.

Not enough of the diagramed positions from Tarrasch's collection were of the opening phase, before the tenth move; consequently, it was necessary to use other material to construct the left side of the graph. This material consisted in all the positions from six frequently played openings, somewhat arbitrarily chosen. On these are based the sample data at *i* = 3.0; *i* = 6.0; and *i* = 9.0. They fit well enough with the rest of the graph. In total the graph is based on a move count of 350 positions.

FIGURE 2. The relation between legal freedom of choice, K , and move number, Z_i .TABLE 1. The relation between legal freedom of choice, K , and move number, Z_i .

Move number class (Z_i -class)	$Z_i =$ 2, 3, 4	$Z_i =$ 5, 6, 7	$Z_i =$ 8, 9, 10	$Z_i =$ 11-15	$Z_i =$ 16-20	$Z_i =$ 21-25	$Z_i =$ 26-30	$Z_i =$ 31-40	$Z_i =$ 41-50	$Z_i =$ 51-...
Number of positions investigated (p)	36	34	36	40	40	40	40	40	23	21
Mean move number (Z_i)	3.0	6.0	9.0	12.7	17.3	23.4	27.7	35.5	45.9	63.9
Highest number of legal moves (K_{\max})	38	47	50	52	57	65	50	43	42	36
Third quartile of K-values (Q_3)	31.5	37	39.5	43	46.5	42.5	41	35.5	28	26
Median of K-values	29	35	36	38.5	38	38	35.5	31	22	21
Mean number of legal moves (K)	29.1	34.3	35.8	37.1	39.4	38.0	34.0	29.1	23.1	20.3
First quartile of K-values (Q_1)	28	31	30	32	33	32.5	29.5	22	14	13
Lowest number of legal moves (K_{\min})	22	25	26	24	25	21	13	12	10	7

The following results can be read from the graph:

1. The mean number of move possibilities on the i^{th} move, $K(i)$, varies from 20 – at the 1st move and around the 65th move in long games – to about 40 around the 17th move. Remarkably enough, less than 20 possibilities rarely occurs before a very advanced stage of the game.
2. As the game ‘ages’ the variation in the mean number of legal moves remains within modest limits but does show a characteristic curve which clearly reflects, among other things, the development of the pieces during the first 10 to 15 moves.
3. To estimate the mean number of move possibilities from the complete universe of master game positions, it would be necessary to resort to some method of weighting since there exist far more positions at the 12th move, for instance, than at the 74th move: most games contain more than 12 moves but less than 74 moves. Thus, the frequencies in the first 20 moves should count more heavily. Taking that into account, it can be seen from the graph that the total mean lies somewhere between 30 and 35 move possibilities. Indeed, some rough calculations based on the lengths of games as noted in footnote 16, Section 4, yielded 32.3 as a sample estimate for the ‘average legal freedom of choice.’
4. From the quartile lines which are close together and from the extreme values, it can be seen that the scatter at any given moment is rather narrow. In all of the material studied, excluding check-positions, there is not a single case before the 26th move (out of 226 positions) in which there are fewer than 20 move possibilities (i.e., where $K < 20$) and before the 51st move not a single case of fewer than 10 possibilities. Even further on, the range is still rather small. The occurrence of 65 possibilities (for $i = 21$ to 25) is quite an exception for the next highest is 57 possibilities (for $i = 16$ to 20).
5. On the graph each game could be represented by a broken line with each position as a point. Positions A, B, and C, with which most of the experimentation was done, are represented as such points. Apparently A is an extremely rich position whereas B and C are normal.

A few words should be said about check-positions in which, of course, the legal necessity of getting out of check greatly delimits the number of move possibilities. In the 45 games from the Margate tournament book, 1938, there were 171 check-positions resulting from a total of

3,640 ‘half-moves.’ Therefore there was one check out of 21.3 half-moves. If we generalize we may roughly set the relative frequency of a check-position at 5 %.

The material on which the graph was based contained 16 check-positions which had to be excluded (4.4 %). The mean of the move possibilities was 3.8, the median was 3.5, the range was from 2 to 9. If a larger number of cases had been taken, these values, especially the extremes, would shift somewhat, but in any case it is clear that this group of positions is exceptional, regardless of the stage of the game. During the progress of a game the giving of check suddenly decapitates the gamut of formal possibilities, leaving three or four replies: a checking move is legally coercive, indeed.

Section 8: Objective freedom of choice

What is more important to us, but more difficult to answer, is the question of *the number of good moves* that can be played in a given position (point *b*).

A ‘good move’ is, of course, a relative concept, but it can be more or less rigorously defined. For a workable operational definition, however, we shall have to make use of the judgment of experts¹⁸ who must be able to handle the difficult cases in which comparisons of complicated variations and ‘deep’ evaluations are needed (a so-called *analysis of the position*). The following definition of a ‘good move’ may deviate somewhat from the actual usage of the term, but it possesses the advantage of greater rigor and testability:

A move is good if and only if it is impossible to find another one, after a careful and convincing analysis, that is better.

Or, put another way:

A move is not good if and only if it is possible by an analysis of the position, to demonstrate convincingly that another one is better.

With this, the relativity of the concept and the necessity of comparison with other moves is clearly shown. As to the meaning of ‘analysis,’ practical considerations can be taken into account: a master must be able to analyze the position without months of investigation, and

¹⁸ Since there are as yet no objective or machine definitions available that are both precise and practically usable, the best basis is still intersubjective agreement among experts.

the results should convince any unbiased skilled player. This definition is, of course, not completely rigid since there is some leeway in the depth of the analysis. It can, however, be tightened and strictly operationalized whenever the situation demands it.

But when move Z_1 is actually better than move Z_2 the question still remains: how can its superiority be proved in an 'analysis'? The answer: When, after considering every possible response, one can show that the minimal results are better with move Z_1 than with move Z_2 . This seems like a tautology but in fact gives the means for proving that Z_1 is better than Z_2 . 'To achieve results' means to reach positions whose value can be objectively assessed by a skilled player in terms such as 'won by White,' 'very favorable,' 'drawn,' 'equal play,' 'somewhat less,' etc. One could even rate the position (e.g., won = 10, favorable = 7, even = 5, etc.). Naturally such rating is not entirely free from subjectivity; but the assumption that one can actually prove the superiority of Z_1 over Z_2 implies that in practice it must be possible, in each variation, to reach an end position – possibly a 'dead position'¹⁹ – that can be evaluated with the proper degree of objectivity.

A few examples: Move Z_1 is better than move Z_2 if Z_1 leads to a 'won' position against every opposing move while only 'favorable' positions can be reached via move Z_2 ; if Z_1 yields at least 'even play' against every opposing move and Z_2 no more than a 'less good' position; etc.²⁰ With this the usage is sufficiently standardized.

We are now able to classify chess positions according to the number of good moves that can be made. But it is not so easy to investigate this statistically – every position would have to undergo analysis. It is certain, however, that the frequencies obtained here are of a different order than those obtained under point *a*. At least nine out

19 A 'dead position' is a position in which an *evaluation of the position*, without further calculations, is in order due to the absence of immediate tactical possibilities. 'Absence of immediate tactical possibilities' can, of course, be defined in different ways, e.g., 'no capturing moves can be made.' The term 'dead position' was introduced into the chess machine terminology by TURING (1953, p. 291).

20 If need be, one can expand the usage of ' Z_1 is better than Z_2 ' to include cases in which branch end results show no clear differences. Thus Z_1 is also better than Z_2 if the same favorable result (e.g., a won position or even checkmate) can be reached faster by means of Z_1 than Z_2 . Furthermore, Z_1 is better than Z_2 , even though both moves lead to the same result with the best opposition, if it is much more difficult for the opponent to find the best moves after Z_1 than after Z_2 . In practice these cases are included, but here they shall be excluded so as not to unduly complicate the matter.

of ten legal move possibilities do not come in for serious consideration; in general, there will remain only three or four out of the thirty or forty possibilities.

From the analysis of an old tournament game, Halic v.s. De Groot (Munich, 1936) the average number of good moves per position was found to be no more than two. The maximum was five, and in approximately 40 % of the positions from this rather ordinary, not particularly brilliant or spirited game there was not more than one good move. The importance of this last category is obvious, for which reason it will be attended to first.

One good move. This is at the same time 'the best move': its superiority over the other moves must be demonstrable, by definition. For example, this case occurs:

- a. with a *check* that can be parried in only one way (then there is just one legal possibility, and, a fortiori, but one good move) or which can be parried in only one way without immediate disadvantage;
- b. with an *exchange of pieces* where the piece or Pawn must be recovered;
- c. with the presence of a *threat* that can be properly parried in only one way (e.g., the opponent threatens checkmate in two moves, or attacks a major piece, or threatens to fork two pieces, etc.);
- d. in positions where just *one combination* yields an advantage (e.g., a series of forcing moves which the player calculated beforehand);
- e. in positions where a consideration of *timing* necessitates the choice of one move to execute a specific plan (e.g., if one must be ahead of the opponent and forestall his counter measures); etc.

Thus again we have to do with the same sort of curtailment of the gamut of possibilities as in Section 7 with check-positions. In practice, indeed, it is not only the check-positions that have a forcing character. For instance, an error by the opponent, if recognized as such, morally forces upon the player a definite, immediate reaction: namely, the consequent winning continuation.

The degree of difficulty of the choice-of-move-problems of this group can, of course, differ greatly. In all the cases from *a* to *e* one can give both trivial and extremely complicated examples. They have, however, one thing in common: the existence of a single objective solution. One can prove that a specific move is the best: *the choice of move problem is objectively solvable*. The problem for the player (the subject) is then to 'find' the single solution; it is not to 'choose' a move that may be partially based on personal tastes or motives.

Two or more good moves. It might appear from the discussion above that positions in which more than one good move can be played are necessarily not objectively solvable. This is not the case, however. Or rather, it is better not to connect the concept of objective solvability with the 'number of good moves.' If there are two good moves, Z_1 and Z_2 , by definition it is not possible to prove their inequality, but it may be possible to prove their equality. In that case also we must say that the choice-of-move-problem is objectively solvable. After all, a quadratic equation is no less objectively solvable than a linear equation.

Double and multiple objective solutions are far from rare in chess. They occur, for instance, in:

- a. positions in which the sequence of moves of a combination can be varied without changing the end result;
- b. check-positions in which the King can move in two or more ways but which does not affect the relevant continuation;
- c. positions in which the opponent can be forced into a tight spot by arbitrary 'tempo moves';
- d. defensive positions in which one can undertake nothing oneself, but must calmly await the coming developments by moving the available pieces back and forth.

Apart from these, there is another category, namely, *objectively non-solvable positions*. Here two or more good moves are possible but neither the inequality nor the equality of these moves can be practically proved because the position itself cannot be fully analyzed. Such specimens are to be found, for example, in:

- a. the opening where one can choose between different variations, none of which has as yet been refuted;
- b. the middle game where one can choose, for instance, between different systems (e.g., to open up the position or to keep it closed), whose relative worths cannot be definitely established even through an extensive analysis, etc.

Such objectively non-solvable position problems do not allow of an objectively valid 'right' solution as does a mathematical problem.

²¹ Notice that the difference is relative. A still playable opening variation can later be refuted by the collective experiences and analyses of chessmasters; a currently objectively non-solvable position can later become solvable. Indeed there is an abundance of cases in the opening where the nineteenth century master still had free choice among various good moves, Z_1 , Z_2 , and Z_3 , while his twentieth century colleague is required to choose move Z_1 because the other two, Z_2 and Z_3 , have in the meantime been refuted. The development of opening theory has the consequence that increasingly more 'free' positions are becoming objectively solvable.

At least no one knows the solution or can find it by human calculation²¹ so that it is impossible to arrive at a choice without subjective considerations. Such positions have a character of their own, especially if the number of good moves is large (three or more possibilities).

From a psychological standpoint the *objective freedom of choice* – defined as the number of playable, good moves – and especially the objective solvability or non-solvability of a position is of much more importance than the legal freedom of choice, K . The latter, psychologically, is but a purely formal maximum both with respect to the number of possible good moves and the number of moves that the player actually considers. This maximum is seldom attained and then almost only in check-positions. Between the legal freedom of choice and 'objective solvability,' there is little connection: a robust position with 60 or more possible moves may be objectively solvable, whereas an anemic position in which $K = 10$ may be 'wide-open.'

Just as we can differentiate objectively solvable and objectively non-solvable positions (or to put it more briefly, forcing and wide-open positions) so also can we distinguish two ways in which the chess player can view his problem. If the player regards the situation on the board as objectively solvable, then he must proceed in such a way as to search for the best move, the 'right' continuation. With a 'free' conception of the problem, on the other hand, he asks himself the question, 'What shall I play (choose) now?' He is cognizant of his freedom and of the personal element in his choice; nor does he pretend that his choice will be the 'right' move. Thus we can consider problem situations and play situations as the two limiting cases. It is characteristic of chess that the player's conception of the choice-of-move-problem fluctuates between these extremes.

Naturally, the player's conception of the problem does not always correspond with the objective character of the position. Sometimes the player believes he has found and played the only correct move while, in actuality, he could have embarked upon another equally good continuation. In other cases the player chooses a move according to personal preference, presuming nothing; afterwards the move may appear to have been the only right one, or worse, not the only right one. Furthermore, the player's way of thinking and his attitude towards chess in general help to determine his conception of the problem. Typological differences manifest themselves at this point (cf., e.g., Section 59). This, however, in no way detracts from the psychological importance of the criterion of objective solvability.

D. THE PROOF SCHEMA FOR AN OBJECTIVELY SOLVABLE POSITION: MOVE W_1 IS THE BEST

Section 9: The two part structure of the proof schema

Let us assume in a given position with White to play that there is only one 'right' move, W_1 . It must be shown then that against any and all replies by Black, White will always achieve better results with W_1 than with W_2, W_3, \dots, W_n . By using the ratings introduced in Section 8, it can easily be shown that a complete proof consists of two parts. To prove:

- (1) By playing $1.W_1$, White's minimum result will be, e.g., 8. (This will constitute the *positive part* of the proof.)
- (2) By playing $1.W_2, W_3, \dots, W_n$, White's maximum result will be, e.g., 6. (This will constitute the *negative part* of the proof.)

In the positive part of the proof every possible Black reprisal must be answerable by one sufficiently strong White move; conversely, in the negative part, every White move must be answerable by one sufficiently strong Black move. The existence of more than one sufficiently strong move is redundant to the proof. In other words: In the positive part branches are only expected when it is Black's move (*counter branches*) and in the negative part only when it is White's move (*own-branches*).

The positive part ('By playing $1.W_1$, White's minimum result will be, e.g., 8.')

After the move $1.W_1$ Black's replies $1...B_{11}, 1...B_{12}, 1...B_{13}$, etc., can be answered respectively, by $2.W_{11}, 2.W_{12}$, or $2.W_{13}$. If after $2.W_{12}$ Black responds with $2...B_{121}$ or $2...B_{122}$, then the replies $3.W_{121}$ and $3.W_{122}$ follow, respectively. Finally, it is assumed that branches will eventually terminate and end positions will be reached where unambiguous, objective values can be assigned. $1.W_1$ can then be evaluated by assigning it the minimum terminal value – in this case 8.²²

²² The evaluation of W_1 may or may not be correct: this depends on whether the subsequently chosen W-moves ($W_{11}, W_{12}, W_{121}, W_{122}$, etc.) are objectively 'best moves' or at least 'good moves.' In case they are not, the final evaluation may be too *low*. Accordingly the ratings assigned the other possible first moves (W_2, W_3, \dots, W_n) may be too *high*. It follows that even so the conclusion of the relative superiority of W_1 over other moves remains correct – provided the counter branches in the

The negative part ('By playing $1.W_2, W_3, \dots, W_n$, White's maximum result will be, e.g., 6.')

For all moves $1.W_2, W_3, \dots, W_n$, Black can respond $1...B_2, B_3, \dots, B_n$ in such a way that every second move by White $2.W_{i1}, W_{i2}, \dots, W_{in}$ ($i = 2, 3, \dots, n$) can be correspondingly answered by $2...B_{i1}, B_{i2}, \dots, B_{in}$, etc. From White's viewpoint no terminus can be assessed at more than 6.

If one succeeds in finding the moves $1.W_i, 2.W_{ij}, 3.W_{ijk}$ ($i, j, k = 1, 2, \dots, n$) in the positive part and $1...B_i, 2...B_{ij}, 3...B_{ijk}$ ($i, j, k = 2, 3, \dots, n$) in the negative part, in such a way as is required for the proof, then one has indeed proved that $1.W_1$ is the strongest move.

We shall use a special case for an illustration. Assume that White is able to checkmate his opponent in three moves. The proof is much simpler here because (1) no final numerical evaluation and comparison of results is needed and (2) the length of no variation can exceed three moves.

To prove:

- (1) After $1.W_1$ no reply by Black can avert checkmate in three moves, or less.
- (2) After any other moves, $1.W_2, W_3, \dots, W_n$, Black is able to prevent checkmate in three moves.

This can be illustrated by a branching tree making the simplifying assumption of a small number of possible branches (see Figure 3).

Incidentally, it is not necessary that two counters with different subscripts actually be different; e.g., different defenses are sometimes followed by the same sufficiently strong move.

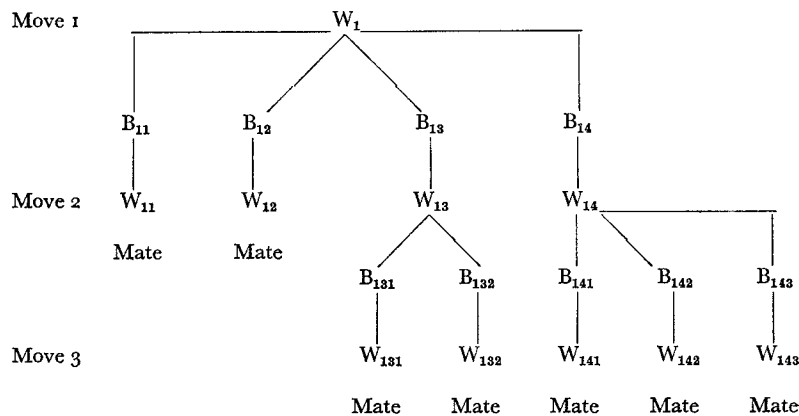
This schema, at least the positive part, may remind the chess player of the way in which chess problems are presented in publications. The problem solver must only find the key move, W_1 , and the successive right White move against every possible Black answer. But the problem composer must solve the negative part too! He must avoid 'cooks' or redundant solutions; he must be able to prove that W_1 is the 'only' best move.

positive part and the own-branches in the negative part cover all (relevant) answers.

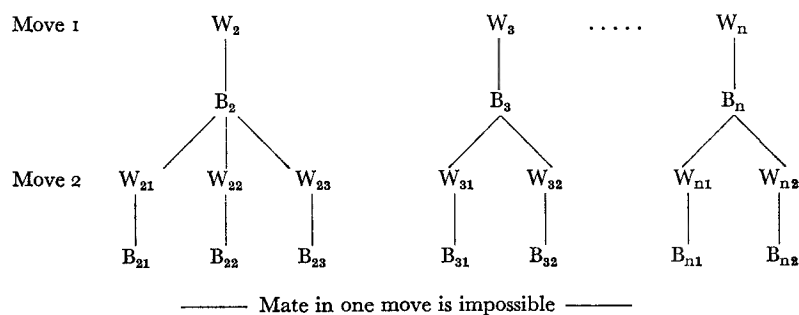
The procedure as described here is, therefore, not completely equivalent to 'blind' *minimaxing* as described in game theory in that it is more parsimonious and less comprehensive. The W-moves in the positive part and the B-moves in the negative part may be intuitively selected; there is no need to consider all possible branches in order to prove the superiority of W_1 over other moves.

FIGURE 3

Positive Part:



Negative Part:



In an actual game the situation is different, of course. A player who sees a chance to force checkmate may not bother to look for other, perhaps even shorter, solutions. Such an indifference towards the negative part can only exist when victory is clearly in sight – immediate winning combinations, threats that cannot be parried, etc. – for in general the negative part must be taken into consideration.

Section 10: The significance of the schema for the thought process

Naturally not every chess player has this proof schema at his continual disposal and certainly not in the form given here. Even in a worked

out *written analysis* this fundamental form is not always easy to disinter, as a result of the fact that groups of variations are considered together or parts of the analysis are omitted. If the opponent has a serious 'threat' in the initial position, all of the moves that do not parry that threat can be omitted from the negative part. With the discovery of a stalemating combination – which legally draws the game – in an apparently hopeless situation, it would be senseless still to ask for the negative proof in order to show that other moves do *not* lead to a draw. If White has a choice between executing a forcing, drastic combination and a number of quiet moves which do not significantly alter the position, the latter group can be treated together in the analysis and rejected at a single glance. Of course, other such simplifying conclusions are also possible.

The *ordinary thought process* follows the lines of such a proof even less. Because of the limited thinking time and the necessity to work everything out in his head, the player at the board often cannot furnish a proof, even where an analysis can. It should be kept in mind, moreover, that in a chess position, what can and must be proved is not given as in a mathematical problem. The player must discover this for himself; the thought process involves both searching and reasoning, both selecting a move and building up an argumentation for the choice. Thus it is comparatively rare for a thought process to constitute a rigorous proof. In the majority of cases a move is played without complete certainty that it actually is the best move.

But, after all, it is a motivated choice, and although the thought process does not consist in a proof it still must involve an argumentation in favor of the move to be chosen. In the present study it was in fact possible to derive from the text of most protocols a reasoning structure comparable to the proof schema set forth above. It was often possible, in particular, to distinguish a positive and negative part in the final argument in spite of the fact that the order of presentation in the protocols was largely unsystematic. Sometimes the many miscarriages of attempted continuations begun with positive intentions collectively form the negative part of the argumentation. In such a case all of the variations previously considered for the opponent become 'dead branches' of the negative analysis. For example: The subject has the choice between the active, combinative move 1.W₁ and the calm continuation 1.W₂. He wants to play W₁ and so calculates four worthwhile retorts for Black: B₁₁, B₁₂, B₁₃, and B₁₄. Against the first three the attack would be driven through, but the fourth possibility, B₁₄, refutes the move W₁. He therefore ends up with 1.W₂. The

negative part now consists of Black's refutation of $1.W_1$, that is, the variant $1.W_1, B_{14}$, etc. The other three Black branches, B_{11} , B_{12} , and B_{13} , are of no further value to the reasoning process; they have become 'dead branches.'²³

Branchings from either White or Black nodes and the wording with which the subject begins a new variation often tell us whether the calculations are an attempt at a positive proof, at a purely empirical investigation into the best possibilities for both sides, or at a negative proof. The first of these three cases is the most frequent, as we shall see.

In any case a familiarity with the general proof schema for the 'best move' has its utility in the psychological investigation of chess thought (cf., e.g., Chapter V, Section 43; and Chapter VII, Section 54).

²³ 'Branches' in the proof schema of a chess game can be likened to the metaphorical 'tree of logic' – perhaps even more profitably here than in a mathematical proof. G. Pólya, moreover, speaks of 'dead branches' in an article on this theme (PÓLYA 1939). In the course of this book the metaphorical 'tree' will be frequently employed.

PSYCHOLOGICAL THEORIES OF THINKING

A. ASSOCIATIONISM AND COUNTER MOVEMENTS

Section 11: The main traits of classical association theory

Although it has often been stated that theories of association are insufficient as a fundament for the description of productive thought processes, associationism still prevails in the terminology and ways of thinking of many researchers. The superiority of Selz's theory, the main subject of the present chapter, has been recognized by some students of thought, it is true, but it has never become very influential. It would appear to be of importance, therefore, to show what Selz has done by relating and contrasting his contribution with a few of the main features of associationism.

(a) In all variations of association theory the concept of association reveals its origin from empiricist philosophies. As a tie between two mental elements an association is supposed to be born empirically, namely, by *contiguity*. No other relationships between the elements – logical, systematic, semantic – are considered; there is no room for mental principles such as 'insight.' *The associative tie between two elements is blind*, so to say; whatever qualities it has stem from the elements' contiguity, i.e., from the fact that the elements were more or less often presented together. If the naïve subject should think, for instance, that the relation between a word and its meaning is more than a blind connection of a word image and a generalized object image, he is supposedly misled by the familiarity of the relationship in question. Moreover, the specificity of the relation is a result of associations.

It is true that even in the classical version of associationism some distinctions are drawn between different kinds of associations. Best known are the following four types of association formation: simultaneity, succession, resemblance, and contrast. But then, there is a general tendency again to reduce these different kinds to one 'blind,' empirical principle. First, two categories can be created for the four ways in which ideas can become associated. The first two (simultaneity and

succession) are 'outer' associations: clearly empirical contiguity types; the second two (resemblance and contrast) are 'inner' associations. It would appear at first sight that some principle of structure or even of meaning is introduced with the second category, but then psychologists immediately tried to eliminate it by reducing the second category to the first. Compare, for instance, WUNDT's statement in his *Grundriss der Psychologie* (WUNDT 1902, p. 268): 'Starting from the simplification into two forms many psychologists finally went on to a reduction to one single principle of association, either by declaring contiguity to be a special form of resemblance or, and this more often, by declaring resemblance to be a result of certain contiguity relations. In both cases often the association itself was in its turn reduced to the more general principle of exercise and/or habit formation.'¹

Associations result from contiguity; the contiguity effect is in its turn explained by some early idea of conditioning or habit formation. The possibility of an inner relationship is denied or at least is not considered.

Mutatis mutandis, the same holds for more modern 20th century researchers who start from an associationist point of view. With them, too, an association is no more than a blind empirical tie of a certain strength. This is hardly deplorable in itself, it is true, since much of the clarity which the concept of association imparts to processes other than those of directed thought would be lost if it were aggravated, so to speak, by too much surplus meaning. But from a point of view of the description and explanation of directed thought the idea that one simple type of tie, undifferentiated and purely empirical, could account for every possible relationship between content elements in thinking can hardly be satisfactory. Thus critics primarily oppose the overestimation of the explanatory capacity of the association mechanism for directed thought (cf. Sections 12 and 13).

(b) Directly related to the foregoing is the *sensationist* conception of the thought process: *a succession of images*. To the strict associationist, thinking is identical with thinking in images, images which succeed each other, largely according to the simple laws of association. Thus,

¹ 'Von dieser Vereinfachung auf zwei Formen aus meinten schliesslich manche Psychologen noch zu einer Reduktion auf ein einziges Assoziationsgesetz fort-schreiten zu können, indem sie entweder die Berührung für eine Spezialform der Aehnlichkeit oder, und dies häufiger, die Aehnlichkeit für eine Wirkung gewisser Berührungsverbindungen erklärten. In beiden Fällen führte man übrigens die Asso-ziation meistens auf das allgemeinere Prinzip der Uebung und Gewöhnung zurück.'

for instance, a concept like a horse or a table is considered to be not only a result of a number of sense perceptions but also a sort of summary or condensation in consciousness, which is of a sensory nature itself. Often the terms 'concept' and 'generalized image' are simply equated.

In this respect a crucial issue is the perception of relations. If a person who hears two tones in succession observes that they are equal in pitch, should this observation be considered a separate mental activity that goes beyond the perception of the two tones? With an essentially positive answer, non-imagerial elements in thought enter the picture. So it is not surprising that consistent associationists like EBBINGHAUS deny it: 'The same perception that provides me with colors and tones gives me the information on the relationships of equality and inequality; I experience the one thing in exactly the same way as the other, just as strongly sensorily and just as immediately without reflection.'² (EBBINGHAUS 1911, p. 513) No separate act of abstraction can be admitted in a theory that views mental processes as a mere succession of images.

Although 20th century associationists have been somewhat more careful in this respect, the fundamental sensualism in the explanation of thought remains a predominant feature of associationism. Thought is considered to be governed by associations and associations are between images.

(c) Another important question: Do the associations exist between image *elements* or between *complex structures*? Extreme representatives of a physiologically colored associationism take the stand that associations are between elements; e.g., a word is split up into phonemes. Most of the more modern associationists are not as atomistic as ZIEHEN was, for instance, but still some degree of *atomism*, that is, a tendency to work theoretically with the smallest possible units, is characteristic of associationism in general. Associations between images are thought of as the blocks from which the mental life of human subjects can theoretically be built up, and it is procedurally characteristic for all association psychologists to start from the blocks and not, for instance, from a global building plan.

² 'Dieselbe Anschauung, die mir die Farben und Töne liefert, gibt mir auch Kunde von den zwischen ihnen bestehenden Gleichheits- und Ungleichheitsverhältnissen; das eine erlebe ich ganz auf dieselbe Weise, ebenso sinnlich lebendig und ebenso reflexionslos wie das andere.'

(d) There is *no large nor basic difference between directed thought and free imaginative thinking* (e.g., daydreaming) in the associationist conception. Like the process of imagination, directed thought is considered to be governed primarily by a competition of associative reproduction tendencies in which the strongest association wins out every time. True enough, some other possibilities were considered by association psychologists: for instance, the possibility of *emerging images* (*frei-steigende Vorstellungen*) – which might be partly explained by the effect of *perseveration tendencies* (MÜLLER and PILZECKER 1900, p. 58). Furthermore, some researchers considered the possibility of truly ‘inner’ associations, in particular of the resemblance type, to be operative without any pre-formed associative relationship. Element *a* may then lead to the reproduction of the similar element *a*¹ even if no tie between *a* and *a*¹ exists on the basis of previous experience (e.g., in the case that *a* is something completely new to the subject). Finally, it was, of course, well known that the set (*Einstellung*) of a subject was indeed quite influential in determining the imagerial flow although it appeared to be difficult to describe clearly how this influence should be theoretically integrated.

In principle, however, all of these factors were considered to be effective in free imagination as well as in directed thought.

The difference between the two types of mental processes was explained solely by the presence, in directed thought, of a general *goal image* or ‘direction’ in addition to the preceding image which acts as a stimulus. Obviously this distinction between goal image (*Zielvorstellung*) and stimulus- or cue image (*Ausgangsvorstellung*) derives from the traditional setups for thought and memory experimentation. On the one hand there is a general *Aufgabe* and on the other a specific stimulus word. According to the then current theoretical conception the *Aufgabe*, or the general task, results in the subject’s forming a selective set: a specific subgroup of images is prepared for activity. Then the stimulus word triggers off the thus directed stream of consciousness (images) – albeit the stream remains in the confines of its bed. This conception is reflected in the then current name of these experiments: ‘restricted associations’ (*eingeeengte Assoziationen*). In other words, the task is supposed to lead to a certain *constellation* or structurization in consciousness by means of which an appropriate selection of images to-be-reproduced among the many associative possibilities is brought about. There are some variations in theoretical details, but in principle it is only through such a structuring effect (constellation) that the specifics of ordered thought, as distinguished from free imagination, are to be explained.

(e) It should be clear from the above that in association theory there is but little room for *the activity of the subject* himself, not only in imaginative processes but also in productive thought. It is mainly for this reason that associationism has been reproached with being of a ‘mechanistic’ nature; the ‘mechanism’ is supposed to work without any organizing activity on the part of the subject. Remarks of this tenor had already been made by early critics of associationism such as JOSEPH GEYSER: ‘In associationist psychology the whole mental process and every relation between elements are considered to result from passive reactions and not from a spontaneous activity.... Thinking itself is only a reactive phenomenon, not an activity carried out by a mind.’³ (GEYSER 1912, pp. 110–111)

With regard to each of the five features mentioned it is true that twentieth century association psychologists were somewhat more reserved than their nineteenth century predecessors. Indeed few associationists around 1910 would agree with THEODOR ZIEHEN that ‘the possibility of also reducing the so-called higher thought processes to the association between images has been proved....’⁴ (ZIEHEN 1911, p. 225) The fact, however, that even most advocates of associationism accepted it in but a rough and non-literal way, without clearly delineating the limits of its validity and without trying to repair its old generality by basic theoretical completions of their own, would seem to be an incisive indictment of its senile weakness.

In the nineteenth and particularly the beginning of the twentieth century each of the above five characteristics of associationism, either alone or in combination, became the subject of lively discussions that were more and more conducted on the basis of experimental findings. The methods, arguments, and results of some of the counter movements, in particular, the Würzburg school of *Denkpsychologie*, will be discussed in the next four sections.

Section 12: Relations are not reducible to images

The classical concept of association explains how images become connected, but it neglects, as we have just seen, the character of the

³ ‘Vor allem ist (in der Assoziationspsychologie) alles psychische Geschehen und jede Beziehung, in welche die Elemente geraten, eine passive Reaktion, keine spontane Tätigkeit. ... Auch das Denken (ist) nur eine reaktive Erscheinung, nicht eine von einer Seele ausgeübte Tätigkeit.’

⁴ ‘Die Möglichkeit der Zurückführung auch der sogen. höheren Denkprozesse auf die Assoziation der Vorstellungen ist ... bewiesen.’

relation between the elements. The relation between a word image and the indicated concept, between the parts of a complex image, between parts and the whole, between an object and its use, between contrasting, analogous, or co-ordinate concepts, between cause and effect, goal and means, and many more – this whole variety of relations, of such obvious importance in processes of thought and imagination is assumed to be reducible by contiguity to simple ‘outer’ empirical associations, more or less strengthened by repetition. This means that the specificity of each of these relations must be solely explained by the presence, number, and strength of other blind connections, namely, the associations that connect both of these elements with other elements. It appears to be an impossible task, however, to explain the *enormous qualitative variety of specific relations* that we know from experience with but the following three factors:

- (1) the purely ‘outer’ connection itself,
- (2) the position of the association in complexes or nets of similarly unspecified ‘and-connections’ (*Und-Verbindungen*), and
- (3) the strength of all these connections as a result of repeated contiguous presentations.

This difficulty was not crucial for classical associationism, so long as it remained purely *sensationistic*, i.e., so long as it was possible to deny the (imageless) consciousness of a relationship as a separate phenomenon. If a *relation* between two elements is nothing but an (imagerial) combination of the (imagerial) elements themselves, possibly charged with the overtones of other (imagerial) associations but on principle given simultaneously with the elements themselves, then there is little reason to bother about a classification or typology of relationships. They all reduce to structures of associatively linked imagerial elements anyhow.

However, the idea that the consciousness of a relationship is encompassed in the perception or imagination of the related elements (see above, p. 35) appeared to be untenable. This can be shown without any experimentation, just by an analysis of the phenomenon, as has been done, e.g., by GEYSER (1912, pp. 523–24). But more convincing than analytical arguments are experimental findings like those of GRÜNBAUM’s (1908) pioneering investigation on the perception (abstraction) of equality of figures.

The main result of Grünbaum’s experiments was the discovery that even such a simple relationship as the equality of two visually perceived figures can occur as a separate phenomenon in consciousness, more or less independently of the perception of the figures

themselves. Let F1 stand for the act of perceiving one figure and F2 the other, and let E stand for the act of perceiving their equality. Grünbaum found that the following cases occurred with his subjects, according to their introspective reports:

F1 and E are present, but F2 is not;

F1 and E are present before F2;

F1 and F2 are present, but E is not, or occurs only later.

Grünbaum concluded, particularly on the basis of the third case, that something new must be added to the perception of the elements in order for the relationship to become conscious. ‘A specific activity is needed which is not included in the perception of the elements.’ (GRÜNBAUM 1908, p. 449)

Naturally, Grünbaum’s point of view was upheld by the other Würzburg psychologists who felt that their experiments had proved the possibility of *thinking without images*. As is well known from the expositions by the reviewers of the Würzburg school (BURLAUD 1927; HUMPHREY 1951; JOHNSON 1955), the development of ever more differentiated ideas about the possibilities and categories of ‘wordless’ and ‘imageless’ thought was the central theme in their work. WATT (1905) found the images, as they were described by his subjects in thinking experiments, to be dependent, fragmentary, inconsistent, vague, and sometimes, so to say, semi-abstract, i.e., half image, half ‘knowing that...’ MESSER (1906, p. 180) stressed the frequent occurrence of completely abstract ‘states of consciousness’ (*Bewusstseinslagen*). BÜHLER (1907), who was the first to work experimentally with much more difficult tasks (philosophical questions, interpretive problems, etc.), was then able to distinguish between *states* of consciousness proper such as surprise or doubt, on the one hand, and ‘thoughts’ (*Gedanken*), on the other. The latter concept corresponds roughly with what ACH had called *Bewusstheiten* and BINET *pensées*.⁵ These can

⁵ Alfred Binet was an important forerunner in this field, too. In a number of not very rigorous but simple and natural experiments he came to an even stronger conclusion than the Würzburgers, namely, that there is a marked contrast between thinking and imagery (BINET 1903). In particular, the rich detailed image is hardly ever found in directed thought processes, but rather in dreams and day-dreams. One of his subjects was his little daughter Armande, who at one time stated the problem quite well: ‘In order to have images, I should have nothing (else) to think about’ (‘Il faut que je n’aie plus rien à penser, pour que j’aie des images,’ BINET 1922, p. 124). Binet’s characterization of the relative unimportance of images in a process of directed thought is very much to the point: ‘... with a hundred thousand dollar thought one has images of five pennies’ (‘... avec une pensée de cent mille francs, on a des images de quatre sous’).

best be verbally expressed by such statements as: 'I know that...', 'I am aware that...', 'I see (understand) that...'. One most interesting subgroup of thoughts was called by Bühler 'intentions.' This group of elements of consciousness corresponds to situations where the subject just 'means' some object of thought, 'thinks of it,' 'intends' it, without any accompanying representation of the object itself – while he knows quite well what it is he means.⁶ According to Bühler, every object, however sensory in character – e.g., some hue of color – can in principle be thought of, conceived, *bedoeld* in the form of an intention without any object- or word image.

From a descriptive introspective point of view it is no exaggeration to say that the Würzburgers discovered, in their protocols, in addition to imagerial elements, a host of other, quite abstract elements of thought. This discovery makes it extremely difficult to maintain any reasonably consistent associationist model of thinking. If so many non-imagerial elements occur in thinking, the position that relations between elements can be reduced to exclusively sensationistic components becomes untenable. So there is no longer a theoretical excuse to disregard the variety of relations that may exist between two elements; moreover, such relationships (and relations between relationships) may themselves be elements in new combinations. For that matter, as soon as the possibility of a relationship-perception as a separate, new, possibly non-imagerial mental phenomenon is accepted, the endeavors of classical associationism to cover descriptively all sorts of relations by means of the one concept of association become futile. It is like trying to make a new language out of one letter.

Section 13: Reproductions are not associative

To a certain extent, OTTO SELZ (1913 and 1922) started his systematic work by attacking the same weakness of associationism. He was able to show that experimental reactions that used to be described simply as associative reproductions, appear, on a more thorough analysis, to consist of much more complex mental processes.

Selz gave his subjects, in the same way as WATT (1905) and others

⁶ In the Dutch language the untranslatable expression *iets bedoelen* (literally: 'aim at something, meaningwise') is quite near to what is meant by intentions. Bühler tried to make clear how empty of representational content an 'intention' is by saying that an intention is 'the act itself of meaning something, not that which is meant.' ('Das Meinen selbst, nicht das was gemeint wird.' BÜHLER 1907, p. 346)

had done, tasks of the following type: 'Death – Co-ordinate concept?' (i.e., the subject has to find a co-ordinate concept to 'death'); 'Hate – Superordinate concept?'; 'Guilt – Effect?' The first word is the stimulus, the second provides the general task; in some experiments the sequence was reversed. Now according to the older associationist conception, the stream of possible associative reproductions is merely directed, delimited by the *Aufgabe*. It had already been shown in Watt's research, it is true, that the effect of the task is more specific. Even so, Watt still believed in a mainly associationist explanation of the thought process. In particular, if the subject in an obviously reproductive solution process was unable to report anything about processes that might have occurred *between* the stimulus word and his own reaction, Watt considered this reaction a purely associative reproduction.

Selz showed that even in these cases there is more to it than just a simple associative reproduction. Often the reaction appeared to be accompanied by a consciousness of the factual relation (*Sachverhältnis*); that is, while the subject is giving his reaction he already *knows* that the two words 'life' and 'death,' from the example above, are co-ordinate concepts as required. Such knowing about the relation is not explained by the associative reproduction mechanism. In other cases, the finding of the required concept was even preceded by a clear consciousness that one *knew* such a concept. Here the *actualization* of the subject's *potential* (or *dispositional*) knowledge (*Wissensaktualisierung*), as Selz calls it, proceeds stepwise. Here again we have a separate, non-imagerial element in the thought process that cannot be explained if the subject's response is considered a direct result of an association.

Furthermore, Selz showed that it is not possible to maintain that it is the stimulus word that actualizes the response word. In a large number of his experimental protocols the reproduction appeared to be preceded by a *process of integration* of the general task and the stimulus word. Thus it is rather the *Totalaufgabe*, resulting from the integration of the *Aufgabe* proper and the stimulus word, which actualizes – not the response word, for that matter, but – the subject's complete factual relation knowledge 'that death and life are co-ordinate concepts.' So the process is demonstrably more complex than a reproduction on the basis of word associations would be.

The apparently simplest cases in which there are absolutely no traces in the protocol of any consciousness of the complete knowledge complex (*Wissenskomplex*) were interpreted by Selz as routine actualizations of a knowledge complex (*geläufige Wissensaktualisierungen*).

The evidence for this interpretation consists of a series of protocol fragments that show gradual transitions from the case with consciousness to the case of an immediate reproduction, where the subject is unaware of reproducing a knowledge complex.

Some psychologists such as G. E. MÜLLER (1919) criticized Selz because his analysis only emphasized aspects of the matter that were well known to every expert psychologist. This may be true, but the main importance of Selz's analysis was his demonstration of the fundamental insufficiency of an associationist model even for the simplest reproductive processes. Even here the conceptual weapons of the associationist appear to be defective: according to introspective protocols, new concepts are needed to describe, in general terms, what goes on.

If the activity of the subject does not consist of associatively reproducing one element when another element – the stimulus word – is given, how can it be described more adequately? According to Selz's terminology, the subject completes a 'knowledge complex' (*Wissenskomplex*), that is, a memory structure for a 'relational fact' (*Sachverhältnis*) consisting of (at least) two elements and their specific relation. The subject carries out the 'operation' of 'complex completion' (*Komplexergänzung*). Thus having a 'knowledge' (*Wissen*) means that the subject is conscious of the existence of a relational fact. Further, 'actual knowledge,' i.e., 'now knowing that...', can be distinguished from 'potential (or dispositional) knowledge,' i.e., knowledge present in memory, possibly to be actualized in the solution process. And so forth (cf. Sections 18 ff.).

Not all of these concepts come from Selz. For instance, a relational fact (*Sachverhältnis*) is nearly the same as MEINONG's *Objektiv*: the fact that *life and death* are (or can be considered) concepts of the same order is a relational fact (MEINONG 1904). The two elements (or concepts) and their relation are both included.

We shall not now enter upon a further discussion of the detailed conceptualizations of Selz and his predecessor Meinong. For the present purpose the main result has been to show the descriptive insufficiency of the concept of association. In order to arrive at an adequate description of thought processes it appears that this one concept will have to be replaced by a detailed theory. It is interesting to note here that Meinong, because of the curious position of his 'theory of objects' (*Gegenstandstheorie*) between logic and psychology, regarded his theory as the beginning of a new science. Even if this is an exaggeration, the fact remains that a detailed description and classifi-

cation of different types of relational facts is of crucial importance in the construction of a good theory of thought and memory.

Section 14: 'Atomism' is not heuristically fertile

Terms like 'atomism' and 'elementalism' have mainly been used in the 19th and 20th centuries by philosophers and psychologists who wanted to oppose the tendency, when dealing with methods of analysis and theory formation, of dividing psychic phenomena into the smallest possible units – as is done in physics by dividing matter into atoms. The term atomism was then used in a derogatory way to refer to the lack of regard for continuity or structure in mental life. Such one-sidedness was assumed to be a necessary consequence of any atomistic conception. Insofar as WILLIAM JAMES and HENRI BERGSON placed their stress on the unity of consciousness, the overlapping of different phenomena, and the continuous stream of inner experience, they can be considered the earliest opponents of atomism in psychology. Certainly one of their targets was classical association theory with its conception of conscious life as a chain of loose, more or less mechanically linked and mechanically reproduced imagerial elements.

Much more fundamental and certainly much more influential in the last analysis, however, was the attack undertaken by VON EHRENFELS (1890) and, in particular, by the Gestalt theorists inspired by him. Their counter position is well enough known: they maintained the priority of the whole with regard to the parts, primarily in perceptual processes, but also in purposive action and thought. As a result, they used much more *macrostructural variables and distinctions* in their analyses – analyses, e.g., of problem solving processes in anthropoids (KÖHLER 1917, transl. 1925), of a child's purposive behavior (GOTTSCHALDT 1933), of solving of mathematical and practical problems by humans (WERTHEIMER 1925; DUNCKER 1935, transl. 1945).

The importance of the Gestaltist expositions on problem solving cannot compare with their great successes in the field of perception – for which field the conceptual apparatus of Gestalt theory was primarily built up. Nonetheless a number of interesting and often inspiring specific contributions were produced. Most important, however, was the radical change of emphasis and direction of study the work of Gestalt psychologists brought about, in the study of thinking as well as in other fields. The well-known statement that the whole is more than the sum of its parts is by no means a deep truth – it is not even

a logically tenable statement (DE GROOT 1944). But as a slogan it has been of great importance in combating the atomist prejudice that the smallest indivisible elements must be the most adequate building blocks for the construction of a theoretical picture of the whole. Moreover, the Gestalt point of view, that 'the whole' is the first 'element' to be considered, encouraged experimental psychologists to investigate complex processes that had previously been avoided.

The Würzburg school did not particularly oppose the atomism of the association theory; on the contrary, they too were in search of 'elements of thought.' But the Würzburg school did not presuppose – quite differently from classical associationism – hypothetical elements in order to theorize; rather they tried to find them by experimental observation, and further to determine and classify them by a careful analysis of experimental protocols. Curiously enough, one of the main criticisms the old master WILHELM WUNDT (1907) voiced, in particular against what he called BÜHLER's (1907) 'questioning experiments' (*Ausfrage-Experimente*), was that the phenomena investigated were far *too complex*. BÜHLER (1908) retorted that the supposition of complexity in the 'real' intellectual processes might well be largely due to a sensationistic prejudice, namely, the *a priori* hypothesis that in one thought a series of images *must* be condensed. If we look at Bühler's work now, the very fact that he had the courage to study intellectual processes of a less trivial ('atomistic') character than those his predecessors had investigated, makes his work of importance to us. In this respect, too, he was quite knowingly freeing himself from the old association theory.

The expression 'atomism' denotes, primarily, a certain direction of study, a certain point of view of the researcher; as such it is therefore not refutable. The only thing that can be said is that an atomism of hypothetical little blocks that cannot stand the test of reality is relatively fruitless, the more so if the building plan is too primitive to be theoretically adequate. That much has certainly been convincingly demonstrated by the experimental studies of Gestalt- and *Denk*-psychologists.

Section 15: Associative models fail for directed thought

The associationist conception of the difference between directed thinking and free imagination (daydreaming, flow of ideas) had already been criticized by some of the nineteenth century psychologists. To

them the concept of *constellation* could not provide a satisfactory explanation. Clearly the directional or goal representation – or, freed from sensationism, simply the *goal* – had to be a decisive factor for the course of the process, but it remained unclear, in the constellation conception, as to just how this was supposed to happen. To many students of thought the proposition that the only underlying mechanism here was one of selection between 'competing' associative reproduction tendencies was unacceptable. Even less acceptable was the assumption that such a selection process itself be determined by purely associative mechanisms.

It appeared difficult, however, to replace the associationist conception by another explanation of directed thought. Some authors, like WILLIAM JAMES (1890) simply acknowledged the insufficiency of current psychological knowledge and methodology; or like BERGSON (1889), retired into metaphysics: human thinking is too complex or rather too subtle and sublime to be explained by laws. Among those who did not abandon the task were some investigators who tried to use logic as a point of orientation. One form of 'logicism' – another 'ism' from which an autonomic psychology of thought had to be emancipated – was the assumption that the primary task of a psychology of thought consisted in analyzing the 'psychological characteristics' of concept, judgment, and conclusion. The influence of such 'logicism' is, for instance, apparent in the work of MARBE (1901), WATT (1905), MESSER (1906), and later STÖRRING (1908) and even LINDWORSKY (1916). In this respect, it was again BÜHLER (1907) who first approached the problem from a more purely psychological point of view. BURLOUD (1927) made this quite clear in his excellent review of the work of the Würzburgers: 'Another novelty of Bühler's work is that the (study-) object was not restricted to one predetermined category of intellectual operations. Watt's objective was to study the directed evocation, Messer had started from the idea that thought is composed of concepts, judgments, and reasonings (conclusions). Nothing proves, however, that this division should have the same value for the psychologist that it has for the logician.'⁷ (BURLOUD 1927, p. 116)

It was, after all, largely as a result of the embryonic stage of *Denkpsychologie* that the Würzburg psychologists could contribute so little

⁷ 'Une autre nouveauté du travail de Bühler est que l'objet n'en est pas limité à une catégorie déterminée d'opérations intellectuelles. Watt s'était donné pour objectif l'évocation dirigée, Messer était parti de l'idée que la pensée se compose de concepts, de jugements et de raisonnements. Rien ne prouve que cette division ait pour le psychologue la même valeur que pour le logicien.'

towards formulating dynamic laws of thought. True enough, WATT (1905) emphasized the importance of the *Aufgabe* for the course of thought, but for the rest he remained a disciple of associationism. ACH (1905) went one step further by introducing his 'determining tendencies,' but it can hardly be maintained that the explanatory or descriptive power of this new concept was very strong. In criticizing Selz's work G. E. MÜLLER (1919, p. 108) remarked, not without some justification, that in the same manner digestion could be explained by means of a number of 'digestive tendencies' in the bowels. Müller overlooked, however, that the acknowledgment of the necessity to introduce such a concept in itself clearly showed the insufficiency of existing explanatory principles. Thus a new domain for study was at least opened up: the next question was *how* is the course of thought 'determined,' that is, what laws are effective in determining it.

Finally, Messer and Bühler were so much involved in their endeavor to describe and classify the separate phenomena of thought that they could not devote much attention to its structure and dynamics. Messer's main contribution in this direction was that he made an empirical categorization of images according to their 'developmental stages' (MESSER 1906) – which at least provides a starting point for a dynamic analysis. Analogously, BÜHLER (1907) categorized the different degrees or stages of understanding of experimentally presented sentences or words. On various occasions he drew attention to the unity of the thought process, in particular to the unifying effect of the so-called connections between experiences (*Zwischenerlebnisbeziehungen*). But only a few of such scattered indications of thought dynamics are found in his work.

It is again Burloud who most clearly elucidates the restrictions which are apparent in the work of the Würzburgers: 'These psychologists who have been quite unjustly reproached with being metaphysicians, have on the contrary voluntarily confined themselves to the description of facts and have constantly taken care to avoid any interpretation beyond the records of their experiments. This is, as we shall see, both their strength and their weakness.'⁸ (BURLLOUD 1921, p. 154)

With regard to the methods of systematic introspection Burloud

has but one criticism to make, viz., their exclusive use of it: 'Psychological causality cannot, or can only in part, be uncovered by interior observations: in order to reveal it, it is necessary to construct hypotheses and later on to confront them with the facts.'⁹ (*Ibid.*, p. 140) This obviously means that there is a need for a *theory*.

Later on BURLLOUD (1927) specifies this need in more detail (cf., in particular, pp. 188–189). According to him the key to understanding the dynamism of thought is to be found in working out a psychology of tendencies – i.e., empirically differentiating Ach's panacea, the 'determining tendency' – and of relationships. Here again he states the necessity of going beyond a pure description of phenomena. He is of the opinion, furthermore, that good, effective hypotheses can only be found with the help of the genetic method, i.e., by analyzing the origin and development of those tendencies that are found to be of importance in the thought dynamism. This position is, of course, debatable, but we shall not now enter upon a detailed discussion of the possibilities for adequate hypothesizing about thought processes.

In summary, up to and including Bühler, *Denkpsychologie* was not able to replace the old associationist model by a new theory of directed thought. It did demonstrate quite clearly, however, the need for a new model.

The only psychologist of the early 20th century German tradition who undertook to build a new theory was OTTO SELZ. Correspondingly it was his task to evaluate the explanatory capacity of the associationist conception of the thought process which he himself called 'the only consistently elaborated endeavor thus far, to represent the dynamic laws of thought in the framework of a closed theory.'¹⁰ (SELZ 1924, p. 5) We shall conclude the present section by a brief discussion of the two main points wherein SELZ (1913) found serious shortcomings in the association theory of directed thought.

1. The idea of *associative reproduction* underlying classic association theory is too vague a concept to be able to explain the strict specific succession of mental events (ideas, operations) which is found in

8 'Ces psychologues, auxquels on a bien injustement reproché d'être des métaphysiciens, se sont volontairement cantonnés dans la description des faits et se sont constamment gardés de toute interprétation dépassant les procès-verbaux de leurs expériences. En cela consistent à la fois, nous allons le voir, leur force et leur faiblesse.'

9 'La causalité psychologique ne tombe pas ou elle ne tombe qu'en partie sous l'observation intérieure: il faut, pour la déceler, construire des hypothèses et les confronter ensuite avec les faits.'

10 '(den) einzigen konsequent durchgedachten Versuch, die Verlaufsgesetze des psychischen Geschehens im Rahmen einer geschlossenen Theorie darzustellen.'

directed thinking. Such a process cannot be adequately described as a somewhat ordered train of 'diffuse reproductions' of the same type that are found in free association. Judging from protocols of thought processes, the impression is that we are dealing with strictly determined successions. The assumption of an underlying *competition* (*Konkurrenz*) between different reproduction tendencies, as would follow from the associationist model, is not supported by the facts.

The experimental evidence Selz produced in order to show this consisted, first, of an analysis of errors as they occur in the protocols and, second, of some statistics on the occurrence of loose, unconnected images without any apparent functional meaning in the thought process. If, according to his reasoning, the theory of competing reproduction tendencies were correct, this would have to result in:

(a) the possibility of explaining errors, as they occur, largely as unconnected associative reproductions evoked by separate elements in the thought process; and

(b) a relatively high frequency of loose images of a purely associative or perseverative origin, without functional meaning, i.e., without any systematic connection with the task at hand.

In neither respect did the predictions based on the associationist theory come true. After a careful analysis of a large number of experimental protocols it appeared that the errors could not, in general, be ascribed to 'non-task-connected reproduction tendencies' (*aufgabewidrige Reproduktionstendenzen*). A much more natural interpretation resulted if they were considered products of a *partial effectiveness* (partial comprehension or conception) of the *Aufgabe*. Errors are, or result from, solution trials with regard to the task, with the reservation that the task is somewhat misconceived: there is only a partial correspondence between what is required and the faulty solution (see also DUNCKER 1945, p. 3). The errors were not senseless, not disconnected; they appeared to fit quite well into an *Aufgabe*-directed process. Selz explained the fact that they only partially reflected the goal as an effect of temporary fading away, blurring, or distortion of some parts of the goal representation in consciousness, or, in Selz's terminology, of the subject's *schematic anticipation* of the goal (cf. Section 18). Thus the corresponding solution trial no longer fits with the real *Total-Aufgabe*.

As to the images as they are reported in the experimental protocols, Selz carefully listed and analyzed a total of 1,487 cases. The great majority of them were fairly easily interpreted as having a definite functional meaning in the total process. Even when the insufficiently

clear cases were added to those that could possibly be considered disconnected, associatively evoked images, their total number was no more than 22, that is, 1.5 % of all images reported. Only this small subgroup could be considered to fit with an associationist interpretation – without any guarantee, moreover, that such an interpretation is correct or even more plausible than other ones.

2. Another issue wherein Selz's criticism was of a basic nature is that of the so-called *constellation mechanism*. In their classical form all constellation theories are based on the idea that the course of a thought process is determined by two independent, essentially associative effects, viz., one emanating from the stimulus (-image) and another emanating from the goal (-image). The stimulus – the stimulus word in most experimental setups – is supposed to evoke purely associative and therefore non-*Aufgabe*-connected reproductions, but then the *Aufgabe* brings about a selection and some kind of order in the process by activating for reproduction a specific sphere of representations. These two rather independent effects – associative reproduction on the one hand, (associative) selection on the other – are together supposed to determine the flow of mental events, namely, the thought process.

Selz's careful refutation of such a classic conception is one of the few parts of his work that have become well known. (Cf. WOODWORTH 1938, pp. 795–800; HUMPHREY 1951, pp. 132–149; and JOHNSON 1955, pp. 5–6.)

We shall confine ourselves here to a brief summary of his argument.

First, the theory does not provide an explanation for the fact that with a task like 'the Netherlands – Part?' the subject tends to give a right answer, such as 'Friesland' and does not respond with 'Europe,' for instance. If one considers, on the one hand, the collection of words that are linked by association to 'the Netherlands' and, on the other, the selective influence independently exerted by the task 'Part,' it is obvious that a solution providing a whole of which the Netherlands is a part, would be just as adequate. The answer 'Europe' would be in order since the Netherlands are part of Europe. The fact that such erroneous solutions do not occur shows indirectly that the process cannot be explained by a combination of two independent influences.

Second, apart from this indirect argument, Selz was able to show directly from his protocols that stimulus word and general task are

actually combined, related to each other: they fuse into one 'unified total task' (*einheitliche Gesamtaufgabe*). Whenever the subject in his experiments is exposed not only to a new stimulus word but also to a new general task, the protocols clearly show traces of a fusion process. It is then the unified total task which determines the further course of events. It cannot be split up into reproduction tendencies evoked by the stimulus word and a constellation effect of the *Aufgabe*. Selz's very explicit and precise expositions, substantiated by numerous protocol fragments (SELZ 1913, p. 194 ff.) leave no room for doubt on this score.

Thus Selz showed that the dynamic principles of the old associationist model were basically insufficient, not to say erroneous. Apart from the need for new descriptive concepts it appeared that even for the simplest processes a completely new dynamic model was necessary for *Denkpsychologie*.

Section 16: *The subject carries out operations*

The argument that there is little room for the activity of the subject in the model of associationism was, of course, mainly taken up by those who, for reasons of scientific ideology or belief, did not want to partake in a 'psychology without a soul.' To them the neglect of the subject's organizing activity was just another consequence of the fact that the model was detestably 'mechanistic.'

We should not confuse two different issues here. It is one question – a now somewhat antiquated one, for that matter – whether one does or does not like 'mechanistic' models in psychology; it is another question whether a model, mechanistic or otherwise, sufficiently takes account of the obvious phenomena of active organization which we find in thought processes. With regard to the latter of the two problems serious objections against the association theory of productive thought can be raised. It is not its mechanistic character as such which is criticized but, in fact, its rather low descriptive adequacy and heuristic fertility.

In this respect the theory of thought processes devised by Otto Selz brought about a fairly radical change, not only in comparison with associationism, but also with the older Würzburg *Denkpsychologie*. While association psychology considered thinking to be a train of *images* and while the Würzburg psychologists focused on the analysis and classification of *content elements* of thinking, the main unit in Selz's psychology is *the operation* – an activity of the subject. In his theory the concept of a *solving method* is central. Of primary importance

are the chain of events, the basic processes in the organization of thought and not so much states and content elements of consciousness. Selz read protocols in a different way: he searched for the procedures (methods) by which the subject made progress, regardless of whether the subject applied such 'methods' with an awareness of their being methods. Selz tried to explain the course of the thought process as a *strictly determined succession of operations* carried out by an obviously active subject.

As to the first question, it both has been and can be maintained that Selz's model is also of a 'mechanistic' nature, inasmuch as he describes the thought process as a 'system of specific reactions' (*System von spezifischen Reaktionen*, SELZ 1924, p. 31). Some critics have interpreted his usage of the word *reaction* – instead of *action* or *act*, for instance – as an exclusion of 'real' activity on the part of the subject and/or as an exclusion of the possibility of individual differences. These interpretations are incorrect, however. The use of the word 'reaction' is just a consequence of Selz's deterministic conception in which he considers every process-phase to be completely determined by the preceding phases. In other words: His choice of the term 'reaction' expresses no more and no less than a striving for an explanation according to general laws. As to the matter of differences among individuals it should be kept in mind that the statement that a process-phase is 'completely determined' by previous phases is meant to be valid only with some implied qualifications. The determination is theoretically complete only if the *personality of the thinker* as a system of specific reaction dispositions is given in addition to the developmental state of the thought process, including the subject's involvement with the task and all preceding results. Moreover, a complete absence of disturbing external influences is required. So it appears that in Selz's conception the subject is not so passive and 'impersonal' as he may at first seem on a superficial inspection.¹¹

¹¹ In machine programming terminology the contents of the final paragraph of this section can be roughly summarized as follows:

- (1) A subject's 'action' becomes a 'reaction' as soon as it is programmed (incorporated in the theory). The difference between the two is relative and not of any basic importance.
- (2) There is room for differences in personality structure to the extent that possible differences in specific problem solving programs of individuals are taken into account.
- (3) Selz's theoretical model is considered to be valid only within the limits of its program domain, which requires, among other things, undisturbed solution attempts of one problem at a time.

B. THE MAIN FEATURES OF THE THEORY OF SELZ

Section 17: Otto Selz and his conceptual model of directed thought

According to the present writer's evaluation OTTO SELZ is the most important and the most generally underrated psychologist of the *Denkpsychologie* group that was inspired and headed by OSWALD KÜLPE. Selz was born in 1881, somewhat later than other representatives of the Würzburg school. He studied law, philosophy, and psychology, wrote his dissertation on 'Die psychologische Erkenntnistheorie und das Transzendenzproblem' ('The Psychological Theory of Knowledge and the Problem of Transcendence') and was in 1912 admitted as *Privatdozent* at the University of Bonn. His first book, *Über die Gesetze des geordneten Denkverlaufs* (*On the Laws of the Directed Thought Process*) appeared in 1913. His work was then interrupted by the First World War during which time he served in the German army. As a result the second and main part of the report on his experiments could not be published until 1922: *Zur Psychologie des produktiven Denkens und des Irrtums* (*A Contribution to the Psychology of Productive Thought and of Error*). In the meantime he had received a professorship in Bonn which in 1923 he relinquished, however, for a chair in psychology and philosophy at the *Handelshochschule* in Mannheim. Most of his later experimental investigations as well as those of some of his pupils originate from the Mannheim *Institut für Psychologie und Pädagogik* of which he was the director. Here his main interest lay in the applications of his theory and experimental methods to the fields of education, didactics and learning, particularly in the school situation. Some of the publications from his school, primarily his early experimental reports on raising the levels of intelligence of school children (SELZ 1935) are still of fundamental theoretical importance. It was during this period, too, that JULIUS BAHLE started, under his direction, his thorough and highly interesting experimental investigations on the composition of music.¹² In spite of the fact that the terminology is quite different and that Selz's name is hardly mentioned in his 1936 and 1939 books, Bahle's studies are to be considered as a direct outgrowth of Selz's ideas.

The second interruption to Selz's scientific productivity came about gradually. Although a Jew he was allowed to keep his chair in 1933,

¹² BAHLE's works (1930, 1936, 1939) like Selz's studies, have never been translated from the German.

probably because of his record as a soldier in the First World War and due to his political and personal harmlessness. His sphere of influence, however, was more and more narrowed down: few serious students, various restrictions, publication problems – Jewish authors were not even to be quoted. Finally in 1938 he fled to the Netherlands. No official university position was available but in Amsterdam he could at least work, give occasional lectures, and exert some further influence mainly in educational circles where KOHNSTAMM had already taken up some of his ideas.

Although the main series of chess experiments reported in the present work was carried out in those same first years of Selz's stay in Holland, the author's personal contacts with him were few and contact with his work was but superficial at the time. It was only in later years (1943–44) when there was time available for a detailed analysis of the chess protocols, on the one hand, and for a thorough study of Selz's voluminous, stylistically formidable works, on the other, that the present author realized more and more his admiration for and indebtedness to Selz.

By that time, however, Selz was dead, presumably. In 1943 he was interned by the Nazi occupation authorities in a concentration camp at Westerbork, Holland. Later he was transported to Poland, where he, too, was devoured by the notorious 'final solution to the Jewish problem.' Unmarried, in general an *Einzelgänger* (a lone wolf) throughout his life, he left no family and few intimate friends. But he did leave an important intellectual heritage.

Selz's theory of thought can be considered his life work. It is true that he published a number of philosophical and psychological essays on various other subjects – learning, typology, perception – but neither in scope, nor in thoroughness, nor in fundamental importance can these publications compare with his *Denkpsychologie*. His main works on thought processes have, however, been read by few, misunderstood by many, and completely overlooked by most psychologists, even in Europe. Apart from the fact that continental psychology was kept in check in the twenties and thirties by other more spectacular movements such as Gestalt psychology, the curious and intricate verbal formulations of his theory and experimental analyses are in part responsible for the neglect with which his work has met. It seems in order, therefore, to start the following exposition of the main features of his theory with a few remarks on its structure and scope.

What Selz has done can be best described, in a somewhat more

modern terminology, as the construction and elaboration, on an experimental basis, of a *conceptual model for thought processes*. He does not himself use the term 'model' but he often speaks of a 'closed theory,' meaning more or less the same thing. It was his express ambition to replace the association theory of directed thought by another, more adequate and differentiated, but still uniform system of concepts and statements which were general enough to cover all sorts of problem solving and productive thought processes. He tried to make his theory as precise and explicit as possible, to produce a 'nomothetic explanation' (*gesetzeswissenschaftliche Erklärung*) of directed human mental activity. Wherever he states his intentions he sounds much like modern model builders, who first of all strive for a system of general laws, logical consistency, and precision – if possible even in axiomatic form.

Curiously enough, however, the formulation of his laws is by no means logical or mathematical. He defines very few of his concepts in explicit statements, although he appears to use them quite consistently. His 'laws' are formulated verbally, in the way of a jurist – his primary training – rather than in the way of a natural scientist. This exceptional feature makes his work difficult to read: neither different kinds of concepts (explanatory or descriptive, hypothetical or empirical) nor different types of statements (postulates, definitions, empirical statements) are clearly distinguished in his large edifice of words. It is, therefore, in spite of Selz's obvious striving for consistency in word usage, a puzzling and sometimes rather frustrating task to disentangle his constructions and to penetrate to his meaning.

According to his theory, the subject's thinking is considered one continuous activity that can be described as a *linear chain of operations*. Given the person's system of reaction dispositions, each succeeding operation is determined by the outcome of the preceding, provided that a number of general laws of linking, on the one hand, and the (subjective) goal in its present stage of partial completion, on the other, are taken into account. In general, the course of a thought process, according to his model, is considered to be determined by:

- (1) the 'intellectual personality' of *the subject*, that is, his stock or *repertoire* of solving operation dispositions;
- (2) the features of *the problem*, as it is perceived and conceived by the subject; and
- (3) the subject's *determination* (intention) to solve the problem: the energetic or motivative factor that initiates the process and keeps it going.¹³

The main restriction on the domain of Selz's model arises from

the requirement that it be valid only for *undisturbed* processes of *goal-directed* thinking where there exists a (conscious) *determination* to solve the problem at hand. Degrees of or vacillation in determinedness, distractions, competing motives are not considered; thus to ensure a purely Selzian explanation of difficult tasks, the subject is required either to be highly motivated or to have a high power of concentration. In his theory the motivation aspect is hardly worked out – 'determination' being the only requirement as well as the only motivational concept. This does not mean, however, that some sort of ideal 'constancy' of available energy must be assumed. In Selz's model the laws by which operations are linked automatically regulate the energy flow. Thus, for example, a subject who cannot solve a problem is not required by the theory to continue trying forever: there is a definite 'stop rule.' In Selz's terminology: The operation, 'delay until later,' appears among the 'solution methods.' This method is assumed, like any other method, to be linked by laws to certain kinds of problem situations – namely, those in which all other resources have been exhausted.

From a point of view of content, Selz's theory can generally be discussed only by an exposition of the main linking principles he postulated and/or derived from his experimental protocols and by an explanation of the corresponding basic concepts.

Section 18: General linkings and the schematic anticipation

Selz replaced the classical concept of association by what he called general linkings or couplings (*generelle Verknüpfungen*). Whereas an association is supposed to link dispositions to concrete images, Selz's general linkings are of a much more abstract nature. They are supposed, for instance, to link certain situations with mental reaction dispositions, in particular, problem situations with general solving methods. They connect specific tasks with the corresponding specific intellectual operations. These 'reflexoidal connections' as Selz called them are partly inborn but for the most part develop in the course of one's life through individual experience. The former category contains only a few basic solving methods; by far the largest part of an individual's (cognitive) reflexoidal general linkings are acquired by learning processes. Insofar as these linkings are of an individual nature

¹³ So far it is obvious that the correspondence with modern machine conceptions is complete.

they could very well be called *fixed thought habits* or *thought habit dispositions*.

By these abstract general linkings a thought process is directed, with each next step determined by the situation resulting from the preceding step. Thus the determination to solve a certain problem may in many cases actualize a very general solving method like the following: try to find a specific approach by means of an analysis of the task (SELZ 1922, p. 415) or, in simple words, 'How shall I tackle this?' The next operation is the first step by means of which the subproblem, the analysis of the task, gets solved: for instance, try to relate some elements of the task. From such a beginning the thought process then proceeds like a closed chain, in which each succeeding operation is determined (caused) by the situation resulting from the preceding operation. Such causation is only possible, of course, if one assumes that the original 'determination' of the subject is transferred from the first operation to the second, and so on. This assumption is expressed in the 'law of consecutive determination,' according to which every existing goal-directed determination carries over to operations that then actualize means – means that are supposedly instrumental in reaching the goal (*Ibid.*, p. 150). Here the word 'supposedly' refers to the personal system of reflexoidal linkings or fixed thought habits of the subject in question.

This law implies, for instance, that whenever some means to reach the goal is found, that is, is recognized as a means, the application of this means becomes the subject's next subgoal, to which his energy is transferred.¹⁴ Let the task be to find a co-ordinate concept to 'death.' Now the search for a specific solving method may evoke the method: 'Look in your memory.' This may be recognized as a possibly successful means – i.e., the subject may know that he knows something appropriate that could be used – so he applies it.

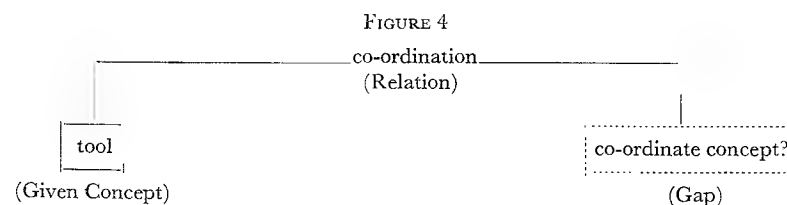
Let us now assume that this operation is successful too and that it leads to the evocation of the reaction word 'sleep' as a solving proposition. Typical of such a situation, says Selz, are the subsequent steps. First, the subject puts the word 'sleep' into the task context; in other words, he substitutes 'sleep' into the complex to be completed (cf. Section 13, p. 42). Now the solving proposition has the form of a proposed relational fact (the fact that 'death' and 'sleep' are co-

¹⁴ In machine terminology the general concept of 'determination' is roughly equivalent to: (1) the problem is fed into the machine, (2) control is transferred to the problem solving program, (3) the problem is accepted by the program. Conceived this way, the law of consecutive determination is rather self-evident – for a machine.

ordinate concepts). Second, the proposed solution is *checked*. With the sophisticated subjects Selz used in his experiments,¹⁵ such checking processes are general, i.e., they occur in all types of tasks, usually before the reaction is given to the experimenter. Selz has called the linking mechanism that connects the 'proposed solution' with the operation 'check its correctness': the law of correction (*Das Gesetz der Berichtigung*, SELZ 1913, p. 261 ff.). If the result of the checking process is again positive, the reaction, i.e., actually saying the word 'sleep,' follows according to another fixed connection.

So much for an illustration of the way in which every next step in the thought process springs lawfully from the preceding one.

To render a fuller description and explanation, the concept of *schematic anticipation* is of basic importance. According to Selz, in the setting of a concrete goal, by which a determined solution process is started, a 'schematic anticipation' of the consequence of reaching the goal is implied. In the act of setting the goal *the subject anticipates the goal-as-attained*. The anticipation is not complete – since in that case there would no longer be a problem to be solved – but incomplete or 'schematic.' If, for instance, the task is to find a co-ordinate concept to 'tool,' the goal-as-attained can be considered to consist of the correctly completed complex: 'tool and X (e.g., 'method') are co-ordinate concepts.' This goal is schematically anticipated, i.e., the subject has a *schema* in his mind in which the given concept 'tool' and the relation (co-ordination) are both present while there is a gap (*Leerstelle*) for X, the sought-after concept (see Figure 4).



A drawing such as Figure 4 is of course misleading in some respects since the visual and spatial character of the schema it suggests are too specific.¹⁶ A schematic anticipation is not necessarily a concrete imagerial structure. The schema may be quite abstract, for instance, when the task is to read and to understand some sentence or instruction. Even in such cases, however, Selz's protocol analyses show

¹⁵ Among his (nine) subjects were Külpe, Bühler and six other Doctors of Philosophy.

¹⁶ A simpler representation is the verbal form: 'Tool' is co-ordinate to '...'

up *some* structural aspects in the goal-as-set. The subject always experiences some sort of *schema* with a *gap* that is to be completed. There is also an *experience of tension* corresponding to the open gap. It is here that Selz links his descriptive notion of schematic anticipation to his dynamic notion. *This schematic anticipation, with gap and tension, is considered to be the starting point for the further course of the thought process.*

The simplest cases for illustration of this idea are those in which the solution springs directly from the schema. The solution may be found, for instance, by an immediate complex completion (cf. Section 12; and below, point 4) as a result of which, for the task of finding a co-ordinate concept to 'death,' the answer 'life' is routinely produced. In such a case, a knowledge complex in the subject's memory is supposed to be evoked by the anticipatory schema by means of some sort of *induction effect*. This effect is based, according to Selz, on a basic tendency in human thought, the *Tendenz zur Gleichheitserregung*, that is, the tendency to evoke and reproduce from memory a structure (partially) identical with that of the anticipatory schema.¹⁷

'Likeness evocation and reproduction' is not the only *basic operation of human thought* by which a schematic anticipation may lead directly to a solution. Selz distinguishes four kinds of basic operations, all of them called 'determined' (viz. by the corresponding schematic anticipation):

1. *Likeness evocation and reproduction (Gleichheitserregung).*
2. *Abstraction.*
3. *Combination.*
4. *Complex completion.*

Although 'abstraction,' (2), is not explicitly defined, Selz seems to have had in mind a definition like the one given by OSWALD KÜLPE (1904, p. 56): 'Abstraction is the process by which the subject succeeds in bringing to the fore certain partial contents of consciousness and in letting others retreat to the background.'¹⁸ If, for

¹⁷ In the terminology of KARL DUNCKER (1945, p. 19), who has adopted this idea from Selz's theory, the same supposed effect is called the *resonance-effect of signals*. The physical metaphor is quite analogous to the one used by Selz: the sought-after knowledge complex is aroused (to 'sympathetic vibration') because of its structural identity with the schematic anticipation, i.e., the *signalment* of the required complex. Being metaphors, they do not explain much, but they may point to possible physiological and/or machine models for the process of direct memory retrieval.

¹⁸ 'Man versteht im allgemeinen unter der Abstraktion den Prozess, durch den es gelingt, einzelne Teilinhalte des Bewusstseins hervorzuheben und andere zurücktreten zu lassen.'

example, the subject's goal is to find and name the relation between 'death' and 'life,' a process is needed to abstract the relation of co-ordination that is implied in the complete knowledge complex. In such a case a visual representation of the schema would most of all show the anticipated 'bringing to the fore' of a specific aspect of the whole complex. Considering the dangers of too explicit drawings, the visualization is left to the reader.

Examples for the case of 'determined combination,' (3), are found in experiments with a stimulus word and a general task, e.g., 'Hate-superordinate concept?' In order for the unified task – that a superordinate concept to 'hate' is wanted – to come into being a process of combination, either immediate or with intermediate operational steps, is needed (cf. the description in Section 15, p. 49). Immediate combination processes occur quite frequently and cannot be reduced to other types of basic operations; thus combination itself is a basic operation.

The basic operation of 'complex completion,' (4), has been discussed in Section 12. An interesting question is whether or not it must contain an operation of likeness evocation, as in the case discussed above. In any case the complex to be completed need not be of such an abstract nature as in the examples thus far presented. It may also be a concrete visual complex. Good examples can be found in the results of tachistoscopic reading experiments in which a subject appears to be able to identify and read a stimulus word from a few cues, e.g., its first (capital) letter and its general visual structure:

FIGURE 5



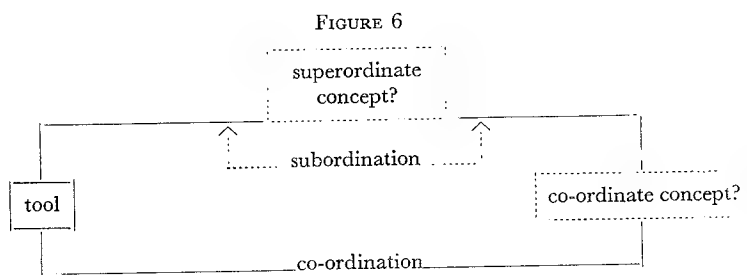
The tachistoscopically presented word is Validity. The subject is assumed to have perceived a schematic picture such as the above drawing. From then on the subject's task is to find (and read aloud) an English word that corresponds with the given 'schematic anticipation' of the word's visual structure.

For basic operations (2), (3), and (4), Selz formulated the *general law of anticipation*: the ease and speed with which an operation is carried out is in direct proportion to the degree of completeness of the schematic anticipation of the goal.¹⁹ (SELZ 1922, p. 512) Or in

¹⁹ 'Die Verwirklichung eines bestimmten Operationserfolges vollzieht sich um so schneller, je mehr sich seine schematische Antizipation einer vollständigen Antizipation annähert.'

Duncker's terminology: The more complete the signal, the stronger the resonance and the higher the certainty and/or the speed with which the operation will succeed (see DUNCKER 1945, pp. 79-80). The law is only strictly valid *ceteris paribus*, i.e., when both the same subject and the same general task are used. Selz's formulation is a generalization of N. Ach's so-called law of specific determination (*Gesetz der speziellen Determination*, ACH 1910, p. 255).

The concept of schematic anticipation is a very general one. Neither its presence as a phenomenal aspect of the goal-as-set in consciousness nor its purported dynamic function²⁰ as a starting point for any goal-directed process is restricted to elementary operations or to simple cases of problem solving. In order to give one example of a somewhat more involved structure, suppose that the subject who is required to find a co-ordinate concept to 'tool' does not have an answer immediately available. That is, the schema of Figure 4 can not be directly completed. It appeared from Selz's protocols that another solving method was generally actualized in such cases: a detour via a superordinate concept. This solving method implies a transformation of the task (*Aufgabetransformation*) leading to a new, transformed goal that implies a schematic anticipation of the type of Figure 6.



Here again the schema initiates the first solving method: 'First look for a superordinate concept,' in order to find later (next operation with a changed schematic anticipation) the required co-ordinate concept.

²⁰ Although Selz has never stressed the point, it would seem best to represent a schematic anticipation as *dynamic*. If the process of 'induction' (1), 'bringing to the fore' (2), 'fitting together' or 'fusing' (3), and 'filling the gap' (4), is anticipated, there is a clear analogy with well-known perceptual phenomena, namely, those of recognition (1), figure-ground-phenomena (2), fusion (3), and closure (4), respectively.

According to Selz's analyses the presence and dynamic importance of anticipatory schemata are no less fundamental in the most complex goal-directed processes than they have proved to be with regard to relatively simple, experimental tasks. Schematic anticipations are implied and are effective in productive and creative goal-settings – such as thinking out a good move in chess, composing a song, or, for that matter, a symphony (cf. BAHLE 1936 and 1939) – as well as in simple reproductive ones. In Selz's own work, this is made particularly clear in his discussion of the *most general solving methods* in thinking, i.e., those methods that are applicable to the largest variety of problems. In his book on productive thought 85 big pages are devoted to a discussion of these methods. The next section is largely an extract from these most important pages.²¹

Section 19: The most general solving methods

From the fact that Selz published two different books, one on reproductive, the other on productive thought, it appears that the distinction between reproductive and productive thinking was of basic importance to him. Here, as with so many other issues, Selz gives no clear-cut definition of the distinction, however. From his expositions it seems that reproductive thought consists primarily of the execution of mental operations by which relational facts that are present in memory are activated and reproduced as such. Thus one could call the mental operation of *recall* (*Besinnen*, SELZ 1924, p. 11) the most general reproductive method. With Selz, however, the term 'most general solving methods' is meant to refer to productive thought, that is, to cases where no previously formed knowledge complex can provide an immediate answer. Within this group of (productive) methods a distinction is made between methods used for the *finding of means* (*Mittelfindung*) and for the *applying of means* (*Mittelanwendung*).

The *Mittelfindung* group is by far the most interesting of the two since it contains the most general human *heuristics* (cf. DUNCKER 1945, p. 20) as they are used in mental production, invention, creation, etc. In the description and classification in the ensuing paragraphs we have tried to stay as close to Selz's meaning as possible.

²¹ Another source was Selz's little book: *Die Gesetze der produktiven und reproduktiven Geistestätigkeit, kurzgefasste Darstellung* (1924), in which he himself summarized the main points of his theory.

1. *Methods of means finding (Mittelfindung).*

In the simplest case, one is able to solve a problem since one 'possesses' the necessary know-how. That is, the solution itself is not at one's disposal but rather the means to reach the goal is available in some form. In principle, two cases can be distinguished:

- (1) the subject may *consciously know how* to proceed, or
- (2) he may have an *automatic solution complex* available.

In the first case the subject can at any time – before, during, or after the solution – actualize a knowledge of method. In the second case too the subject finds and applies the means on the basis of previous experience; but to a direct question such as 'How did you do it?' or 'Which means did you apply?' the subject is not immediately able to give a verbal answer.²²

Examples of automatic solution complexes, (2), are particularly frequent with regard to all sorts of motor skills and achievements. When walking, one cannot verbally explain how it is done, let alone specify which muscles are aroused; it has been learned 'automatically' by experience, on the basis of inborn dispositions. To some degree the same holds for certain intellectual tasks, e.g., in matters of verbal formulation and in some respects for the know-how of the skilled chess player, as we shall see.

In both (1) and (2) the connection between means and ends is supposed to be already there, as a disposition in memory that has only to be actualized. Therefore, the two subtypes of the *method of determined means-actualization*,²³ as Selz called it, are both of a reproductive nature. They are nonetheless of great importance in productive thought (cf. below p. 67).

In case (1), in which the subject knows the relation between means and ends, another distinction can be made. The subject may or may not have previously applied the means in question. If he has not, the actualization is a purely cognitive one, while if he has it is a matter of routine. Thus apart from case (2) above (the automatic actualization of means on the basis of experience, not knowledge) Selz distinguishes

between a *purely cognitive actualization* (without application experience) and a *routine actualization of means* (with both knowledge and application experience: *routinemässige Mittelaktualisierung*).

The actualization of means with which the subject is solely cognitively – i.e., not by practice – acquainted is already very near to, and sometimes rather difficult to distinguish from, the case of reproductive abstraction of means that will be discussed shortly.

Although it is by no means necessary to apply new means in order to produce new results, it does of course happen that new means are found in productive thought. Methods for finding new means fall under the heading of *determined abstraction of means (determinierte Mittelabstraktion)*. In his 1924 publication Selz calls this the second main type of directed productive thought activity (*Zweiter Hauptfall der geordneten produktiven Geistestätigkeit*, SELZ 1924, p. 21) and he distinguishes three subtypes:

- a. reproductive abstraction of means (*reproduktive Mittelabstraktion*);
- b. coincidental abstraction of means (*zufallsbedingte Mittelabstraktion*);
- c. immediate abstraction of means (*unmittelbare Mittelabstraktion*).

In all of these operations, the direction of the abstractive process is supposed to be determined by a corresponding schematic anticipation of the result of the abstraction. The result of abstraction is schematically anticipated as an, as yet unknown, solving method to be successfully applied to the original problem.

a) We are dealing with a case of *reproductive abstraction of means* if the subject knows that A leads to B, or at least has seen $A \rightarrow B$ occur, but has never hit upon the idea of using A as a means to attain or reach B. Only now does its serviceability dawn upon the subject. Here and now the means in question is abstracted as a result of the anticipatory schema in the goal-setting. The schematic anticipation: 'a means X is sought in order to attain B' activates the complex ' $A \rightarrow B$ ' and, at the same time, abstracts the complex's property (*Komplexbestimmtheit*) 'A can be used as a means to attain B.' This mental operation is reproductive in that it is based on the actualization of a memory-complex, but the realization that A is usable as a means is a new element.

Clearly, for a subject who has gone through this process once, the second use of the same means in an analogous task situation is most probably a result of another general method. The finding of the means may now result from an operation of actualization since a

²² In such a direct question we have a relative but nonetheless *operational* and *specifiable criterion* for the distinction between *knowledge* and *experience*. We shall have occasion to use such criteria repeatedly in the text of this book. They do not in this roughly operational form originate with Selz, however.

²³ For the case of *means-actualization* the distinction between *Mittelfindung* and *Mittelanwendung* is difficult to maintain; they generally occur inseparably. In particular in the second case, that of automatic actualization of means, 'actualizing' the means is both finding and applying.

general linking between type of task and solving method may have grown out of the first experience. Especially with a human subject, after such an abstraction process the chances are high that later the same means is simply actualized if needed. Most probably such a means-actualization will then be of the routine-type, in which the previously acquired awareness of the means-end relationship is not lost.

b) The time order is reversed in the case of *coincidental means abstraction*. Here it is supposed that the subject once tried to find a means to reach his goal, thus far in vain, however. Later, at a time when he is still under the influence of the uncompleted task (or the uncompleted complex), even though he is not actively engaged in it, a coincidental, accidental perception of $A \rightarrow B$ may suddenly lead to the abstracted realization that *A can be used as a means* to reach B, his temporarily forgotten goal.

This striking phenomenon has often been described. It appears to occur primarily when the subject is strongly involved in, if not obsessed with, the (unsolved) problem in question. In other words (those of Selz): Although at the time of the perception of $A \rightarrow B$ there is no active determination, there is a relatively strong determination disposition so that very little is needed to re-actualize the subject's determination. (A resonance-effect does it, to use Duncker's terminology.) It is the perception of a partially related event or structure, $A \rightarrow B$, which re-actualizes the determination, the problem, the uncompleted task, and in particular the schematic anticipation. And now, immediately afterwards, or even simultaneously, A is seen as a possible means to attain B: *a mental operation of means abstraction*, in fact.

We know from experience that there is no guarantee that such coincidental findings will have an exceptional value; on the contrary, they may be very trivial. It appears, however, that the conditions for discovery (means abstraction) are rather favorable when the coincidentally evoked problem reappears in the mind of the subject. Indeed, if something new is to be found or discovered, it has often been asserted that a temporary distraction from the problem may have a facilitating or fertilizing effect.

For this phenomenon Selz offered the following explanation: At the moment of reorientation, after the distraction, when the determination is re-actualized, the task is only present in its general outline; so now the implied schematic anticipation is less detailed and works in a more general way. This may have important advantages. A highly specific schematic anticipation is useful only if a

correspondingly specific solution exists; if this is not the case and if the specifying conditions are not all necessary, the specificity of the anticipation may prevent the subject from finding a good solution. The effect of the temporary distraction then, apart from the general increase in receptivity (for useful coincidences), is to alleviate the subject's coarctation. The task, the *Aufgabe*, the schematic anticipation is only partially effective: unnecessary details and conditions will drop out.²⁴ On the other hand, of course, the subject may arrive at pseudo-solutions since partial effectiveness furthers the occurrence of errors as well (cf. Section 15).

Since it is also possible *to think* of something *by coincidence*, a distinction should be made between inner and outer coincidences. 'By coincidence' in the preceding sentence means that *the way* by which the subject comes to the mental experience in question is not related to the problem and does not evolve from a consciously organized thought process. Just as the occurrence of outer coincidences or serendipity can explain many cases of sudden inventions, so can the effect of 'inner coincidences' explain the phenomenon of *inspiration*.

JULIUS BAHLE (1939) showed that such a conception is tenable in musical composition. Indeed, there is in the composer an extraordinarily strong general determination disposition (motivation), namely, that of composing musical works of certain types, in a pre-conceived, aimed-at style (according to a *Leitbild*). This general determination disposition may then specialize into the plan of expressing some particular feeling or fundamental experience in music. Experiences of this sort – Bahle calls them productive experiences – are liable to persevere strongly, that is to say, the corresponding determination disposition may be evoked very easily by some chance thought, haphazard image, or perception. At such a fortuitous recurrence the conditions for a means abstraction of the inner coincidental type may be very favorable. But outer coincidences too, such as noises, intervals, and rhythms in daily life may be instrumental in triggering off an inspirational process. One of the things Bahle showed is that composers often create for themselves the most favorable conditions in order to encourage such fruitful coincidences: by choosing specific working conditions, by setting up schedules and certain daily routines, by embracing special food and drink habits, by selecting the right climate and scenery, etc.

²⁴ The practice of 'brain storming' is obviously based on the same ideas: get rid of a too specific conception of the problem, forget restrictions, take up whatever is – coincidentally – offered by others.

c) The finding of means sometimes results from a structural 'insight' into the nature of the task; this is characteristic of the case of *immediate means abstraction*. The word 'insight' is used here in the Gestalt-psychological sense of the word (e.g., KÖHLER 1917, transl. 1925). The means to be applied is 'read' from the very structure of the task, possibly after an immediate restructuring (*Umstrukturierung*) or possibly during the act of perception or understanding. Immediate means abstraction is sometimes applied in the kinds of logical tasks used in Selz's experiments. Most typical, however, are immediate means abstractions in problem areas where visual structures or special schemata play a role: practical, technical, and geometrical problems (cf. DUNCKER 1945) – and chess, as we shall see. In everyday language immediate means abstraction in problem solving is often spoken of in terms of 'immediately seeing that...', regardless of whether 'seeing' is considered to refer to a perceptual or to an abstractive process.

The extent to which this method can be applied is restricted by an element of experience. Often it is especially the expert in some field who, just from an inspection of the data, is apt to 'see immediately' how to proceed. For him the materials structure themselves, so to speak, so that he is in a position to read from such a structure the solving method which must be applied. Often immediate means abstraction operations are highly important in directing, from the very beginning, the course of a solution process. Thus, for instance, given the task of proving some theorem, an experienced mathematician may immediately see that it should be proved through an indirect procedure. In the same way the chessmaster may 'immediately see' that in a given position on the board he should play for a King side attack, or had better push his Pawns in a hurry, etc.

Although the two conditions mentioned above, a visual or spatial character of the problem data, on the one hand, and experience on the part of the problem solver, on the other, certainly facilitate the occurrence of immediate means abstraction, they are by no means necessary. It may be possible in different problem situations to describe the structure from which the means to be applied is 'read' in spatial or visual terms as well as in logical, causal, systematic, conventional, or other terms. It may even be possible that no objective structure whatsoever can be indicated, the structure being present only in the personal way of thinking (and ordering of thoughts) of the subject. Because of this dependence on structures-as-perceived or -as-conceived by the subject as opposed to objective structures, in

analyzing protocols it is often very difficult to recognize and to pin down clear-cut cases of immediate means abstraction; in particular, to distinguish them from reproductive means abstraction and from the cases of means actualization.

Immediate means abstraction, like the other means abstraction types for that matter, is apt to be replaced by means actualization methods after repeated applications in similar situations. As the subject's experience and knowledge in the field continue to grow, means actualization methods, in their turn, are apt to be replaced by purely reproductive methods of recall of a knowledge complex.

The question has often been posed: How is it possible for a machine-like conception of human thought, such as the theory of Selz, to account for the element of human creativity? If the course of a thought process is adequately and completely described as a chain of mental operations (solving methods), each of which is *reproductively* actualized, triggered off by the result of the preceding one, how can new results emerge? In other words, how is it possible that processes of a reproductive nature can lead to productive achievements? To this question Selz gives a precise answer.

First, the methods of means-actualization (p. 62), whether automatic, cognitive, or routine, may lead to results that can already be considered new. The simple fact is that applying old means to new materials may lead to new results. There is a restriction, of course: the 'new materials' cannot be generically or categorically new to the subject since otherwise there could hardly be an actualization of a *known* means-end relationship. Even so, means-actualization can clearly be productive of new results – albeit not ordinarily of new 'creations.'

Second, even though means finding methods (i.e., the three kinds of means abstraction) are themselves actualized by a reproductive process, it is obvious that their applicability to the problem at hand is something newly found by the subject. Hence the result must be new to him too; whether the result is new to the world is not a psychological question, but there is no reason why it could not be new. Some examples have already been provided in the discussion on the composer's inspiration. Neither Selz nor his pupil Bahle (in his works on musical composition) found it necessary to assume a *generically* different thought process in artistic or scientific creation, nor in 'great men' or geniuses, as a current romantic conception would have it.

Selz, of course, realized that he did not yet provide a complete picture of the methods of directed productivity by attributing the mental production (creation) of something 'new' to the operations of means-actualization and the three types of means abstraction. In particular, the really creative mind is not creative only when at work. Selz had already in part accounted for this fact by his description of coincidental means abstraction (cf. p. 64). But this is not the only way in which mental productivity is furthered outside of specific problem solving processes. First, of course, the repertoire of solving methods in a particular field is built up by learning processes, both from books and by experience. Theoretical study leads to cognitive mastery of means that can then be applied through 'cognitive means-actualization,' while experience 'in the trade' continually extends and differentiates the subject's gamut of automatic and routine actualization methods.

While taking these learning processes for granted in his 1924 publication, Selz draws attention to one specific way by which a creative person is apt to build up methodically his creative power. Even at times when he is not involved in any specific creative undertaking, he may be continually *trying to find and store means which can be used later*. The difference from regular learning processes lies in the creative 'set,' in the self-imposed, permanent motivation to build up and improve the repertoire of solving methods, a set which pervades and penetrates the whole of a creative person's life. Means are collected and stored from every possible sector of life: there is no restricted learning context.

Alongside the two cases of means-actualization and means abstraction Selz considers this *productive utilization of previously abstracted means-end relationships* (*produktive Auswertung von vorher entdeckten wertvollen Wirkungs-zusammenhängen*) the *third main case of directed productive mental activity*.

This third case, the third part of his answer to the question of how productive thought and the creation of new products are possible, is of particular importance in the realm of artistic creation. Writers and poets continually use their own real life experiences in their later creative work. They are liable to develop the set to look for and then to store materials and expressive means which they may find potentially useful, regardless of whether specific creative plans are in mind. Such a set may even go so far as to interfere with the artist's ability to experience real emotions and to partake personally of life. Complaints of this very nature have in fact been voiced by writers and poets.

With his exposition of general solving methods and their effectiveness in creating new results, Selz built up a very strong position against those philosophers and psychologists who wish to maintain that creativity and mental productivity are essentially irrational processes that can never be explained by laws.²⁵ We do not propose to enter upon a discussion of Selz's conceptions of creativity and cultural development, however (see OTTO SELZ: 'Der schöpferische Mensch' 1932). For our purpose, the discussion of chess thought, enough has already been said about the most general operations of means finding.

II. In Selz's system, the *methods of means application* (*Mittelanwendung*) are supposed to be the counterpart to those of means finding (*Mittelfindung*). This group of general solving methods includes, according to Selz, such operations as ordering, trying out and checking. Application methods do not appear to be very problematic. First, although they are highly important in thought processes, they are not supposed to help solve the intriguing problem of creativity. Second, in the case of means application, the term 'method' is more readily understandable since its meaning is nearer to everyday usage than, for instance, is the 'method' of immediate means abstraction.

We have already pointed out that the methods of means-actualization generally involve both finding and applying of means (cf. footnote 23, p. 62). In the various cases of means abstraction, however, it is generally quite possible to distinguish the abstraction of the means proper from the application which follows. As we shall later see, in more complex thought processes such progressions can be traced throughout the whole course of the thought process. In the protocols of such processes we find a continuous alternation of phases of abstraction and realization (application), or of transitional and elaborative phases as we shall call them in chess.

It should be noted that the distinction between finding and applying means is not an absolute one – or rather the terminology is not free from ambiguity. If a subject searches for a means to a goal, he is certainly in for an operation of means finding. But if he does so by a systematic ordering or trying out procedure he is in for an operation of means application as well. That is to say, he applies a means for finding a means. There is in this respect some inconsistency and obscurity in Selz's terminology that has never quite been cleared

²⁵ Selz's answer is not final, of course. But it is possible nowadays to make his argument much more convincing and explicit by machine simulation of human productive thought processes (see NEWELL, SHAW, and SIMON 1958 and 1959).

up in his writings (cf. Chapter IX). We shall see, however, that for the purpose of analyzing protocols of chess thinking the distinction can be made sufficiently clear and operational.

Section 20: Combination and linking of solving methods

1. As an instance of the way in which different solving methods are linked and combined into a composite solving process, Selz discusses the example of a subject copying an irregular polygon from a given model. Such a process is not one of pure thought, it is true, nor is Selz's treatment of it quite realistic; but nonetheless the example is useful as an illustration of structural qualities that are typical of actual thought processes.

Selz supposes the subject to perform his task by copying the sides of the polygon one by one, taking care that each side has the right direction and length. Thus he should first look at his model, particularly to study the direction and length of the side on which he is going to start (a process of *abstraction*). Next he will reproduce the side in question as adequately as possible (a process of *motor realization*). If we now leave checking processes and other complications aside, the total course of the process is characterized by alternating phases of abstraction (a-phases) and realization (r-phases).

The next question is how we can account for the *transitions* from one phase to the next. The transition from an a-phase to an r-phase ($a \rightarrow r$) is not problematic, since in principle it is a matter of first finding (a) and then applying a means (r). The reverse transition ($r \rightarrow a$), however, cannot be considered to result solely and directly from the completion of the preceding operation, that is, the drawing of the previous side. Certainly the next a-phase is cued off by this completion, but something more is needed. In order to explain the transition $r \rightarrow a$ (next side) we must suppose the existence of a *determination to realize the composite operation*: copy the (complete) polygon. The successful completion of the preceding side acts as a stimulus which re-activates this 'higher order' determination. After the re-activation a new abstraction and corresponding realization phase can follow.

The point of this armchair analysis is that after the successful realization of a part of the copying job the subject must somehow *return to the main problem*. The information or cue 'part completed' is not a sufficient condition for the continuation of the job – a fact well known from employment situations. A re-activation of the original

determination is needed. It is characteristic of composite solving processes that such *recursions to higher order goals* and finally to the main goal occur after successful completion of parts of the task. The phenomenon is well known from naïve introspection; after the solution of a subproblem or the completion of a part of the task: 'Let us now get back to the main goal, to what we were trying to do (to prove, to complete, to solve, to produce).'

Protocols of thought processes yield much less direct evidence for such returns to a higher order problem than might logically be expected. With expert subjects in a certain field – such as experienced chess players in our experiments – the ever-recurring returns to higher order goals (or returns to the main problem) have become automatic. The result then is that the record of the thought process consists largely of seemingly automatically linked phases, regardless of whether the transition is of the $a \rightarrow r$ or of the $r \rightarrow a$ type.

2. In the example of copying a polygon we have to do with a sequence of operations in which the *successful completion* of the preceding operation cues off the following one. In Selz's terminology: The operations are *cumulatively linked* and so are the subject's operation dispositions (the solving methods in his 'program'). Solving methods (and actual operations) can also be connected by *subsidiary linkings*.* In that case the next solving method replaces the one that has just *failed* to attain the aimed-at goal or subgoal. Such subsidiary replacements may also lead to a sequence of operations and therefore to complexity of the thought process. Whether two or more consecutively applied subsidiary solving methods can be considered to constitute one composite method remains somewhat open to question, however. The replacement may or may not have been planned or, minimally, foreseen by the subject.

In any case here, too, the new solution attempt must be preceded by some sort of return to the original task. After the failure of the first solving method the same goal or subgoal must now be approached in a new way. This generally implies a renewed goal-setting. Most important here is the fact that while we are apt to speak of 'the same goal,' the schematic anticipation of the goal-as-attained (cf. Section 18, p. 57) must have changed somewhat since the subject now has some information on how *not* to proceed and what can *not* be achieved (cf. Chapter VI: 'The Development of the Problem').

Further complications may arise if one partial operation in its turn requires a combination of solving methods. The structure of

* We have retained Selz's term 'subsidiary' but point out expressly that it means 'alternative' or 'substitutional' rather than 'subordinate.' See p. 110, n. 8.

the complete process can become extremely complex and hierarchical. Of course, the same is generally true of the subject's system of available solving methods if it is adequately to govern the course of the solution process. If the complexity becomes difficult for the subject to handle, he may repeatedly and expressly return to his starting point – and here we see how 'returning to the main problem' can be used as an important general solving method in problem situations that become confusing to the subject.

We have seen that if an operation fails to lead to the fulfillment of the schematic anticipation, the corresponding solving method is replaced by another one, the 'subsidiary method.' However, some qualifications are again necessary. The structure found in a protocol, for instance, may necessitate a distinction between *provisional failure* and *definitive failure* of a method. In the first case the method is not discarded but is tried again on, e.g., different materials. Two, three, or more of such applications of essentially the same method may occur before it is finally scrapped and the subject makes the transition to another subsidiary method. It may happen that all of the methods tried fail to lead to the hoped for result. Even such an exhaustion of the set of subsidiary methods is not final, however; one other method is still available: *Delay until later!*

So much for the main concepts.

3. Among the composite solving methods where linkings are cumulative, Selz makes a further distinction between *complementary* and *subordinate* methods. In the example of copying a polygon we are dealing with a composite solving method consisting of complementary submethods. The subgoals and suboperations are complementary to each other, they are co-ordinate, they complete each other and only in combination lead to the goal.

In the case of a composite operation containing subordinate methods there is a *hierarchy among the suboperations or submethods*. In the course of a sequence of (successful) operations the subject may, for instance, discover that he must first make a minor correction, completion, or detour, in order to be able to continue. In such a case the sequence of solving methods used becomes more complex in the process, as a result of the inclusion of *subordinate corrective operations*. In protocol terminology such unplanned for inclusions are characterized by expressions like: 'Wait a moment, first I'd better...' or 'Oh, I must first... (carry out a subordinate method).' Selz called this kind of operations 'corrective completions' (*ergänzende Berichtigung*, SELZ 1913,

p. 267). As with subsidiary operations, whether or not we have to do here with one 'composite solving method' remains open to question. Certainly the process and structure of the protocol become complex, but in order to speak of a 'composite solving method' one would like to require the subject at least to realize its possible complexity beforehand. Since this is primarily a matter of definition, we shall not now discuss the issue any further.²⁶

4. We already know that in Selz's conception the complete thought process is a sequence of operations resulting from strictly lawful combinations and linkings – either cumulative or subsidiary – of solving methods. The question of which solving method will be actualized at a certain point in the process depends, first, on the system of general linkings in the subject and, second, on a number of strictly definable cues in the current stage of the problem situation. There is no room for probabilistic processes like associationism's 'competition of reproduction tendencies' nor for haphazard 'trial and error' – how the latter differs from (the method of) 'trying out' will be discussed in detail in Section 56.

Selz himself formulated his ideas on the strict deterministic order of directed thought processes in somewhat different words. A particular solving method within the repertoire of a certain subject will only be actualized if:

(1) there is a corresponding *determination* – either immediately originating from the goal-setting (schematic anticipation) or originating from a chain of 'consecutive determinations' (cf. Section 18, p. 56);

(2) there is a *stimulus* in the present situation – again either in the original problem situation or in a problem situation in some later phase of the process brought about by the preceding operations.

It should be clear from the foregoing that these Selzian 'stimuli' have very little in common with the perceptual or imagerial stimuli that were theoretically used in associationism. 'Stimuli' that lead to the actualization of a certain solving method may be highly abstract – and rather complex.

26 It is interesting to note that KÖHLER's (1917, transl. 1925) criterion for a solution process based on 'insight,' the criterion of a rather suddenly begun but then gapless, closed, continuous process, appears to require a truly composite solving method composed solely of complementary submethods (suboperations). From a machine point of view the main question is, of course, what kinds of subroutines for corrections, completions, and detours should be written into a particular program.

Even so, Selz's general conception of the dynamics of thought is quite analogous to the laws of 'reflexiology.' It is not surprising, therefore, that Selz tried to combine some of the notions of his *Denkpsychologie* processes with those of the early (1920) reflex psychology.

In particular, he regards the psychological reflex proper as a subspecies of what he calls the *Reflexoids*. Reflexes are inborn specific reaction dispositions: *inborn 'reflexoids.'* 'General linkings' between solving methods and specific problem- (goal-, schematic anticipation-, partial result-) situations are acquired specific reaction dispositions: *acquired 'reflexoids.'* All motor and intellectual achievements of man are to be considered a result of systems of partly inborn but largely individually acquired reaction dispositions, each of which is linked to specific (types of) situational stimuli – provided the word 'situational' includes the subject's inner situation at any stage of the solution process.

In a current terminology one could summarize: All achievements of man are based on well-organized programs, general and specific and, in principle, specifiable.

Section 21: Phase structure

It follows from the above that we can expect to find a *phase structure* when analyzing productive thought processes as soon as we start to consider somewhat more complex tasks. As a matter of fact, such a structure has been reported by all investigators who have worked with protocols of actual thought processes. Even in the solutions to Selz's relatively simple tasks where the time required may be no more than some fraction of a minute, phases can often be clearly distinguished. As with the concept of schematic anticipation, the principles ('laws') that govern the combination of solving methods and therefore the structure of the thought process itself can be interpreted and applied both 'macroscopically' and 'microscopically' with equal success.

If we take, for instance, the composition of a piece of music, it both can and has been shown that the process is started by a schematic anticipation of the goal-as-attained (the completed work). Often this initial schematic anticipation is highly abstract, that is, it includes hardly any musical notes (BAHLE 1936 and 1939). From here on the subject's 'total goal conception'²⁷ develops continuously. Each solving attempt at the main problem or at subproblems results in certain completions, additions, or modifications, regardless of whether the

endeavor fails or succeeds. The total goal conception by which the completed goal (work) is anticipated – or, adopting Selz's terminology, the anticipatory schema implied in repeated goal-settings – is gradually filled out. The outcome of each solving method contributes to its completion.

A characteristic feature in this process is the *alternation and interaction between the anticipations*, on the one hand, and *the results of attempts at solving subproblems*, on the other. Bahle worked this out for the particular case of musical creation. There is no room for any doubt, however, that we have here a feature of general importance in productive thinking. The influence in one direction, namely, from anticipation to elaboration – compare the $a \rightarrow r$ transition discussed in Section 20 – is already well known from Selz's theory. In Bahle's investigations Selz's findings are confirmed although Bahle speaks not of 'schematic anticipation' (or of 'total goal conception') but of 'specific work problems' that the composer poses himself. Specific work problems can be shown to determine and direct the ensuing elaborative phase. On the other hand, the reverse influence is equally clear: the influence that outcomes of partial elaborations have on the total goal conception (de Groot), on specific work problems (Bahle), on the schematic anticipation that is to govern the next phase (Selz). Not only is the schema 'completed,' not only does the work problem for the composer 'specialize' and 'differentiate' itself, not only does the total goal conception become more and more 'detailed,' but also substantial changes other than completions may and often do occur. The structure of the goal conception and with it the schematic anticipation may change quite substantially and even radically under the influence of successful and, especially, unsuccessful elaborations. This 'interaction between whole and parts' as it is called by Bahle in his *Ganzheits-psychological terminology* is, of course, not only characteristic of musical creation but of all sorts of creative activity. Bahle named this interactive principle 'the principle of creative form-making' (*das schöpferisches Gestaltungsprinzip* – BAHLE 1939, p. 292).

27 For a discussion of this concept see Chapter VI, Section 44. In contrast to Selz's 'schematic anticipation' the 'total goal conception' is more *descriptive* of thought development than *explanatory*. Inasmuch as a schematic anticipation is supposed to evoke or to cause the next operational step at a certain point in a thought process, the schematic anticipation is a (dynamic, active) *part* of the total goal conception. Briefly, the 'total goal conception' of a subject *at a certain moment* in a process of thought or creation is the answer he would provide if interrogated at that particular moment about his total goal conception!

Thus by a macroscopic analysis of the composing process, a typical phase structure appears in which a clear alternation can be distinguished between elaboration of subproblems, on the one hand, and gradual completion – possibly restructuring – of the goal conception, on the other. Compared to the simple and short thought processes which Selz studied, the analysis of creative processes that extend over a much longer period of time affords a greater likelihood for substantial revisions (restructuring) of the total goal conception to show up. Also, in creative work the goal is self-set and not fixed as it is, for instance, in tasks involving the solution of a mathematical problem. It is hardly surprising, therefore, that it was Bahle and not Selz who explicitly drew attention to goal changes under the influence of outcomes of subproblem elaborations. Moreover, in musical composition the linkings – the subsidiary ones in particular – do not always function so immediately. Bahle reports frequent pauses in the creative activity. During such pauses (*schöpferische Pausen*) the active production is discontinued but the problem development is not. The composer's specific determination disposition is still strong, that is, easily evoked.²⁸ Meanwhile, his mind is more open to external influences and internal inspirations than in the ordinary, stressed working state of mind, so that coincidental means abstraction processes are strongly favored.

All of these phenomena, but briefly sketched here, will be discussed in more detail in the context of chess thinking.

²⁸ It is also often evoked between his other occupations ('distraction'); the composer mentally continues with analyses of work problems. Bahle argues strongly against romantic beliefs that call the composer's progress (after a pause) a complete miracle or that see it solely as a result of 'unconscious integration.' There are few fields in which unwarranted romantic views are more tenacious than the study of inspiration, musical composition, and creativity in general.

METHOD AND EXPERIMENTAL DESIGN

A. DISCUSSION OF METHOD

Section 22: Methods used by other investigators

In an old but still excellent exposition, JOSEPH GEYSER (1909) discusses in some detail the methodology of introspective investigations on thought processes. He draws attention to a fundamental difficulty which one inevitably encounters in such work, namely, the impossibility for the subject to think and introspect (report) at the same time. Some of the suggestions he gives as to how this problem might be optimally approached and some of his remarks and advice are just as valuable today as when they were written. Hence it seems worthwhile to bring some of Geyser's points to the fore again.¹

Geyser viewed *denkpsychological* experiments as a co-operation between experimenter and subject. The experimenter provides the stimulus materials, the experimental design, and has a certain theoretical aim in mind; the subject does not know the aim but gives his co-operation by providing 'protocols.' The systematic organization of the investigation makes it possible to avoid, to a certain degree, subjectivity and biased confirmation. The fact that the experimenter's theoretical aim is not known to the subject is essential in this respect. On the other hand, the subject cannot be expected to report on the relevant experiential phenomena unless he has some idea of what to expect and what to look for. Thus he should at least be given some instructions and some training in the right mental set, i.e., in the perception of certain categorically specified phenomena. In addition to his reporting, however, the subject has a problem solving assignment to carry out, so he must in some way divide his time and attention between the two. Since it appears to be impossible to do both things simultaneously, there must be some sort of alternation. According to Geyser the best way for the experimenter to avoid confusion is, first, to restrict the investigations to relatively simple and *elementary processes* of short duration, and, second, to have the

¹ For a discussion of the value of introspective techniques in general the reader is referred to Chapter IX.

subject report on his introspections *immediately after* the problem solving task proper is carried out. Thus the two processes are at least separated in time. However, to be reportable in the right vein, the thought process itself must take place in the shadow of the coming introspection, that is, in the right mental set. This is essentially what *a good instruction* should be able to achieve. In particular, the experimenter should select carefully which information must *not* be given to the subject in order to avoid bias in the design and, inevitably in that case, in the later interpretations of results.

This method of so-called *systematic introspection* was used practically without exception in the German investigations on thought and memory around the beginning of this century. Selz's experiments were designed according to the same system, with the notable difference that the tasks and the corresponding thought processes were often far from simple and elementary – one of the factors that accounts for the importance of his work.

As a matter of fact, the requirement that the task and the processes be elementary is the part of Geyser's exposition which in its generality is most antiquated. But the scientists around 1910 had not yet the courage, nor had they adequate methods at their disposal with which to investigate directly the more complicated processes. Moreover, it was considered somewhat superfluous to do so since every process was still thought to be a series of simple associative reproductions. A thorough understanding of the associative mechanisms, on the one hand, and of the specific effects of 'direction' (determining tendencies, constellation), on the other, was felt to be sufficient, in principle. Higher processes were not considered to differ in any fundamental way from simple reactions, only in 'greater complexity.' In brief, the hoped for creative synthesis (*schöpferische Synthese*) à la Wundt had not yet been given up.

This conception was not broken down until Gestalt psychologists, as well as Selz, started to criticize the elementalist notions and corresponding methods by experimental means. Gradually, both the necessity and the possibility for a direct experimental attack on intelligent behavior and higher mental achievements were realized. Köhler's famous experiments with chimpanzees (KÖHLER 1917, transl. 1925) and those of his followers with normal and backward children were instrumental in this development. One of his followers, Kurt Gottschaldt, remarked that what is needed for the investigation of higher functions is not *schöpferische Synthese* but rather *schöpferische Analyse*, that is, creative analysis with the complex purposive behavior

process as a whole as a starting point (GOTTSCHALDT 1933). Although this statement may harbor some Gestalt psychological exaggeration, it reflects an important change, in both object of study and method. More and more scientific psychologists were induced, at least for some time, to leave the somewhat overworked area of nonsense syllables and other laboratory-produced, artificial elements in order to study objects nearer to real-life. Selz himself is a case in point. Next to his laboratory experiments he developed methods for classroom experimentation and brought his investigations to bear on the practices of school learning and intelligence (SELZ 1929 and 1935; KINDLER 1929; SAND 1930).²

Some of the Gestalt psychologists also set out to grapple with directed thought processes. It is true that WERTHEIMER's well-known early study 'Über Schlussprozesse im produktiven Denken' (1920) did not yet contain much more than a number of highly interesting observations,³ but his pupil, Karl Duncker, carried out a thorough investigation of thought processes involving mathematical and practical problems (DUNCKER 1935, transl. 1945). Due to the nature of the problems that he chose for experimentation the duration of the thought processes of his subjects had to be rather long; therefore, he used quite a different method: he let his subjects *think aloud*. This deviation from the old methodology was an important novelty. By this relatively simple technique it appeared to be possible experimentally to investigate and systematically to analyze complicated, lengthy processes – *provided* the experimenter is content with a somewhat rougher, more macroscopic overall picture of the thought process.

It was Selz's pupil Julius Bahle, however, who went after the biggest game. He invented an experimental method to analyze the process of musical composition – or at least song composition (BAHLE 1936 and 1939). After some highly interesting laboratory experiments BAHLE (1930) sent eight carefully selected poems to a large number of well-known composers. He then requested them to put one or more of the poems to music and, additionally, to make daily protocols on the progress of their work. In order to facilitate the latter task and, in

² In this respect Selz had a definite influence in the Netherlands. KOHNSTAMM (1948) took up some Selzian ideas for use in school practice and experimentation, while his co-workers, in particular NIEUWENHUIS and PRINS (1939, 1941, 1951) experimented with various didactic methods on a *denkpsychological* basis.

³ See also WERTHEIMER (1945): again a stimulating contribution but certainly not an endeavor to theorize on the organization of thought processes as a whole.

particular, in order to furnish the composers with the point of view from which such an uncommon task had to be carried out, he enclosed an elaborate instruction (*Richtlinien für die Selbstbeobachtung*) as well as a questionnaire. The questions referred to the composer's motives in choosing a certain text, possible phases in his developing the composition, etc. Moreover, the composers were requested to turn over all sketches and notes which they had made during their work. As a result of the skillful choice of the poems in addition to the carefully worded questions and instructions – that were obviously adequate from both a psychological and a musical standpoint – Bahle's experiment by correspondence (*Fernexperiment*) became a great success. No less than 32 composers co-operated, together producing a wealth of valuable data. Some of the participants, it is true, could not be inspired by any of the eight poems, so they instead reported on a song composition using a text of their own choice. The materials collected in this way were compared with historical data from letters and biographies and worked into an integral conception of musical composition.

Bahle proved by his work that a systematic, experimentally based analysis is possible even in seemingly unapproachable areas of human creativity as long as an appropriate method of psychological experimentation is employed.

Section 23: Pros and cons of 'thinking aloud'

Our problem, the investigation of the process of solving the choice-of-move-problem in chess, is most nearly comparable with Duncker's work. The experimental positions were for the most part chosen in such a way as to require some 'real' chess thinking of the subjects; that is, the positions posed problems difficult enough to elicit thought processes of 10, 20, or even 30 minutes duration. For such processes retrospection is out of the question as a technique: one cannot reproduce the course of the process with any reliability as soon as the thinking time exceeds a few minutes. The only way of working with 'systematic introspection' would have been to interrupt the process after, say, every two minutes in order to have the subject introspect, and then continue. A few preliminary trials, however, with the author as subject showed this technique to be relatively ineffective as well as extraordinarily troublesome. After each interruption one feels disturbed and cannot continue normally. Apart from being unpleasant for the subject the technique is highly artificial in

that it disrupts the unity of the thought process. So it was discarded in favor of 'thinking aloud.'

At first sight it appears that by this decision more than one of GEYSER's (1909) warnings was thrown to the winds. Not only is the object of investigation far from elementary but, also, the distinction between thinking proper and self-observation, so essential for 'systematic introspection,' is abandoned. The latter problem is serious enough to warrant brief examination. How far do thinking and reporting on thinking interfere with each other if the subject has to 'think aloud'?

Duncker does not go into this question. He only emphasizes that 'thinking aloud' cannot actually be called introspection: 'While the introspector makes himself-as-thinking the object of his attention, the subject who is thinking aloud remains immediately directed to the problem, so to speak allowing his activity to become verbal. When someone, while thinking, says to himself, "One ought to see if this isn't..." or, "It would be nice if one could show that..." one would hardly call this introspection; yet in such remarks something is revealed which we shall later deal with under the name of 'development of the problem.' (DUNCKER 1945, p. 2) This means that the two tasks are not so strongly conflicting in thinking aloud as they are in systematic introspection.

Even so, verbalizing one's thoughts unequivocally adds *an extra burden* to the subject's task. On the one hand, the added instruction to think aloud necessarily influences the thought process to some degree; on the other, concentrated thinking on the problem itself must somewhat hamper its reporting. Duncker's conception of the issue seems a bit too optimistic.

The author's experiences on this two-way influence, gleaned from questioning the subjects, are condensed below:

1. *Deficiencies of a Protocol.* The expression and reproduction interferes, in fact, with thinking itself; thus the protocol does not represent the true course of thought.⁴ Evidently the most salient deficiency of the protocol is its *incompleteness*. 'A protocol is relatively reliable only for what it positively contains but not for that which it omits. For even the best-intentioned protocol is only a very scanty

⁴ It is difficult to define objectively the 'true' course. The above statement refers to opinions of subjects only: they did not *feel* that what they produced was identical with (what they felt to be) 'the true course.'

record of what actually happens.' (*Ibid.*, p. 4) The most important causes of this incompleteness are the following:

(a) Phases and/or steps in the course of the thought process remain *under the threshold* of the subject's awareness and are therefore not found in the protocol.

(b) Phenomena that come to consciousness – meaning that they could be recalled without difficulty if an immediate interruption were effected and systematic introspection required – may still be omitted. Quite often *thoughts move so quickly* that the spoken word cannot keep up with them. The subject is then either forced to skip steps or to deliberately slow down his thinking (if possible) which thereby disturbs the thought process (see below).

(c) *Not all thought is immediately verbalizable* thought. It is true that 'wordless thoughts' can sometimes be filled in afterwards by careful introspection, but they cannot be adequately expressed by the subject while thinking. The formulation would require an extra effort and more time than the subject can afford to spend given that he does not want to disturb completely his flow of thinking.

(d) Finally the subject may *intentionally suppress* one or more steps. For example, if he notices a mistake in a train of thought before rendering it verbally. Although the subjects were expressly instructed to report as fully as possible on the process as a whole and in no way to feel inhibited by the fear of possible errors in their thinking, this instruction was not always followed.⁵

All four mechanisms may lead to incompleteness in a protocol, quite apart from 'incompleteness' of supernumerary details on mental processes, such as the way in which an envisaged position is represented in the subject's mind (cf. Section 2). Information of this kind can never be expected from a thinking aloud procedure, nor was it sought. 'Incompleteness' refers to possible gaps in the reasoning process as it develops. On this basis, there are two criteria for (in)-completeness: first, the degree to which *the subject is satisfied* that his protocol is a rendering of his thought process; second, the *understandability*, to the experimenter and any other expert reader, of the

⁵ In a few cases the tendency to avoid possibly erroneous statements was clearly detrimental to the reliability of the protocol text. E.g., Subject G6 (in position C) for a long time abstained from any judgment; instead he produced a number of descriptive, 'literary' remarks on the combat on the board, comments that were fit to print – Tartakower was a well-known chess journalist – but non-committal from a chess player's point of view. The first part of this protocol is more the result of searching for nice covering up formulations than a report on the problem solving process itself.

protocol text, as a gradually developing argumentation leading up to a certain choice (cf. Chapter IX, Section 68). These criteria were in fact not always met, i.e., gaps did occur here and there.

Even so, in agreement with Duncker we can state that most protocols did give an acceptable *outline of the macro-structure of the process* from which the 'development of the problem' could be roughly read. Of course, some protocols were better than others, more complete and richer.

The reader of this book may judge for himself in how far the understandability criterion has been met (see Appendix II).

2. *Disturbing influence of thinking aloud.* According to most subjects the instruction to report all thoughts out loud had the effect of markedly *slowing down the thinking itself*. The verbal expression of what one thinks takes extra effort and time, particularly when movements on the chessboard must be specified. Since the setup – without a tape recorder – required that the experimenter be able to follow and jot down every variation the subject calculated, the subject could not get by by imagining the moves and squares as he would normally do. It is true that talking to oneself while thinking is certainly not an uncommon phenomenon for a player in an actual game, but then it goes something like this: 'If he does that, then I can do this; he takes; then I take; check...', etc. Obviously if talking in the experimental situation is done in terms of 'thats,' 'theres,' and 'thens,' it is not sufficiently informative. Therefore the subject was required to designate the moves algebraically or in some other unambiguous manner. Even to the most accomplished master such 'spelling out' means some extra effort. For less practiced players it can become very burdensome.

A second effect of the thinking aloud instruction, less commonly mentioned but undoubtedly significant, was that the subject was obliged to *think more explicitly*, at a higher level of conscious organization than he would otherwise do. The obligation to report must have had the general effect of furthering an active, organizing attitude in the subject and to some degree of preventing his thinking from being intuitive and receptive.

Understandably, the degree to which the instruction to think aloud was felt as an impediment varied among subjects. To a subject who habitually operates at a high level of conscious thought organization the added task is no more than a little additional inconvenience, whereas to the more intuitive type of thinker it is an unnatural

impediment to his concentration, i.e., to his habitual immersion in the problem.

Dr. Max Euwe was an extreme case of the first type of player. He did not feel strongly disturbed by having to think aloud nor by the artificiality of the experimental situation (cf. Section 24). He said, moreover, that his protocol gave a good, fairly representative picture of his thinking in actual play. Indeed his protocols were very rich, quite readable, and understandable – therefore probably complete. Salo Flohr, on the other hand, had great difficulty in following the added instruction. He found the formulation of his thoughts very difficult and several times tended to fall back on normal silent thinking. Under these circumstances the protocol must remain rather incomplete. Nico Cortlever, with whom the largest number of experiments was carried out, was of the opinion that thinking aloud markedly slowed down his thinking but for the rest hardly disturbed it. Like Dr. Euwe he felt that his protocols gave a rather good, representative picture of his actual thinking.

Although the chosen method certainly has deficiencies, all in all it can be said to answer the requirements of the research purpose.

In order to avoid one-sidedness and to check on possible blind spots in the thinking aloud procedure, other methods, closer to systematic introspection, were used in this study. In a number of cases there was an interruption after the first ten seconds with the intention of finding out somewhat more about the first Phase of the thought process. The subject was given due advance warning of this and asked not to concentrate on the interruption (and subsequent retrospection) but to consider it as an introduction to what still remained his main task: to choose a move and to think aloud. Further, a special series of experiments was carried out in co-operation with Nico Cortlever in which he and the author were alternately subject and experimenter. Less complicated positions were used in order to produce shorter thought processes that lent themselves well to classical retrospective procedures. These auxiliary techniques enabled the author to extend his analysis of the thought process and to check on and increase the value of his findings in the main series.

Section 24: Experimental versus real play situation

In the trials of the main series the experimental situation was made to resemble a match or competitive situation as much as possible.

The subject was expressly asked to regard the position as though it were his own and to decide upon a move in the same way that he would in a serious game. Nonetheless, and quite apart from the added instruction to think aloud, the psychological situation in the experiments was far from identical with that in a real game.

First, when the experiment begins the position is entirely *strange* (and new) to the subject – quite unlike the reality of chess play. The subject must first familiarize himself with a position for which he is in no way responsible. In the experimental thought processes there is, as a result, a ‘first Phase’ of a special and interesting but in fact artificial character. Even later on in the process some indications of unfamiliarity may remain. Throughout the process the subject is less ‘at home’ in the position presented and *less involved* in the choice-of-move-problem that must be solved.

The *previous course of play* is also unknown; Paul Keres said: ‘One does not know what kind of plans he had before.’

T. D. van Scheltinga in position 1...Q-K2, line 87, said:

87 ‘Really the whole thing is to know what Black has been aiming at. The plan, I should like to know his last move.’

However well-intentioned, the subject cannot identify as much with this problem as with a position built up by himself. Again Keres expressed it: ‘One is not so interested in a position that one has not developed oneself.’

Related to this is the *amount of energy* the subject is prepared to expend in solving the problem. The motivating factors are different in any case: the vital will-to-win is replaced by the willingness to participate in an experiment and the determination to solve a problem set by the experimenter. The most important stimulating factors are the desire to cut a good figure and the spontaneous interest aroused by the position itself. Anyone who knows chess will realize that these motives cannot compete with the personal drive that a player can have in a competitive game. For the grandmasters, in particular, the problem situation was not challenging enough for them to marshal all of their thought energy, as they would in a tournament game. They did not want to and/or they did not manage to concentrate on the choice-of-move-problem for more than five to ten minutes. Noteworthy is the qualification at the end of Alexander Alekhine’s protocol of position A, line 40: ‘Well, in case of time *pressure* I would play 1. BxN/5.’ Implying: ‘In a serious game I should go to much more trouble.’ This is no comment on his willingness, for it may actually be

impossible for him in a cool, objective experiment to bring to bear the same energy which he exerts in a serious game.

The counterpart of this relative weakness of motivation in the grandmasters is the tendency of some of the less strong amateurs *to take the situation too seriously*. In spite of the experimenter's careful assurances that he was concerned only with studying thought processes and not with evaluating skill, some of the subjects had the feeling that they were 'taking an examination.' In other cases the subject remained convinced, in spite of the instruction, that a position had been placed before him that contained a hidden 'winning move' which he was supposed to but could not find. Then the result was that he had great difficulty in coming to any choice of move at all. Whenever such evidences of wavering appeared, the experimenter made some remark like 'your clock is beginning to run out.' This usually worked well. The effect of the admonition was similar to that of a glance at the clock in an actual game so that it actually reinforced the real play situation.

Thus, apart from the obligation to think aloud, the main factors that disturbed the likeness between the experimental and the real thought process were, in summary, the difficulty to identify with the position, the grandmaster's somewhat facile attitude towards the experiments, and the converse overserious attitude of some of the less strong players. On a closer inspection of the protocol materials, however, these disturbances cannot be said to have been seriously detrimental. Since we can safely assume that familiarization with the experimental situation diminishes the influence of these factors, it is of particular importance that some of the Dutch masters served as subjects for several positions. Their protocols do not show any systematically different contents from the bulk of others. Both from their comments and the author's own experience it appears that the experimental thought processes are quite analogous to the kind of decision making that goes on at the board.

Finally, widely different types of positions were included in the experimental series in order to avoid generalizations made on a one-sided or too restricted basis.

There remain a few external factors of actual chess thinking that could not be simulated in the experiments. First, the presence of the clock: the time pressure that may arise in practical play. Second, a player's standing in a tournament and his score in the table of results generally has some effect on the risks he is willing to take and consequently on his thought process. Third, the strength and playing style

of the opponent may be allowed for in the decision processes. All three of these factors were lacking in the experimental situation. This is hardly a serious limitation, however, since these factors are only of substantial importance in exceptional cases. For the rest they must be supposed to have but a negligible borderline influence. Even though chess be no science, the motivation underlying the player's choice of move is still formed by a backbone of objective, rational considerations independent of time, place, tournament situation, and opponent.

B. DESCRIPTION OF THE EXPERIMENTS⁶

Section 25: Subjects

The following subjects gave their much appreciated co-operation to the experiments:

Six grandmasters:

- G1: Paul Keres
- G2: Dr. Alexander Alekhine
- G3: Salo Flohr
- G4: Dr. Reuben Fine
- G5: Dr. Max Euwe
- G6: Dr. Savielly Tartakower

Four masters:⁷

- M1: Lodewijk Prins
- M2: Nico Cortlever
- M3: Samuel Landau
- M4: Theo D. van Scheltinga

Two lady players:

- W1: Mrs. F. Heemskerk (1939 Women's champion of the Netherlands)
- W2: Mrs. C. Roodzant (1938 Women's champion of the Netherlands)

Five strong Amsterdam *hoofdklasse* players (roughly corresponding to experts in the United States Chess Federation rating system):

⁶ See also Chapter VIII for a description of supplementary experiments.

⁷ A protocol of the American master and recent International Master Raymond Weinstein is included in Appendix II. The protocol, taken in the summer of 1961, has not been analyzed at all and is mainly presented as an example of a protocol obtained with a tape recorder.

W. Koomen (1935 champion of the Amsterdamsche Schaakbond)

J. A. J. Polak

J. C. Sterk (1938 champion of the Amsterdamsche Schaakbond)

M. B. Stodel (1937 champion of the Amsterdam secondary schools)

W. Tegelaar (1939 champion of the Amsterdamsche Schaakbond)

Five skilled players ranging from weak *hoofdklasse* (Class A on the USCF scale) to 'second class' (Class C) players, of whom three were psychologists:

Prof. H. C. J. Duijker

Dr. M. J. W. de Groot

Prof. H. W. Ouweleen

J. Verheek

Dr. E. S. van der Vleugel

Some of the Dutch masters co-operated in taking the role of the experimenter and in selecting positions. The author was then enabled to don the robes of subject M₅. Even though M₅'s protocols were classified with those of the other Dutch masters, care was taken to avoid a bias in the analysis of the thought process. The use of these personal protocols was restricted to illuminating the findings derived from other protocols.

From now on each subject will be denoted by a letter and a number. The numbers indicate nothing more than the chronological sequence in which the players served as subjects. The letters denote the playing strength of each group:

G = Grandmaster

M = Master

W = Women's champion

E = Expert

C = Class player

The names of the E- and C-players above are simply listed alphabetically.

Section 26: Chess positions

The positions given to the subjects were, with the exception of position P-B6, taken from actual games. A total of 34 positions of varying character were used, 17 of these (A, B, C and 14 other positions) in the 'main series' (cf. Section 28). A was used with 19 different subjects, B with six, and C with five. The other positions were used only once.

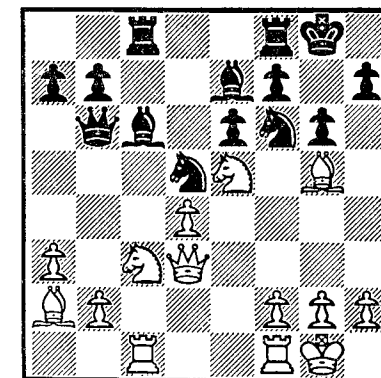
Position A (see diagram). Taken from a game between A. D. de Groot – C. Scholtens, Amsterdam, 1936. White is on move. (See Appendix I for the complete game.)

This position presents problems of a mainly tactical nature. Through his last move (...Q-N₃) Black has created a 'hanging position' for his Bishop on K₂: it is defended only by the exchangeable Knight on Q₄ so that the Black Knight on B₃ is somewhat tied down. There are all sorts of exchange possibilities in the center, and the question is whether or not it is possible for White to profit from the tactical weaknesses in Black's position. If no such possibility should exist, White could best strengthen his position with some calm move.

From a thorough analysis, however, it appears that White is in a position to get the better of it; there is even a forced win. The winning move is 1.BxN/5. After 1.BxN/5, 1...NxN is shown to be unplayable as follows:

(1.BxN/5), NxN?; 2.NxN with an attack on the Queen so that Black must immediately take back: 2...PxN; 3.BxB winning a piece. (White must not choose the wrong order to exchange on Q₅, however. If 1.NxN, then 1...NxN follows and the tactical tension in the center is dissipated. E.g., 1.NxN, NxN; 2.BxN, and Black saves himself through 2...BxB/N.)

Nor does recapturing with the Bishop help after 1.BxN/5: (1.BxN/5), BxB; 2.BxN, BxB; 3.NxB, PxN; 4.N-Q₇ winning the exchange.



POSITION A

White on move

Thus Black must recapture with the Pawn: (1.BxN/5), PxN. But now the Knight on KB₆ is immobilized and the Bishop on QB₆ no

longer has an open diagonal. White can, for instance, continue with 2.Q-B3 and have a superior position.

For the grandmasters these considerations were already sufficient to play the move 1.BxN/5; further analysis was superfluous. The move does lead to a forced win, however, as can be shown with the following variants:

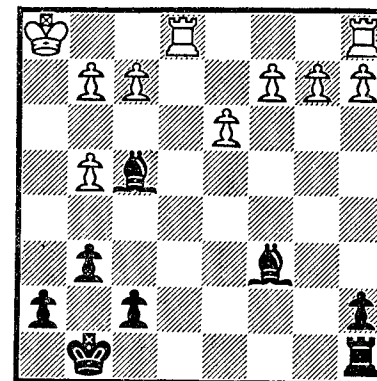
1.BxN/5, PxB; 2.Q-B3, Q-Q1 (or 2...K-N2; 3.N-N4, NxN – on 3...Q-Q1 follows 4.B-R6ch etc. – 4.BxB, KR-K1; 5.B-B5 and 6.QxN); 3.QR-K1! Now Black still cannot free himself of the pin, for after 3...N-K5; there follows 4.B-R6! N-N4!; 5.Q-N3, R-K1; 6.NxB, PxN; 7.BxN, BxB; 8.RxRch, QxR; 9.QxB winning a piece. And on other Knight moves 4.BxB and 5.NxB follow, also winning a piece. Therefore Black must do something else, e.g. 3...R-K1. But then the Knight on B6 can no longer move so that White can quietly strengthen his position, for example, with 4.R-K3 and KR-K1. Or he can even wind up immediately with 4.NxB, RxN (on 4...PxN follows 5.RxB and 6.BxN); 5.RxB (anyway!) QxR; 6.NxP, NxN; 7.BxQ, NxQ; 8.P-Q5 and wins.

The position is thus 'objectively solvable'; the analysis bears out that White can win. This does not mean, however, that the player at the board is able to find the forced win. As a matter of fact the relevant variations are not easy to find and are rather deep. On the other hand a complete analysis is not needed in order to decide on the choice of the best move, 1.BxN/5. For most of the less prominent players the real difficulty did not lie in the depth of calculation but rather in thinking of seriously considering a move that exchanges the 'strong,' 'attacking' Bishop on R2 for a Knight. In many of the protocols of weaker players the move 1.BxN/5 is not even mentioned.

Position B (see diagram). Taken from a game between J. M. A. Wind-A. D. de Groot, Utrecht, 1935. Black is on move. (See Appendix I for the complete game.)

This is an endgame position which is in many respects the counterpart of position A. Black (on move) has a serious problem to solve. But this time it is more of a strategic problem: he has to set up a plan for the future course of play. The White Pawns are dangerous and their advance must be prevented either by a counterattack on the White King or by some effective blockading defense. Black's next two moves – both of which were required in position B – should both fit into a long range plan that must be decided on in the present position.

An objective solution of this position cannot be given. It is probable, however, that a King's side attack is preferable; for example, in the following way:



POSITION B

Black on move

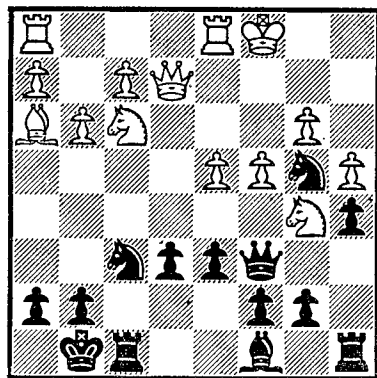
1...R-N1; 2.P-N3, R-N4; 3.P-B4, R-N4; 4.P-B3, P-KR4; 5.PxP, RxPch; 6.K-N1, B-K4; 7.QR-Q1, B-Q5ch; 8.K-B1, R-R8ch; 9.K-K2, R-R7; etc. This variation only serves as an illustration of further possible developments; in the protocols numerous other variations are to be found. The attack with the Rook and the two Bishops is stronger in any case than one would think at first sight. Compare, for example, the following cute variation: 1...R-N1; 2.P-N3, R-N4; 3.R-K7, R-N4; 4.P-KB3, P-KR4; 5.RxRP, PxP; 6. R-R6, PxP!; 7.RxB, R-R4ch; 8.K-N1, B-K6ch; 9.K-B1, R-R8 mate. White must obviously play with care.

An alternative execution of the same idea is to play 1...K-N2, followed by 2...P-KR4, as was done by subject C5 and considered by some of the other subjects. Less strong is the immediate 1...P-KR4 or 1...R-N1 and 2...P-KR4, as was played by M4. It is impossible, however, to produce a complete analysis. No really advantageous solution for Black can be pointed out. On the contrary: Black should be happy with a draw.

Like position A, position B appeared to be well chosen for the experiments. It represents, so to speak, a turning point in the whole game: Black has to make up his mind about the strategy to follow. In practically every game such crucial situations do occur at least once. 'Planning a strategy' belongs to the typical mental activities that should be (and will be) studied in an investigation of chess thought.

Position C (see diagram). Taken from a game between Dr. J. H. Pannekoek-A. D. de Groot, Santpoort, 1935. Black is on move. (See Appendix I for the complete game.)

This position has an entirely different character from A and B. Black has on K3 an embarrassingly weak Pawn that hampers him in his movements and forms a good target for the White pieces. This strategic disadvantage is somewhat compensated for by the 'tactical weakness' of White's Knight on the open KB file and the somewhat opened up position of the White King. Black has several ways to try to make use of these minor weaknesses. First moves worth considering are: 1...N-K5 (opening up the KB file and threatening 2...NxBP); 1...Q-K5; or, in an endeavor to force a solution, 1...P-K4. It would seem that the latter move is the best one but it is very difficult to find the correct continuation over the board. It is small wonder that none of the subjects succeeded here, although some of them did consider and even played 1...P-K4 (but for the wrong reasons!).



POSITION C

Black on move

Black can free himself as follows:

1...P-K4; 2.BxB, QRxB!; and now I. 3.N-R7, Q-K5; 4.QxQ (on 4.KR-K1, there follows 4...QR-K1; 5.PxP, QxQ and 6...N-N5, just as in II), NxQ, 5.NxR, RxN/6!; 6.PxP!, RxBP and Black even has winning chances. The game actually went on as follows: 7.N-K7ch, K-R1; 8.N-Q5, N-R7ch; 9.K-N1, PxP; 10.QR-K1!, N/7-B6ch; 11.NxN, NxNch; 12.K-B1. Drawn.

It might be stronger, however, to accept the other sacrifice: II. 3.PxP. But now follows: 3...Q-K5!; 4.KR-K1, QxQ; 5.RxQ,

N-N5!; 6.PxP! (the only chance), RxN; 7.PxP (or 7.P-Q7, R-Q1; 8.R-K8ch, R-B1; 9.NxP, NxBP!; 10.RxRch, KxR; 11.N-K6ch, K-K2; 12.NxR, KxN), N-Q6ch; 8.K-B2, N/5xBP or 8...N/5-K4 and apparently Black has nothing to fear anymore. In case White does not choose to embark upon such possible complications he can, not disadvantageously, play 6.N-K1; on the other hand 6.N-K1 presents no dangers for Black either.

The other possible first moves, in particular 1...N-K5, may lead to analogous complications. It is very difficult to handle the position correctly. In the protocols it appears that many more calculational errors are made in the analyses of position C than in A and B – probably a consequence of the somewhat atypical tactical complexity of position C.

The other 14 positions of the main series (and the four special positions of the 'combinatorial tasks' – see Section 28) are written out below. Those positions with diagrams elsewhere in the text are so indicated.

Position R-Q1 (See diagram on p. 237) (White on move).

White: K = KN1; Q = Q4; R = KB1; N = K3; Ps = QR2, Q5, K4, KB3, KN3, KR2.

Black: K = KN1; Q = QB2; R = QB4; N = QR4; Ps = QR2, QN3, K2, KB2, KN3, KR4.

Position ...N-B3 (See diagram on p. 279) (Black on move).

White: K = KN1; Q = QN3; Rs = QR1, KB1; Bs = QB1, QB4; Ns = QR2, K5; Ps = QR4, QN2, Q4, K3, KB2, KN2, KR2.

Black: K = KN1; Q = K2; Rs = QR1, KB1; Bs = QB2, K5; Ns = QN1, KB3; Ps = QR2, QB4, K3, KB2, KN2, KR2.

Position ...Q-K2 (See diagram on p. 249) (Black on move).

White: K = QB1; Q = Q3; Rs = QB2, KR1; B = QN3; N = Q2; Ps = QR2, Q5, K4, KB3, KN5, KR2.

Black: K = KR2; Q = Q2; Rs = QR4, QN5; B = KN2; N = Q3; Ps = QB2, QB4, K4, KB2, KN3, KR4.

Position ...Q-K4 (Black on move).

White: K = KN1; Q = QN5; Rs = QB1, Q1; B = Q2; N = QB3; Ps = QR2, QN2, K3, KB2, KN2, KR2.

Black: K = KN1; Q = QB2; Rs = QR1, KB1; B = KN2; N = QR3; Ps = QR2, QN2, K3, KB2, KN3, KR2.

Position ...N-K1 (See diagram on p. 206) (Black on move).

White: K = KN1; Q = Q3; Rs = QR1, KB1; Bs = QR3, KN2; N = QB3; Ps = QR4, QN3, QB4, Q5, K3, KB2, KN3, KR2.

Black: K = KN1; Q = QB2; Rs = QN1, KB1; Bs = QN2, K2; N = KB3; Ps = QR3, QN3, QB4, Q3, K4, KB2, KN2, KR2.

Position K-N2 (See diagram on p. 247) (White on move).

White: K = KN₁; Q = KB₃; Rs = K₁, KB₁; Bs = Q₂, KR₃; N = QB₃; Ps = QR₄, QN₃, QB₄, Q₅, K₄, KB₂, KN₃, KR₄.

Black: K = KN₁; Q = Q₁; Rs = QR₂, K₁; Bs = QR₃, KB₃; N = KB₁; Ps = QR₄, QN₃, QB₄, Q₃, K₄, KB₂, KN₂, KR₂.

Position N-Q2 (White on move).

White: K = K₁; Q = Q₁; Rs = QR₁, KR₁; Bs = QB₁, K₂; N = KB₃; Ps = QR₂, QN₂, QB₂, Q₅, KB₂, KN₂, KR₂.

Black: K = K₁; Q = Q₁; Rs = QR₁, KR₁; Bs = Q₃, KN₅; N = QN₁; Ps = QR₂, QN₂, QB₃, KB₂, KB₃, KN₂, KR₂.

Position N-N6 (White on move).

White: K = KN₂; Rs = Q₄, K₂; N = Q₇; Ps = QN₄, QB₂, QB₅, KB₂, KR₃.

Black: K = KN₁; Rs = QN₇, KB₂; B = KB₅; Ps = QN₂, QB₆, KN₃, KR₂.

Position ...P-N4 (Black on move).

White: K = KN₂; Q = K₂; Rs = QN₃, K₁; B = Q₃; N = Q₄; Ps = QR₅, QN₄, QN₅, K₅, KB₂, KN₃, KR₂.

Black: K = KN₁; Q = QB₂; Rs = QB₁, QB₈; B = Q₂; N = K₂; Ps = QR₂, QN₂, Q₄, K₃, KB₄, KN₂, KR₂.

Position B-R7ch (See diagram on p. 210) (White on move).

White: K = KN₁; Q = QB₂; Rs = KB₁, KR₃; Bs = QB₁, Q₃; Ps = QR₂, QN₂, QB₃, KN₂, KR₂.

Black: K = KN₁; Q = Q₁; Rs = QR₁, KB₁; Bs = K₂, K₃; Ps = QR₂, QN₂, QB₄, Q₄, KB₃, KN₂, KR₃.

Position ...P-QR3 (See diagram on p. 242) (Black on move).

White: K = KN₁; Q = QR₄; Rs = QB₇, KB₁; B = QR₂; N = QN₅; Ps = QR₃, QN₂, Q₄, K₃, KB₄, KN₂, KR₂.

Black: K = KN₁; Q = QN₃; Rs = QN₁, K₁; B = KB₁; N = Q₂; Ps = QR₂, QN₂, Q₄, K₃, KB₄, KN₃, KR₂.

Position ...P-R5 (See diagram on p. 243) (Black on move).

White: K = QB₂; Q = KB₄; Rs = KR₁, KR₆; N = QB₃; Ps = QR₂, QN₃, QB₄, Q₅, K₄, KB₂.

Black: K = KN₁; Q = Q₂; Rs = QR₁, KN₂; N = K₄; Ps = QR₄, QN₂, QB₂, Q₃, KB₆, KR₂.

Position P-B4 (See diagram on p. 180) (White on move).

White: K = QN₁; Q = KN₇; R = Q₁; N = QN₅; Ps = QR₃, QN₂, QB₂, KN₃, KR₂.

Black: K = K₁; Q = QN₁; R = QB₁; B = K₃; Ps = QR₂, QN₂, Q₄, K₅, KB₄.

Position Q-Q1 (White on move).

White: K = KN₁; Q = Q₂; Rs = QB₁, KB₂; Bs = K₃, KN₂; N = QB₃; Ps = QR₂, QN₂, K₄, KB₅, KN₃, KR₂.

Black: K = KR₁; Q = QR₄; Rs = QN₁, KB₁; B = KN₂; Ns = QB₃, Q₅; Ps = QR₂, Q₃, K₄, KB₃, KN₃, KR₂.

The following positions do not belong to the main series but were used in the special series of 'combinatorial tasks' (see p. 98).

Position P-KN4 (White on move).

White: K = KN₁; Rs = QR₈, Q₁; B = QB₁; N = K₄; Ps = QN₂, KB₃, KN₂, KR₂.

Black: K = K₁; Rs = KB₄, KR₁, B = K₂; N = Q₁; Ps = QN₄, QB₂, KB₂, KN₂, KR₂.

Position ...RxPch (See diagram on p. 243) (Black on move).

White: K = KN₄; Q = QR₇; R = KB₂; B = Q₂; Ps = QR₂, QN₂, QB₃, KB₄, KN₂, KR₂.

Black: K = KN₁; Q = Q₆; Rs = K₁, KN₈; Ps = QB₄, Q₃, KB₂, KN₄, KR₃.

Position P-B6 (Endgame study: White to play and win).

White: K = KB₃; N = Q₇; Ps = KB₅, KR₆.

Black: K = QR₆; B = QN₇; Ps = QR₄, K₂.

Position R-K2 (White on move).

White: K = KN₁; Q = QN₇; R = K₁; B = QN₄; Ps = QR₂, K₃, KN₂, KR₂.

Black: K = Q₁; Q = Q₆; Rs = QB₁, KR₁; Ps = QR₂, QB₂, KB₄, KN₂, KR₂.

Section 27: Place and Time

In order to profit from the presence of several foreign grandmasters in the Netherlands at the time of the A.V.R.O. tournament, the experiments were begun in the fall of 1938. They were continued aboard the ships that transported the chess players to and from the Tournament of Nations at Buenos Aires in 1939. After that, the author was compelled to put aside this work for several years because of other duties until, in 1943, a new series of experiments was begun with Dutch players. Even then the experiments were far from the ideal of a continuous series of sessions at a fixed place. Both the wishes of the subjects and the author's own schedule had to be taken into account so that often the availability of half a day a month for experimentation was considered fortunate. All in all the times and places of the different experiments were quite varied. Protocol (G₃; A) was produced in a room in the Amstel Hotel at 11:00 a.m.; (M₂;...N-K₁) at 10:00 p.m. on the Atlantic Ocean on board the 'Copacabana'; (G₆; C) in the harbor at Weymouth at the end of October, 1939, where the ship had been taken for contraband inspection. The protocol (C₄; P-KN₄), on the other hand, was one of the few that was produced in the calm atmosphere of the Psychology Laboratory of the University of Amsterdam.

All in all, these circumstances were, of course, far from ideal. However, the differences should not have caused a seriously disturbing influence. After all it seems unlikely that the general laws governing the thought processes of a chess player are so very different at the equator and the North Pole or are different during the day and night.

Section 28: Instructions and experimental conditions

1. The main series: thinking aloud.

The subjects for the main series were instructed as follows:

'In a moment I shall show you a position taken from an actual tournament game. You get the side of the player on move. You are requested to think of a move and then to play it on the board as if the position had arisen during one of your own tournament games. In addition, however, I should like you to do all of your thinking out loud, insofar as is possible, so that I can follow and record the way in which you arrive at your move. The idea is for me to be able to study the thought process as it develops, so no special achievement is expected from you. Please voice everything that comes into your head regardless of its correctness. I should like to follow the course of your thought, so the unsuccessful tries and variations that you may later find faulty are just as important to me as the correct ones.'

As soon as the subject appeared to have understood what was expected of him, he was asked to turn his back long enough for the experimenter to construct the position on a normal chessboard with regulation size chessmen. When the experimenter was 'ready' the subject was told to turn around and the experiment could start. The subject was seated behind the board in the usual manner while the experimenter sat across from him like an opponent.

If the occurrence of an abnormally long pause seemed to indicate that the subject had temporarily forgotten the instruction of thinking aloud, the experimenter reminded him of this task by asking: 'What are you thinking now?' or something equivalent. For the rest the experimenter did not interfere with the course of the thought process. The experiment was not considered finished until the subject actually made a move; thus such statements as 'I would play this' or 'I play such and such' were insufficient. As a matter of fact quite a number of subjects repeatedly made such remarks during the thought process without, however, playing the move mentioned! Such utterances were not considered as proof that the decision had really been made; as in a real game the requirement for the closure of the experiment was the execution of the move on the board. After that, the experimenter often asked questions on certain parts of the protocol that were not quite clear to him. In a few cases the subject and experimenter went through the whole protocol together.⁸

⁸ As a matter of course, the experimental design would nowadays include a tape

2. Interruption after 10 or 15 seconds

In order to get some specific data on the interesting first Phase of the thought process, the procedure was varied in eight of the experiments from the main series: the thinking aloud period was preceded by 10 or 15 seconds of silent looking and thinking, followed by introspection. The instruction in these cases was as follows:

'You need not think aloud now, but after 10 (or 15) seconds I shall ask you to report chronologically, as precisely as you can, on your thoughts during this time. I will want you to tell me then what you have seen, what has struck you, what you think of the position, etc. However, please do not especially prepare yourself for the interruption. Just start looking at and thinking about your position as you would if it was your move in a tournament game.'

After the 10 or 15 seconds the experimenter said 'Stop' and requested the subject to close his eyes or to look in some other direction. Also he was expressly asked not to continue with 'blind' analysis of the position or, if he could not prevent himself, at least to indicate clearly when he crossed the border from the report of what he saw during the exposure time into his later analysis. The subject's reporting was, in part, stimulated by means of such questions as: 'Is there anything that struck you?'; 'Have you seen any possibilities for action?'; 'Have you any judgment on the position?'; 'Who is better off?'; 'Have you seen or considered any specific moves?'; 'Do you know, without counting, the material situation?' Finally, at the end of his reporting, the subject's knowledge of the position was checked: he had to try to dictate the position from memory.

Actually, the subject was not required to give a very detailed psychological introspection. Reporting on the process in understandable chess terminology was sufficient. But this experiment differed from the generally used procedure of thinking aloud in that not only could the relatively slow, consciously organized investigations be reported but also, to some degree, the results of perceptual processes ('seeing' relationships and possibilities on the board). Since perception predominates the first Phase, it is here that a more refined method was especially needed. Moreover, an interruption is much less disturbing in this Phase than later on in the thought process when the subject is engaged in actual analysis. For this reason interruptions were not made during later moments of the thought process.

recorder; these instruments were not yet available in Holland in the years 1938-1945, however. Having a tape recorder also helps to get rid of incomprehensible passages, since the subject can be expected to be quite willing to listen to and clarify his own words afterwards.

3. *Two moves required*

The first Phase is quite interesting in itself, but its experimental evocation is not the same as it is during the thought process from an actual tournament game. In order to have available for psychological analysis a few protocols in which there was no such artificial first Phase, the subject was required to play two moves in the experiments with position B; that is, as soon as the subject made his move in the first part, the experimenter immediately replied with the opponent's move, and the experiment continued with the new position. As a matter of course, the subject was informed at the start of the experiment that this was going to happen.

4. *Short processes*

A series of rather randomly selected positions in which the problems were less complex and difficult offered a good opportunity to strive systematically for short thought processes in which the subject would be able to give a reliable report *post factum*. Therefore, in these positions the subject was forced to decide quickly on his move as though he had only a little time at his disposal. With several of the positions from this series there was no fixed time, but the subject was expected to play his move within about one or two minutes; with the rest of the positions he was asked for his move after 20 seconds. Both instructions appeared to be quite workable – with a chessmaster subject – and the protocols that resulted from this specific experimental setup form a valuable complement to those of the main series. In the one to two minutes experiments the subject was required to think aloud and also to report after the process while in the 20 seconds experiment his main task was, of course, the report following the move decision.

5. *Combinatorial tasks*

In this subseries of experiments the author himself was the only subject. Here the deviation from the experimental method of the main series consisted in the fact that the positions were 'objectively solvable,' and the subject knew it. So the task was always: 'White (Black) to play and get an advantageous position by means of a combination,' or even 'White to play and win,' as with an endgame study. The main reason for including these experiments was to explore the degree to which such a specialized task influences the structure and dynamics of the thought process.

THE EXTERNAL STRUCTURE OF THE THOUGHT PROCESS

Section 29: Introduction to the analytical part

The second part of this book, consisting of Chapters IV, V, and VI, treats the main results of what can be called the structural analysis of the protocols. For a reasonably complete descriptive analysis it was necessary to analyze the complete set of protocols of the main series from many different viewpoints. A consideration of length, however, prohibits the presentation of the complete evidence on which the findings are based. For that reason a somewhat less space consuming method of presentation is used: fragments of protocols serve as illustrations rather than proofs. Only a few protocols are treated in their entirety.¹ In order to provide the reader with some idea of the relative importance of the described phenomena, frequencies are often given. These frequencies refer to 43 protocols of the main series: 19 of position A, 6 of B, 5 of C, and 13 from other positions – (M5; Q-Q1) was produced after completion of these statistics.

All of the items discussed in the following pages must be regarded as typical for thought processes in chess – fully apart from the question of whether the phenomena are also characteristic of other forms of thought. Thus, for instance, the treatment of subproblems within the main part of the thought process, although based on a complete classification of problem and goal formulations from the protocols, is confined to cases or types of subproblems that appear in *at least five protocols*, not all of them from one subject nor from the same position. Such an objective criterion for the exclusion of atypical cases cannot, of course, preclude all subjectivity on the part of the author because the phenomena themselves must be classified and coded largely by his judgment. This judgment was based, first, on many years of personal chess experience; second, on common sense; third, on psychological and theoretical distinctions, terminology, and insights –

¹ Thirty-five protocols appear in this, the English text, in the Appendix.

all applied in a continuous endeavor to judge as objectively as possible. In a systematically descriptive study such as the present one in which no stringent guarantees exist against biases in the processes of observation, abstraction, and clarification, it would seem that this is about the upper limit of 'objectivity' that can be attained. The reader should keep in mind, for that matter, that the purpose of this study is not to present the results of hypothesis testing but rather to establish a conceptual framework and corresponding classifications – albeit of a somewhat preliminary nature – from which it is hoped that rigorously testable theories and hypotheses can originate.

A. THE EXTERNAL PHASE STRUCTURE

Section 30: The protocol (M2; B)

In studying protocols of chess thought processes (that are not too brief) the typical structure of the solution process in phases is one of the most striking features. This by itself does not lead to any strong corroboration of any one particular psychological theory – phases are distinguishable in even the most primitive trial and error processes. Nonetheless it is the newer psychological schools which have paid attention to the peculiar phase structure of thought processes.

Indeed, this point deserves much more emphasis than it has had thus far. The macroscopic structure of the whole is in fact one of the first things to consider in studying thought processes. It is not too difficult by way of experimentation to obtain concrete and relatively reliable information with regard to the structure of the process; the external form then provides important cues for hypothesizing on the inner, operational structure of the process, i.e., on the dynamics of the development of problem and solution, of rational choice and decision.²

First, a complete protocol, (M2; B), is set forth in order to exhibit in concrete form the different properties of the structure of the thought process.

² Also it would appear advisable to analyze and to develop schemata for different (types of) structures as they occur in various kinds of productive thinking, in order to have a concrete basis for comparison.

(M2; B) FIRST PART (M2; B1) S: NICO CORTLEVER E: DE GROOT T = 15 MINUTES
OCTOBER 17, 1939

- (S counts the Pawns.) Difficult: this is my first impression. The second is that by actual numbers I should be badly off, but it is a pleasant position. I can do a whole lot of things – as usual. Get my Rook into it, at the Pawns. Nowhere for his Rook to stand on the King's file, except on my K2. And that I can always prevent with K-B1. 1...K-B1 is impossible. If I, for example, play 1...R-N1; he 2.QR-N1, B-Q4 – then he can still get in. I might try to block his Pawns; then I can't win, but that won't be too easy anyway. Also seek something to do along the Rook's file: P-N4 – P-KR4 – K-N2 – R-R1. Then he must play something like P-KB3. Exchange of Rooks in general not good; must avoid that.
- The first move under consideration is 1...K-B1 to keep the Rook out. Then 2.R-K2 and double them. ...doesn't help matters either then. But if he then advances with his Pawns: P-B4, P-N4, or like that; then hard to stop them. Straight off, that won't work, after 2.P-B4 then R-Q1 and R-Q5 follow and I blockade him.
- 1...P-KR4, I don't like the idea of that very much. If immediately 1...R-N1, then 2.R-K7 is annoying. I should play 1...K-B1, or perhaps 1...B-KN4.
- Oh, no, 1...R-N1; 2.R-K7 won't do because of 2...R×P. Consequently he must reply 2.QR-N1 or 2.P-QN3. Then perhaps 2...B-Q4; but then comes 3.P-B4, so that is not so good. I do not stand well after all. He can always play P-QB3. Can I prevent that?
- 1...R-N1; 2.QR-N1 and now B-Q7-QB6. Or perhaps 1...R-N1 and then 2...K-B1; then he still cannot play P-N4, so maybe 1...R-N1; 2.QR-N1, K-B1; 3.P-QB3. No, doesn't suit me though. Maybe still something better; still on the King's wing:
- 1...B-KN4 or something like that, 2.R-K5, P-B3 – no, a touch of fantasy. 1...P-KR4 immediately, 2.P×P, P×P or 2...K-N2; no, then 3.K-N1 – not worth much either.
- 1...R-Q1 and 2...R-Q5 – no good either. 1...R-N1; 2.QR-N1, P-KR4; 3.P×P, then 3...R-N4; 4.K-N1 – or 4.P×P – R×RP; and 5...R-R7. Maybe not so crazy.
- 1...R-N1; 2.QR-N1, P-KR4; 3.P×P – 3.P-KB3 is not good, then the Rook comes to KN4 – therefore 3.P×P, R-N4; 4.P×P. Am I doing anything to him then? I also can immediately play 2...R-N4 and then P-KR4.
- The Bishop, can that do anything? 1...B-Q7; (2.R-K7), B-N5. A little slow. And the Bishops are well placed. Yes, a little slow.
- 1...P-QR4 – but that is nonsense. The Rook must do something; other moves are a bit too passive in that position. 1...P-QR4; 2.R-K7, R-K1 – Rook exchange; no, that is nothing. Yes, I begin with 1...R-N1.
- (E: Suppose now that the opponent plays 2.QR-N1 or 2.P-N3. Let's make it 2.QR-N1. What will you play next?)

(M2; B2) Second part, in the new position 1...R-N1; 2.QR-N1. T = 10 minutes

- Let's have a look:
2...B-Q7; 3.R-K7, B-QB6; 4.P-N3 R-N5. ... Ach, no, 3.R-K7 is bad. 2...P-KR4, what then? Or 2...R-N4. Then 3.R-K7, P-KR4; 4.P×P, R×Pch; 5.K-N1, B-R7ch and 6...B-Q3. Then 7.QR-K1 or something like that. If I do nothing, I have the idea

- 10 that I am gradually going to lose. To hold back the Pawns doesn't work so well. No, I must do something on the King's wing. Perhaps to play
 2...R-N5 – doesn't seem so strong to me. The Bishop on B5 must then first
 15 move. No, that stands well.
 No, look again:
 2...R-N4; 3.P-B4; then 3...R-N4 and I win a Pawn; unless 4.P-B3. Then
 4...P-KR4; 5.P×P, R×Pch and
 20 advance with the KN Pawn – not so bad.
 2...R-N4; 3.R-K7, P-KR4; 4.P×P,
- R×Pch; 5.K-N1, R-R7; 6.P-KB3, P-N4; 7.R×RP, P-N5; 8.R-KB1, P-N6, threatens mate! Not unpleasant.
 25 If he now defends the Pawn instead of 4.P×P, thus 4.P-KB3, then take twice on KN5 and R-N4.
 2...R-N4. Then also to offer a Pawn at KN4, in order to stop the attack (after
 30 P-KR4) – possible. But now I can take with the Rook; after P-KB3 I can then move the KR Pawn forward. That also gives good chances.
 Yes, I should play
 35 2...R-N4.
 36

Section 31: The first Phase³

A close look at protocol (M2; B) shows that before an actual attempted solution, wherein a possible first move is sought, a 'first Phase' takes place in which the subject orients himself as to the position (lines 1 to 9). He considers the material situation (line 1) and on the basis of that and the placement of the pieces, he appraises the 'value' of the position. The problem appears complicated to him (line 1), but nevertheless he finds it 'a pleasant position' (line 4). He rather broadly orients himself as to available move possibilities, notices a wide choice (line 5), and then mentions, still vaguely, a broad plan (lines 5 and 6). Next he briefly considers possible continuations by the opponent, first by the most active piece: the Rook (lines 6 to 9).

With this *the first phase in the narrower sense* is over. What follows already implies the first considerations of particular moves (1...K-B1 and 1...R-N1). These moves serve principally, however, as illustrations of the plan which is first clearly formulated in line 12, namely, to block the Pawns. Alongside this blockade concept (*plan B*) a second plan appears for the first time in line 14: 'Also seek something to do along the Rook's file,' (i.e., an attack on the King – *Plan A*).⁴ Since

³ A chess protocol can be divided into any number of phases and subphases. In Section 52 of this book, however, a specific interpretation is offered: the thought process is divided into four Phases of deepening: (1) the first Phase of orientation, (2) the Phase of exploration, (3) the Phase of investigation, and (4) the final Phase of proof. This ambiguity is resolved by introducing a distinction: a capital letter refers to the specific Phase structure of Section 52 and a small letter to the universal word 'phase.'

⁴ Indeed, position B is one of those chess positions in which a general plan of cam-

the whole of the thought process up to and including line 19 can be considered as part of the subject's orientation in and exploration of the problem, it is called *the first Phase in the broader sense*. In this first Phase the subject lets the position impress itself on him; the problem 'sinks in' and becomes structured for him.

Preparatory phases before the first legitimate attempted solution occurs have often been called phases of problem analysis. The term is also applicable here, but it carries with it the danger of misunderstanding. If one understands 'problem' to mean the *objective choice problem* in a given position, that is, the technical problem which when presented to a chessmaster can be objectively solved by a thorough analysis, then one can indeed maintain that it is being 'analyzed.'

In the present context, however, what is relevant is the psychological concept of 'problem,' the problem as it develops in the mind of the player. This *subjective problem*, that is, the player's conception of the objective problem at a specific point in time, can be said to form and concretize itself during the first Phase.⁵ Thus it is preferable to call the first Phase *the Phase of problem formation* or the *Phase of orientation* (cf. Section 52).

In the forms in which the first Phase appears in the protocols it is a somewhat artificial product. During the normal chess game the position changes gradually, move by move, so that in general the player already has a good idea of the objective choice problem. No renewed orientation is needed when it is his move. This does not necessarily mean that the first Phase disappears, only that it is shorter in duration. In protocol (M2; B2) above, in fact, the introductory remarks to the solution attempts are of the 'Let's have a look' (line 1) variety.

Section 32: Alternation of elaborative phases

It appears from protocol (M2; B) that the subject takes up a rather large number of possible solutions, one after another. After the first Phase of orientation every shift in the analysis from one move to a different *first* move can be considered as a transition to a new *solving proposition* or *solving attempt*, that is, a direct attempt to solve the choice

paign is required. As a matter of fact the competition between plans A and B runs throughout the rest of the solution process in (M2; B) and in other B-protocols as well.

⁵ In what follows, it is necessary to maintain a careful distinction between the concept of the *objective choice problem* and the *subjective problem* (cf. Section 38).

of move problem. Each such solving attempt is then worked through, more or less thoroughly, up to a point where the subject can come to a preliminary *result*, such as: 'no, a touch of fantasy' (line 52); 'not worth much either' (line 54); 'no good either' (line 56); 'Maybe not so crazy' (line 60); and so forth. Results are not formulated so clearly at every shift from one first move to another; sometimes no comments at all are made.

In any case, it is possible after the Phase of problem formation to describe the structure of the thought process in terms of discrete *elaborative move-phases* in which the various possible moves are successively taken up and analyzed. In almost all of the chess protocols examined the same kind of alternating elaborative move-phase structure is found. Generally speaking, the more difficult the problem is for the subject and the longer his thought process lasts, the more conspicuous is the move-phase structure.

Another feature of (M2; B), namely, the appearance of more than one elaborative move-phase devoted to the same first move, is a general characteristic in all of the thought processes of somewhat longer duration. In the present protocol, for instance, the move 1...K-B1 is examined in lines (9), 21, and 34; the move 1...R-N1 in lines (11), 32, 37, 44, 58, 62, and finally 78; the move 1...P-KR4 in lines 29 and 53; etc. From now on this phenomenon shall be called *re-investigation of the same solving proposition*.

The move-phase structure of a protocol can be schematically represented by a formula: *the formula of successive solving propositions*.⁶ Moves are represented by small case letters, the 'better' moves receiving letters nearer the beginning of the alphabet. Thus every shift from one first move to a different one shows up in the formula by the appearance of a new letter. Furthermore, whenever the analysis of a proposed move takes a long time, and the subject starts over from the beginning – e.g., 1...R-N1 (line 62) after the previous examination of the same move in line 58 – these 'fresh starts' within the analysis of the same first move are represented in the formula by a repetition of the same letter. That is to say that here too discrete elaborative move-phases are distinguishable.⁷

6 The term 'solving proposition' is used only in connection with moves. In analyzing protocols, however, it sometimes happens that subjects refer to vague ideas or plans, sometimes exclusively so (protocol (M2; K-N2) approaches this extreme); these are hardly to be called solving propositions but are most certainly attempts to solve the choice-of-move-problem. Therefore we use a more inclusive term, *solution proposal*, when we wish to be vague, that is, when the reference is to moves and/or plans.

Taking all this into account, the formula for (M2; B1) becomes:

(M2; B1): c – b – a – c – g – a – a – g – b – h – a – a – d – e – a
line: 21 29 32 34 35 37 44 51 53 56 58 62 69 72 78

where a = R-N1; b = P-KR4; c = K-B1; d = B-Q7; e = P-QR4; g = B-KN4; h = R-Q1.

The second part becomes:

(M2; B2): d – b – a – f – a – a – a – a
line: 2 4 5 13 17 22 29 36

where now a = R-N4; b = P-KR4; d = B-Q7; f = R-N5.

The move actually selected is italicized and does not count as a solving proposition. A closer analysis of the data that this formula can supply follows in Section 36.

There is, however, more to say about the inner structure of the thought process. A move possibility seldom stands by itself; it is almost always connected with a more general goal or plan that the player has in mind. If this is so, the exploration of a possible move is not an independent solving attempt but rather a possible means for realizing a more far-reaching 'board goal': for example, to obtain a direct material advantage (through a combination), 'to embark on a mating attack,' 'to force a break through,' 'to consolidate the Queen's side,' and so forth. Such more general goals are well known both from the chess literature and from discussions among chess players; insofar as they direct the strategy for a certain period one correctly speaks of *plans* and *planning*.

As has already been said, in the thought processes of the stronger

7 A 'fresh start' is rather easily picked out when a new move is mentioned. The only restriction is that the move must be a serious attempt to solve the choice-of-move-problem and not just the mentioning of a move in passing, as sometimes occurs in the first Phase or a transitional phase. It is, however, not so easy to operationalize the concept within the analysis of the same move, that is, within the same solving proposition. That the subject start again with the same first move is a necessary but not sufficient condition, for this can be simply a matter of convenience in a sequential investigation of variations. In addition there must be some indication that the subject's conception of the problem has changed. The relevant criteria for asserting the emergence of a fresh start are

- (1) an intermediate change in the general evaluation of the position;
- (2) intermediate remarks of a non-elaborative nature; and
- (3) a pause in the thought process.

In (M2; B1) obvious examples can be found in lines 44 and 63, where 'transitional phases' (see Section 34) are apparent.

players a conflict exists as to which plan of campaign should take preference in position B: either the attack on the King, plan *A*; or the blockade of the Pawns, plan *B*. If one traces this competition between plans in the protocol, it appears that the alternation of move-phases can in fact be viewed as a subdivision of the more roughly structured *A-B* plan competition. In (M2; B) this alternation begins in the second part of the first Phase with a clear exposition of both plans. In the course of the thought process both *A* and *B* are worked out in more and more detail. The alternation of plans and with it the continual return to the original ideas must not be construed as vacillation or wavering, however, since in the successive elaborations a *progressive deepening and broadening of the investigation* is apparent.

We are able to present the macro-structure of the first part of the protocol:

(M2; B1) MACRO-STRUCTURAL SCHEMA

1-19: First Phase of problem formation	1-9: First phase in the narrower sense		
	9-19: Continuation of first Phase; exposition of <i>A</i> and <i>B</i>	9-11: sample moves	$\left\{ \begin{array}{l} 9: 1...K-B1 \\ 11: 1...R-N1 \end{array} \right.$
20-76: Progressive deepening and broadening of the investigation	20-28: Plan <i>B</i> : Consideration of a defensive system	21-28: 1...K-B1	
	29-30: Plan <i>A</i> : First (superficial) consideration	29: 1...P-KR4	
	31-52: Plan <i>B</i> : Deeper and broader investigation	32: 1...R-N1	
		34: 1...K-B1	
		35: 1...B-KN4	
(58-76: Convergence towards choice)	53-55: Plan <i>A</i> : First calculations	37-48: 1...R-N1 (deeper examination)	
		51: 1...B-KN4	
	56-57: Attempt to combine <i>A</i> and <i>B</i>	53: 1...P-KR4	
	58-67: Plan <i>A</i> : Serious examination of the move	56: 1...R-Q1	
	68-76: Plan <i>B</i> : Elimination of passive moves	58-67: 1...R-N1	
77-78:	Choice of move	68-71: 1...B-Q7	
		72-76: 1...P-QR4	
		78: 1...R-N1	

The schema clearly shows the struggle between plans *A* and *B*. At first the subject has defensive plans: plan *B* is set out in the first Phase of orientation and is worked out in the following lines (9-28), while *A* receives little attention (lines 14-17). Later in the protocol the subject seems close to a *B*-decision: 'I should play 1...K-B1' (lines 33-34). Only much later is plan *A* seriously considered. The B1-part of the protocol is never decisive, for that matter, since the move 1...R-N1 fits into both systems (compare, e.g., lines 36-49 with lines 62-67).

The struggle is continued in the second part:

(M2; B2) MACRO-STRUCTURAL SCHEMA

1: First Phase	$\left\{ \begin{array}{l} 2-3: \text{Plan } B: \text{ blockade attempt} \\ 4-8: \text{Plan } A: \dots\dots\dots \end{array} \right.$	$\left\{ \begin{array}{l} 2-3: 2...B-Q7 \\ 4: 2...P-KR4 \\ 5-8: 2...R-N4 \end{array} \right.$
2-34: Explorations and investigations	$\left\{ \begin{array}{l} 8-12: \text{Plan } B: \text{ is eliminated} \\ 13-33: \text{Plan } A: \text{ deeper examination} \end{array} \right.$	$\left\{ \begin{array}{l} 13-15: 2...R-N5 \\ 17-33: 2...R-N4 \end{array} \right.$
(13-34: Convergence towards choice)	33-34: Plan <i>A</i> : wins out	
35-36: Choice of move		36: 2...R-N4

In comparison with the actual thought process this rough macroscopic outline is, of course, a somewhat artificial simplification. Both the organization and the complexity of the structure can be discerned from it, however.

In many cases a more refined division is necessary for an adequate description. Thus the analysis of 2...R-N4 (lines 17-34) consists of four different variations worked out consecutively; namely:

- 17-21: The continuation 2...(R-N4); 3.P-B4, etc.
Result: 'not so bad.'
- 22-25: The continuation 2...(R-N4); 3.R-K7, etc.
Result: 'Not unpleasant.'
- 26-28: The continuation 2...(R-N4; 3.R-K7, P-KR4); 4.P-KB3, etc.
Result: not verbally expressed but apparently also favorable.
- 29-34: The continuation (2...R-N4; 3.R-K7, P-KR4); 4.P-N5.
Result: 'That also gives good chances.'

Thus the elaborative phase of 2...R-N4 must be split into at least four *subphases*. Or by combining the first two and the last two subphases above, two intermediary subphases could even be formed:

lines 17–25 as the first intermediary subphase, lines 26–34 as the second. The first ‘new’ subphase consists of a first investigation with early counter branchings (i.e., the consideration of different moves by the opponent): 3.P-B₄ and 3.R-K₇. The second subphase is a further investigation but at a deeper level in the tree: the opponent’s three possibilities, (4.P×P), 4.P-KB₃, and 4.P-N₅, are consecutively considered. Here the question of the structure of composite calculations is touched upon; it will later be treated separately (cf. Section 41). For the time being, we shall not go beyond the division into four parts. Thus the structure for the *A*-phase under consideration (lines 13–34) becomes:

13–34: Plan A: deeper analysis	{	13–15: 2...R-N ₅	Result: -
		(1) 3.P-B ₄	Result: +
		17–34: 2...R-N ₄	Result: +
		(2) 3.R-K ₇ ; 4.P×P	Result: +
		(3) 3.R-K ₇ ; 4.P-KB ₃	Result: +
		(4) 3.R-K ₇ ; 4.P-N ₅	Result: +

Therefore 36: Choice of move: 2...R-N₄

If a more sensitive technique than that of thinking aloud could be used for examining the successive thought operations involved in the analysis of one such variation, an even more refined division of phases would be certain to appear. On principle, one should finally be able to reach Selz’s ‘reflexoidally’ linked microphases where, according to his theory, each subsequent operation is completely determined by the outcome of its immediate predecessor. The total structure would then become much more complex. The above example already illustrates clearly enough the high degree of organization of a chess thought process as it appears in the structure of a protocol.

As a matter of fact, such a structure of phases and subphases is certainly a general characteristic of all thought processes concerned with the solution of a difficult problem. In his chapter on composite solving methods SELZ (1922, p. 598 ff.) describes as one of the general procedures that are routinely available to experienced subjects the following series of operations: If the first attempt with solving method *A* does not lead to a satisfactory result, then there follows a second attempt in which *A* is used again but this time on new ‘materials.’ There may possibly follow a third attempt, etc., until the available materials are used up. At that point the alternative (or subsidiary) method *B* is brought in and a similar series may follow, in which the b-set of materials may or may not be identical with the a-set. In such a

situation the structure of the thought process comprises, of necessity:

Method <i>A</i>	{	Material a ₁	Result: -
		Material a ₂	Result: -
		Material a ₃	Result: -
Method <i>B</i>	{	Material b ₁	Result: -
		Material b ₂	Result: -
Etc.			

Obviously, this case is comparable to the progressive probing of different moves (or variations) in the framework of a plan, (e.g., plan *A*) before it is abandoned in favor of another plan (see lines 31–53 in (M₂; B₁)). If the subject is equipped with an internal program similar to what Selz described, a specific phase structure is certain to appear in the protocol. Conversely, the details of the phase structure of a protocol provide the main cues for finding the principles of organization that must be assumed to underlie the thought process. It is for this reason that a systematic and detailed analysis of the formal structure of a protocol is a matter of crucial importance.

Section 33: ‘Cumulative’ and ‘subsidiary’ linking

The result of the examination of a move or variation can be positive or negative, that is, it may or may not ‘satisfy the expectations.’ Leaving aside for the moment the question of where such expectations come from and precisely what meaning they have, we shall now see what the protocols can tell us about the influence of positive or negative results on the course of the thought process.

The first problem that may arise is that the subject does not always explicitly state any evaluation result. In such cases there remains some uncertainty as to whether the result is positive or negative – if we are correct in assuming that in the actual process there was any evaluation result at all. There are no a priori reasons to assume that with the examination of every move the subject has a precise expectancy level in mind which is either reached or not reached after a process of evaluating results. Even if we limit ourselves to move examinations that occur after the first Phase, they cannot always be considered solely as direct solution attempts. Some of the early move explorations in the protocols apparently serve another purpose as well: orientation to the dynamic aspects of the position (‘see what sorts of things can be done’). It is only in the later (‘investigative’) Phase of the thought process that expectancies become specific, evaluations explicit, and

results more or less precise. In fact, it is particularly in the later stages of the thought process that results are explicitly – and variously – expressed in the protocols. Thus, e.g., in (M2; B1):

- line 41: investigation of 1...R-N1 and 2...B-Q4 'not so good'
- line 48: investigation of 1...R-N1 and 2...K-B1: 'No, doesn't suit me though'
- line 54: investigation of 1...P-KR4: 'not worth much either'
- line 56: investigation of 1...R-Q1: 'no good either'
- line 60: investigation of 1...R-N1 and 2...P-KR4: 'Maybe not so crazy.'

With the first non-negative result ('Maybe not so crazy') the subject follows it up with a closer investigation of *the same* variation (line 62–67). In such a case we join with Selz in speaking of *cumulative* linking⁸ of subphases, or of cumulative linking of the underlying general methods:

If results of preliminary investigation of P are positive, continue with closer investigation of P.

The result of this second investigation of the move 1...R-N1, however, is not verbally expressed in our example; it remains a question: 'Am I doing anything to him then?' (line 65). At first sight it would seem to be negative, since the subject goes on to consider other moves – but that does not explain how the subject finally comes to play the move 1...R-N1 without analyzing its consequences any further. A better explanation of the dynamics of this part of the thought process is that the result was neither negative nor especially positive but rather: 'Not so bad. *If I find nothing better*, then I can play this.' As a matter of fact, formulations of this tenor do often occur in the protocols. Although such a formulation is lacking in this particular case, the interpretation appears warranted since it makes the rest of the protocol fully understandable; namely, what follows is a brief search to see if there really is anything better – which appears not to be the case.

In connection with Chapter I, Sections 9 and 10, it is possible to describe the same process in another way. By investigating 1...R-N1 the subject knows how much he can gain with that move. The *positive*

part of his argumentation is clear. What follows is the elaboration of the *negative part*: other moves are not so good (cf. the final conclusion: 'other moves are a bit too passive in that position,' line 73). As a matter of fact, the preceding partial results are all expressly negative:

- line 71: investigation of 1...B-Q7: 'a little slow'
- line 72: investigation of 1...P-QR4: 'but that is nonsense'
- line 76: investigation of exchange of Rooks after 1...P-QR4: 'no, that is nothing.'

We can now, in principle, understand all the transitions from one phase to the next, from line 41 up to and including line 78. It is not difficult to treat the second part (M2; B2) in a similar way. The cumulative linkings present there have already been mentioned.

Protocol (M2;B) is one of the most explicit ones as far as the formulation of evaluation results is concerned. Elsewhere formulations of results are more frequently missing, even in the final most explicit stages where we are certainly justified in assuming that the subject has reached a definite result. In such cases it is often possible to determine afterwards whether the result was favorable or unfavorable on the basis of the formal ties between consecutive branches.

If a particular variant is not satisfactory (negative result), the subject will in general try to improve his *own* play; so formally an *own-branching* comes into being (cf. Section 9): 'This is not good, what else can *I* do?' Conversely, when an own-branching appears, it probably points to negative results from the preceding variation. On the other hand, the presence of a *counter branching* in the second of two consecutive elaborative phases points to a preceding positive result. 'So far it is good for me, but what else can *he* do?'

The simplest case of an own-branching occurs when the subject tries another first move. Indeed this generally means – though not always (see p. 110) – that the previous attempt was unsuccessful.

This criterion of own- versus counter branchings provides another illustration of the fact that the formal structure can yield important cues about the internal dynamics of the process. The above example from (M2;B) shows, however, that this criterion must be handled with due care.

Section 34: Transitional phases

Up until now, for the sake of simplicity the structure of thought process (M2; B) and of all others has been assumed to consist of a first

⁸ The terms 'cumulative' versus 'subsidiary' linking are maintained for historical reasons (cf. Section 20). A solution method may be *cumulatively linked* if the succeeding step builds on the positive result of the preceding: 'Is the result +? If yes, go to next step.' If the previous step was unsuccessful and a new attempt is now made, we speak of *subsidiary linking*: 'If no, go back to previous step' (see also Section 51).

Phase having a character of its own, which is followed by a series of directly linked successive phases of elaboration. This is a highly simplified version of the matter, however. Transitions from one phase to another do not always occur immediately; the linkings are not so 'reflexoidal' as one might conclude from the schematic picture above. The parts of the protocol that we have hitherto called elaborative phases are not always completely devoted to calculations and other direct investigations of possibilities. Remarks of a much more general nature are often found in the text of the protocol that point to a return to problems more general than the elaboration and evaluation of specific solving propositions.

In (M2; B1) an example is found in the long plan *B* phase of lines 31–52. Here the remark (line 41): 'I do not stand well after all' does not express a specific result of a specific solving proposal. It goes beyond that: first, in that it evidently summarizes a number of specific, attained results; second, in that these findings are generalized to produce a (changed) evaluation of the whole position. Such expressions would not appear if the protocols were no more than a mechanical succession of elaborative phases with definite positive or negative results in between.

We find another example in lines 69–76. Not only are certain first moves dismissed but the reasons for doing so are given too: 1...B-Q7 (line 69): 'A little slow,' and later (line 73): 'other moves are a bit too passive in that position.' The subject concerns himself with the general problem posed by the position and gives a general outline to be followed, namely, not to move passively. Why all of the previously attempted defensive moves did not appeal to him is made explicit and summarized in this sentence. Here again is an abstraction, an integration, a generalization, instead of the more or less automatic succession of operations that are executed in an elaborative phase.

Finally we find an interesting example of the same thing in lines 7–12 of the second part of the protocol. Here the subject definitely gets rid of plan *B* – which in the absence of calculations is the only reason that it has been called a *B*-phase in the structural schema. It is not at all a phase of elaboration, however, rather a phase of reasoning out; the subject consciously articulates why plan *A* is the chosen one. His mental activities in this phase are apparently quite different from those in a regular elaborative phase. The subject's set or *Einstellung* is more receptive, less actively organizing; his thought activity is not so much deductive and systematic but rather inductive. The subject's mind is open to discovery and abstraction. In short, his mental set

is closer to that of the first Phase than to that of the elaborative phases proper.

The parts of the thought process where this general description holds will be called *transitional phases*. Their function is to integrate what has preceded and to prepare for the next actively organized investigations.

The external form of these transitional phases can vary, but they are usually characterized by a less rigid, much more 'open-minded' thought process along lines that appear to be less consciously constructed. Here the thinking of the subject consists not so much of ordering, working out (calculating), or checking, but rather of reflecting, integrating, and most of all *abstracting*. By processes of abstraction new 'possibilities' stand out against the concrete situation on the board, or important relationships and properties are abstracted from the results thus far achieved.

The protocols are often rather incomplete on this score. Because these phases tend to be of short duration, and precisely because they are to a lesser degree consciously organized and so more difficult to verbalize, we often find a dearth of verbal material – substantial *pauses* instead. In other cases transitional phases have to be assumed in order to understand the progress of the thought process even though the written protocol gives no indication at all.

During the experiments the temporary changes of attitude on the part of the subjects were often evidenced by external symptoms that were not recorded in the protocol text.⁹ The subject 'looks around' – he sometimes says so: 'Now let's see' – his eyes wander over the board, and he shows a certain passivity. His tone may become somewhat lower or unclear towards the end of what he has to say about his recently completed investigation because his thoughts are already toying with other things. Most convincing and obvious, however, is the shift in attitude at the end of a transitional phase when a decision is reached to renew the active efforts of analysis. On the basis of such observations, aside from theoretical considerations, the frequency of transitional phases must be assumed to be much higher than it would appear from explicit indications in the verbal protocol.

From a theoretical point of view, transitional phases are to be

⁹ The protocols were recorded as literally as possible and do contain, in their original form, indications like '... (pause) ...' here and there. A tape recorder would, of course, have provided much more precise information – but in order to register every possible sign of attitude change one would have needed a motion picture camera and possibly EEG and eye movement registration as well.

expected whenever the partial result of an elaborative phase is obtained but there is no longer available any *pre-established, direct connection from the result to the next elaborative method* (variation, move, plan). Assume, for instance, that the subject considers own-variations to be playable: *a*, *b*, and *c*, all stemming from a first move W_1 that he is inclined to play. Assume further that on analysis he finds all three of them to be unsatisfactory one after the other. Some sort of reconsideration of the problem situation is obviously needed. Since he is 'through' for the time being, the subject is forced to *revert to a more fundamental problem* than that of analyzing specific variations. Later, of course, he may decide on doing different things: look for a fourth variation, *d*; re-examine one of the three somewhat deeper; look for another first move instead of W_1 ; re-examine a previously less preferred move W_2 ; reconsider his preference for the plan or general idea, *A*, to which move W_1 may belong; reconsider his expectancies; etc. (cf. Section 51). But regardless of what the next elaborative phase may contain some process of summarizing or generalizing (results to date), of abstracting or 'looking for' (other possibilities of investigation and/or play), and of reconsidering or reflecting (on a more fundamental problem) must occur. In short: Some process devoted to mental activities that are typical for transitional phases, as found in the protocols, must of necessity precede the elaborative decision.¹⁰ Again this means, this time on theoretical grounds, that transitional phases must be assumed to occur in many cases where there are no direct indications in the protocol text.

A transitional phase most obviously occurs when as a result of continuously negative results the subject runs out of subsidiary methods at some level. But this is not the only time. In the final stages of the thought process, in particular, the same reversion to more general problems may occur after a few cumulatively linked elaborative phases. If, for example, the analysis of a number of the opponent's replies to a considered move reaps continuously satisfactory results, then some sort of conclusion, summarizing, and getting back to the main choice problem is needed here too. After the reversion the move decision may be taken right away; or

¹⁰ In a somewhat more machine-oriented language: Wherever in the solving process the machine gets through a number of ordered but co-ordinate subroutines without success, either a pre-programmed recursion to a more fundamental level (or to the main tree) or a reconsideration and reorganization of the program by the programmer is needed. For the human process these two cases are not distinguishable; they both correspond to what happens in a transitional phase.

it may be delayed. In (M_2 ; B_2) the move in question is played immediately after (line 33): 'That also gives good chances' (the word 'also' evidences a summarization). In other cases the 'variation outcomes' of the preferred move are first checked to see if they are really better than those of other moves. This means that the subject starts on the negative part of his 'proof' (cf. Section 9); as in (M_2 ; B_1) from line 68 on, according to the interpretation given in Section 33, p. 106. In any case the decision must of necessity be preceded by some sort of transitional phase or summarization and a getting back to the main choice problem.

Transitional phases after positive results are apt to be less conspicuous. Mostly, they are of a less problematic nature than those of the negative case, where the subject runs out of subsidiary methods at some level (cf. Section 48). After positive results there is also hardly any need to 'look for other possibilities,' so this characteristic feature in the attitude of the subject is lacking, too.

In general it appears that the structure of the thought process itself must be modified if we take the transitional phases into consideration. From the point of view of move calculations the structure becomes intermittently interspersed with blanks. Fortunately, their addition does not necessitate any major changes in the cumulative or subsidiary linkings between elaborative phases. So our conception of the macro-structure can be largely maintained – provided that we realize that the linkings cannot be taken as direct and automatic: there often exist rather complex intermediary processes.

If we disregard momentarily the hierarchy of main and subphases, the basic structure of a thought process can be seen to consist of alternating phases of direct analysis on the one hand and integration, abstraction, and restructuring on the other. To a large extent this distinction parallels that between the finding of means (*Mittelfindung*) and the application of means (*Mittelanwendung*), as discussed in Chapter II, Section 20. Indeed, one of the functions of transitional phases is to find new means: new moves, new plans, fresh views of the problem, new approaches, new subprograms for analysis and decision making. The typical features of transitional phases – such as the return to the more general, primary goal, the integration of obtained results leading to a sometimes explicit reformulation of the problem, the abstractive activities of the subject, his open-mindedness while looking at the board – all fit well into a functional interpretation of *Mittelfindung*.

Actually, an even more general principle can be considered to be embodied in the alternation of active analysis and receptive abstraction. Parallel to the distinction between receptivity for the new (means) and application of the old (means), one could contrast the following in human thought: induction and deduction; expansion and concentration in the perceptive field; possibly even extraversion and introversion. If we consider that a subject in our experiments goes through a process of learning – gets acquainted with and finally masters a specific situation – it is not surprising that an analogous ‘pulsation of set’¹¹ is found here. This has been described for other processes of adaptation to certain situations as well as for the conquest of (a part of) the external world. An alternation of receptiveness (broadening the horizon) and active organization appears to be essential in all processes of adaptive learning and development.¹² Due to the player’s long experience at the chessboard, the lion’s share of the more receptive thinking can be dealt with in a short first Phase, but even so every now and then short phases of expansion and enrichment are obviously needed: the transitional phases.

B. PROTOCOL STATISTICS

Several protocol variables along with data on their distribution are given a statistical grounding in the next two sections. First, in Section 35, data on the duration of the experiments, the size of the protocols, and both time and size of the first Phase. Then, in Section 36, a number of structural variables are derived from the sequence of solving propositions; thereafter the interrelations and experimental distributions are briefly analyzed.

Section 35: General protocol measures

Table 2 gives the mean values (column 3) as well as the lower (column 4) and upper (column 5) limits of T, the *duration of a thought process* (in minutes), for each of the experimental subseries with positions A, B (B₁ and B₂), C, and the rest group (other positions).

¹¹ *Pulsering van de aandachtskring*, a term coined by the late GERRIT MANNOURY (1947, p. 81).

¹² If one wishes, this ‘dualism’ may even be expanded into a metaphysical principle – but such an extension definitely goes beyond the purpose of the present study.

Also the means for the *number of lines* – from the original, typewritten Dutch protocol – are listed (column 6) and the mean number of lines per minute (column 7).

TABLE 2: *Duration, lines, and lines per minute*

1 Sub-series	2 Number of protocols	3 Mean duration (minutes)	4 5 Duration extremes		6 Mean number of lines	7 Mean number lines/min.
			minimal (minutes)	maximal (minutes)		
A	19	15.3	6	28	35.4	2.4
B ₁	6	13.0	10	15	30.8	2.3
B ₂	6	12.5	0	20	30.7	2.5
C	5	18.6	7	30	45.8	2.5
Rest	13	13.9	3	35	38.1	2.8
Total	49	14.6	0	35	36.1	2.5

N.B. The figures in the table are based on the original typewritten Dutch protocol texts. In the present format – translated and printed – the numbers of lines (and lines per minute) can be multiplied by two, roughly. For instance, a protocol in the present form (see Appendix II) would contain some 70 lines on the average.

It appears from Table 2 that an average thought process with the kinds of positions used in the main series takes about a quarter of an hour. The range is rather large, from a few minutes to more than half an hour. Judging from chess experience, the means and the spread seem quite normal for positions in which a serious and more or less crucial decision problem must be solved. Within this group, the spread does not seem to be highly dependent on the type of position, with the possible exception of position C where the most time is consumed (cf. Section 26, p. 92). That there are but a very few brief processes (duration of less than a minute) is largely due, of course, to the newness of the position to the subject. The exception is position B₂ where indeed the lower limit is zero: subject C₂ made an immediate reply to the experimenter’s move. From the protocols of the series of positions involving combinatorial tasks (cf. Section 28, p. 98) it appears that experimental processes of less than one minute are by no means impossible, even if the position is new and complex (see, e.g., protocol (M₅; R × Pch), p. 243). Not surprisingly, in the randomly selected positions (cf. p. 98) brief processes are common whenever the position happens to be trivial (e.g., if the problem is one of recapturing).

The duration of the process in the A-subseries (mean = 15.3 minutes; average deviation = 5.9 minutes) appears to depend on the

subject's level of skill: for the five G-subjects the average time is less than 10 minutes, for the seven W- and C-players, more than 20 minutes. Position A is apparently easier, although certainly not trivial, for a grandmaster than for a less skilled player.

Columns (6) and (7) give a rough idea of the verbal production of the subjects: $2\frac{1}{2}$ (present format: 5) lines per minute. Different positions do not seem to influence the average volubility, V. Among the subjects, however, there are some noteworthy individual differences (cf. Section 23). Two extreme cases prove illustrative: G5's average production over positions A, B₁, B₂, and C taken together is $3\frac{1}{2}$ (present format: 7) lines per minute, as opposed to $1\frac{1}{2}$ (present format: 3) lines per minute for subject W₂. As previously stated, the ease with which a subject thinks aloud, that is, formulates while continuing to think, does not in general depend on his level of skill – provided he is well enough acquainted with the names of the pieces, squares, positional features, standard maneuvers, etc.; in short, is able to 'talk chess.'

Table 3 provides some information on the relative length of the first Phase – again, based on the original Dutch, typewritten form. The six B₂-protocols and a few others in which the 'experimental procedures (such as 'interruption after 10 seconds,' cf. Section 28, p. 97) interfered with a normal development of the first Phase had to be excluded. Thus the total number of protocols tabulated is somewhat lower than in Table 2.

TABLE 3: *Length and duration of first Phase*

1 Subseries	2 Number of protocols	3 Mean number of first Phase protocol lines	4 Per cent of total number of lines	5 Estimated duration (in minutes)
A	17	7.3	23.4%	3.1
B, C, and Rest	18	7.8	23.6%	3.3
Total	35	7.5	23.5%	3.2

The data of Table 3 do not pretend to give more than a rough idea of some sample findings and estimates on the first Phase – which is in itself largely an artifact of the experimental setup. So the figures should in no way be generalized to other chess or

problem solving situations.¹³ Furthermore, the sample values are not very precise: exactly where the first Phase comes to an end in a given protocol cannot be decided without some degree of subjective arbitrariness. Nonetheless, we can quite safely conclude that in our kinds of task situations the first Phase takes up one fourth to one fifth of the total thinking time, that is, somewhat more than three minutes on the average.

In the A-subseries the same time dependence on the subject's level of skill that we found for the total duration shows up for the first Phase. In particular W- and C-subjects needed more orientation, as could be expected. Again, there is a rather large scatter around the averages. From the main series the two extremes for the 'estimated time' of the first Phase are one half minute and seven minutes (average deviation = 1.3 minutes).

Section 36: Statistics of solving propositions

A series of numerical structural variables can be derived from the formula of successive solving propositions (cf. Section 32). Here the protocol (C₂; A)' will be used for illustration.

After the first Phase of (C₂; A) we find considerations and calculations of the following move proposals:

1.N×N; 1.N×N; 1.N×N; 1.P-KR4; 1.R-B2;
1.N×N; 1.B-R6; 1.P-KR4; 1.B-R6; 1.B-R6.

Finally, after 16 minutes, 1.B-R6 is actually played.

Such a series of moves, of course, reflects the actual progress of the thought process only in a fragmentary way. All solving propositions in the broader sense (plans and other summarizing formulations) are lacking and so are transitional phases. Even so, the formula contains interesting structural data that have the advantage of being statistically manipulable.

Following the practice of assigning a letter to each of the moves that were either played or considered by any of the subjects in position A (Table 8, p. 128), the succession of solving propositions can be represented by a series of letters. The formula for (C₂; A) is as follows:

c - c - c - f - l - c - e - f - e - e - e.

¹³ It is interesting, though, to compare the 20-25% finding in column 4 with the ratio of time and money consumption in research in the 'first phase' of orientative (exploratory, pilot) investigation and the 'main part' of the definitive (hypothesis testing) project, respectively; see also Section 62.

The final linking is cumulative here, as in (M2; B2), but not in (M2; B1) (cf. Section 32, p. 105). The last move considered (1.B-R6 = e) is in fact played. According to the formula subject C2 makes a total of ten clearly defined *starts* or *re-starts* in considering and analyzing the consequences of a move or its more or less independent substructure. Thus, the *number of fresh starts*, N , equals 10. (See Footnote 7, Section 32, for the operational definition of fresh start.)

It can be seen that a fresh start is not always a start on another move; repetitions of the same move occur at c-c-c and e-e. These are cases of *immediate re-investigation* within the same solving proposition. If we combine these contiguous repetitions and count only the number of times the subject makes a transition to a move which differs from the immediately preceding one, we get another variable that will be called the *number of successive solving propositions*, n . We find here $n = 7$. N is always greater than or equal to n . In the difference, $N - n = c$, we now possess a rough numerical measure for the *amount of immediate re-investigation*. In (C2; A) $c = 3$, that is, three of the ten fresh starts qualify as immediate re-investigations.

Besides N and n we may distinguish the *number of different moves considered*, n_0 , that is, the number of different letters appearing in the formula. n_0 is always less than or equal to n ; if the same move (letter) appears more than once, then n_0 is smaller than n . When that happens we speak of *non-immediate re-investigation* (e.g., move c in the formula sequence: c-f-l-c-). The difference $n - n_0 = r$ is then a rough measure for the *amount of non-immediate re-investigation* in a protocol. In (C2; A), $n = 7$ and $n_0 = 4$, so $r = 3$, that is, three solving propositions qualify as non-immediate re-investigations: once c (the sixth letter in this formula), once f (the eighth letter) and once e (the ninth letter).

The way in which different moves contribute is not reflected in the quantities c and r . If $r = 3$ this may, as in (C2; A), result from non-immediate re-investigations of three different moves, but also, for instance, from three non-immediate re-investigations of one single move. If this should actually be the case the move itself would occur four times in the formula: once initially and three non-immediate re-investigations (see, e.g., (M1; A), p. 172). Because of this ambiguity a few more quantitative variables have been derived from the formula.

n_c is the number of different moves that have been immediately re-investigated; n_r the number of moves that have been non-immediately re-investigated. Thus for (C2; A), $n_c = 2$ (moves c and e) and

$n_r = 3$ (c, e, and f). The maximum number of fresh starts within one solving proposition is m_c . In (C2; A), $m_c = 3$, because of the sequence c-c-c at the beginning. Correspondingly, m_r is the maximum number of times that a certain solving proposition appears (always non-immediately). Therefore for (C2; A), $m_r = 2$ because neither the first solving proposition of c's, the second one of f, nor the fifth one of e appears more than twice in the formula.

So we have for (C2; A):

$$\begin{array}{lll} N = 10 & n = 7 & n_0 = 4 \\ & c = 3 & r = 3 \\ & n_c = 2 & n_r = 3 \\ & m_c = 3 & m_r = 2 \end{array}$$

The three quantities, c , n_c , and m_c together present quite a good picture of the range and character of the phenomenon of immediate re-investigation in a protocol. This also applies to r , n_r , and m_r for non-immediate re-investigation. There are certain relations among the variables N , n , n_0 ; c , n_c , m_c ; r , n_r , m_r . For the sake of completeness the most important ones are presented here:

$$N - n = c; \quad n - n_0 = r; \quad \text{therefore: } N - n_0 = c + r.$$

When no re-investigation at all takes place in a protocol, $N = n_0$.

$$\begin{array}{ll} n_c \leq c & (1) \quad n_r \leq r \\ 1 \leq m_c \leq (c + 1) & (2) \quad 1 \leq m_r \leq (r + 1) \\ c \leq n_c (m_c - 1) & (3) \quad r \leq n_r (m_r - 1) \end{array}$$

If $c = 0$, then $n_c = 0$ and $m_c = 1$.

If $c = 1$, then $n_c = 1$ and $m_c = 2$.

For higher values of c there are more possibilities: e.g., if $c = 2$, then either $n_c = 1$ and $m_c = 3$, or $n_c = 2$ and $m_c = 2$; etc.

In Table 4 the means for N , n , and n_0 for each of the subseries A, B₁, B₂, C, and 'rest' are presented. The legal freedom of choice in each position, K (see Section 7), has been added in column 7 for the sake of comparison. Also the total number of moves that were considered in one particular position by all the subjects for that particular subseries is offered for comparison (column 6; given in full in Tables 8-10 at the end of this section). The latter number is also the number of letters of the alphabet needed for coding the moves.

TABLE 4: Means of N , n , and n_0

1 Sub-series	2 Number of Dutch Protocols	3 \bar{N} Fresh Starts	4 \bar{n} Solving Propositions	5 \bar{n}_0 Different Moves Considered	6 Total Moves Considered	7 K Legal Freedom of Choice
A	19	8.4	6.2	4.8	22	56
B ₁	6	6.3	4.8	3.3	10	35
B ₂	5	9.4	7.8	5.2	9	40
C	5	13.6	9.4	5.6	10	37
Rest*	11	8.3	5.7	4.0	—	—
Total	46	8.8) _† 9.2) _†	6.4) _† 6.8) _†	4.5) _† 4.5) _†	—	—

* Protocols (M₅; B-R7ch) and (M₂; K-N₂) are excluded; see p. 127

† Weighted and non-weighted means, respectively.

First, we may read from the table that the mean number of different moves considered by the subjects (n_0) is no more than four to five. This number is very small not only with respect to the number of legal moves (K) but also with respect to the total number of moves considered (column 6). Later on we shall return to this 'inexhaustiveness' in the exploration of move possibilities.¹⁴

Differences between the subseries are not conspicuous and are quite understandable from a chess point of view. Position C has again the largest means for \bar{N} , \bar{n} , and \bar{n}_0 ; as it had for mean duration and protocol length (Table 2, p. 117). If we base our conclusions – for thought processes of some fifteen minutes – on the total means, we can figure on an average of nine fresh starts and six to seven solving propositions. If we subtract three minutes for the first Phase (Table 3), this means that at an average of every minute and a half, the subject starts afresh on a relatively independent part of the investigation (calculation), and at an average of once every two minutes he switches to considering another move (not necessarily a previously unconsidered move).

Since $(\bar{N} - \bar{n}) = (\bar{N} - \bar{n}) - \bar{c}$ and similarly $(\bar{n} - \bar{n}_0) = \bar{r}$, we may immediately calculate from Table 4 the values of \bar{c} and \bar{r} . Further, the sum of \bar{c} and \bar{r} is equal to $(\bar{N} - \bar{n}_0)$. If we compare the columns,

¹⁴ More positively formulated: This has to do with the curiously strong and obviously efficient *selectivity* of the human chess player – in sharp contrast to the amount of 'brute force' employed in most computer programs.

we see that \bar{N} is almost twice as large as \bar{n}_0 ; on the average the difference is about four. This means that $\bar{c} + \bar{r}$, that is, *the number of re-investigations* in a protocol averaged over all protocols of the main series is almost four, the same as the average number of different moves considered. Obviously, the phenomena of re-investigation are highly important.

On closer inspection the distribution of $(\bar{N} - \bar{n}_0)$ appears to be skewed. The median is 3; the mean is raised somewhat by a few extreme values. Thus in subseries A the value $(\bar{N} - \bar{n}_0) = 9$ is attained twice and $(\bar{N} - \bar{n}_0) = 7$ once. We find the highest value, however, in the following three cases: Table 5.

TABLE 5: Maximal values for $\bar{N} - \bar{n}_0$ and corresponding data

Protocol	T (in minutes)	N	n	n_0	$\bar{N} - \bar{n}_0$	c	n_c	m_c	r	n_r	m_r
(G ₅ ; B ₂)	20	16	13	6	10	3	2	3	7	4	3
(M ₂ ; C)	30	21	14	7	14	7	2	4	7	4	5
(M ₄ ; C)	21	16	12	7	9	4	2	3	5	2	4

These are rather extreme examples of non-immediate re-investigation protocols, as is evidenced by the quantities on the right-hand side of the table, but they are not atypical. High values for both c and r occur rather often, which would follow from the relatively high mean for $(c + r) = (\bar{N} - \bar{n}_0)$. The lowest value for $\bar{N} - \bar{n}_0$, namely 0, occurs but twice in the entire material.

TABLE 6: Frequency table for c, n_c , m_c , and r, n_r , m_r (46 Dutch protocols)

1 Variable	2 Immediate re-investigation	3	4	5 Non-immediate re-investigation	6	7
Value	fr(c)	fr(n_c)	fr(m_r)	fr(r)	fr(n_r)	fr(m_r)
0	6	6	0	17	18	0
1	10	21	6	9	11	18
2	11	15	22	6	8	13
3	10	4	10	5	5	10
4	2	0	6	2	3	3
5	3	0	2	2	1	2
6	3	0	0	2	0	0
7	1	0	0	3	0	0
Total	46	46	46	46	46	46

The two kinds of re-investigation will now be considered separately. The clearest picture is attained by a frequency table for the different values of the c - and r -variables; in Table 6, $fr(x)$ means the frequency of the corresponding value in column 1 for variable x .

Concerning the c -variables: $c = 0$ (and thus $n_c = 0$ and $m_c = 1$) is evidently a relative exception (13%). The normal values are $c = 1$, $c = 2$, and $c = 3$; the mode and the median are both 2, while $\bar{c} = 2.4$. With n_c all of the values are concentrated near 1 and 2; with m_c , near 2 and 3. While the phenomenon of immediate re-investigation is apparently a fairly general one, the maximum number of fresh starts within one solving proposition remains limited to two or three (in only 17% of the cases is m_c above 3).

Concerning the r -variables: The cases where $r = 0$ form a group by themselves (37%). The decline of frequency through the higher values of r is slow, however. Similarly, the frequencies for n_r decline more slowly than those for n_c . In other words: If a subject is inclined to return to any previously considered move, the chances are relatively high that this will occur more than once or with more than one move. Indeed, non-immediate re-investigation only comes about when the subject finds his choice difficult. But in such a case the entire thought process is apt to be built up, as we shall see later, in phases of progressive broadening and deepening. That is one reason why $n_r = 3$ or 4 is less exceptional than $n_c = 3$ or 4 and why relatively high frequencies of $r > 4$ occur. Finally concerning m_r : From the table it does not appear to be at all exceptional that some solving proposition occurs *three times*, with other solving propositions in between. Even $m_r = 4$ and 5 is found in 11% of the protocols.

On the basis of the above data it would seem that the phenomenon of non-immediate re-investigation is not quite as general as that of immediate re-investigation. It appears to be restricted *not* to certain persons who might have the habit of 'hesitating' and 'going back and forth' from one solving proposition to the other, but rather to situations where the subject – any subject – finds it difficult to come to a decision. Position B was characteristic in this respect: non-immediate re-investigations occur with all subjects. Since the solving of 'difficult' problems is in many ways psychologically more interesting and important than the solving of simple ones, we must conclude that the phenomenon of non-immediate re-investigation deserves some specific attention. Later on we shall have occasion to study it from a qualitative, non-numerical point of view (see Sections 42 and 47).

To conclude this section there follows a short discussion dealing with the dependence of the values of the above statistics on the position and the subject.

Position A: The data provided by position A agree fairly well with the distributions and the means of the entire material, only \bar{r} is somewhat low (1.4 as against 1.9 for the total mean). It is remarkable that in but one case (5%) – as against 15% for the entire material – is the final linking subsidiary, that is, in but one case is the move actually played *not* the last one investigated. It is apparently a rarity in position A when the subject comes to his decision through the negative part of the reasoning, that is, indirectly through comparison with other moves. Indeed, to the grandmasters and to many other subjects as well, the task was in fact a question of finding and calculating 'the best move' (see Section 8). The negative part of the proof is omissible as soon as a definite advantage is guaranteed – provided the subject is sure enough on the basis of his previous investigations that (other) *direct* moves cannot lead to advantage.

Position B: In the six protocols of subseries B1 we thrice find a subsidiary final linking (50%). Although the sample is too small for statistical generalization, it seems that the different character of position B is relevant. In the protocols indirect arguments like: 'other moves are not better' and 'I don't see much else,' occur more often in position B (and in position C with two subsidiary final linkings out of five) than in position A. Furthermore, the B1-material distinguishes itself by low values for N , n , n_c (see Table 3) and by a low value for c :

$$\bar{c}(B_1) = 1.5 \text{ as against } \bar{c}(\text{total}) = 2.4$$

Only a relatively small number of moves are to be considered, and these do not demand deep calculations nor continued investigation. Plans and a more general analysis of position and possibilities are more important than precise calculations.

Position B2 is different, however: generally, the B2 quantities do not deviate much from the total mean.

Position C: The material from position C distinguishes itself by high values for N , \bar{n} (see Table 3) and especially c , \bar{n}_c , r , and m_r :

$$\bar{c}(C) = 4.2 \text{ as against } \bar{c}(\text{total}) = 2.4$$

$$\bar{n}_c(C) = 2.2 \text{ as against } \bar{n}_c(\text{total}) = 1.3$$

$$\bar{r}(C) = 3.8 \text{ as against } \bar{r}(\text{total}) = 1.9$$

$$\bar{m}_r(C) = 3.2 \text{ as against } \bar{m}_r(\text{total}) = 2.1$$

Thus we see a high number of fresh starts; composite calculations with many immediate re-investigations, affecting two solving propositions on the average; moreover, frequent returns to previously considered moves (non-immediate re-investigations), as a result of frequent intermove comparisons and progressive deepening in the investigation of possibilities. These phenomena are undoubtedly connected with the unusual character of position C. The position is not objectively solvable (like A) nor does it lend itself very well to systematic treatment along lines of plans and general considerations (like B). Only by direct trying out, i.e., by calculating the consequences of various moves, the results of which are weighed against each other, is it possible to reach a subjectively satisfying choice.

Other positions: The means of the heterogeneous group of other positions agree fairly well with the means of the entire material. \bar{N} , \bar{n} , \bar{n}_0 and \bar{c} and \bar{r} are somewhat on the low side due to the presence of some less profound problems, such as positions ... P-QR₃, ... Q-K₄, and N-Q₂.

Finally, as an illustration of the usefulness of the numerical structural quantities, a few words will be said about the idiosyncrasies of some of the subjects and about a few distinguishing features of certain protocols.

We find very low r -values among the weaker subjects (W- and C-classes) although the average thinking times and numbers of solving propositions (n) are normal to large (cf. Section 49, IV, 3). Thus in the four protocols of subject W₂ we find $r = 1$ only in B₁, for the rest $r = 0$; in (W₂; A), for example:

$$\begin{array}{l} T = 28 \text{ minutes} \quad N = 10 \quad n = 9 \quad n_0 = 9; \\ \text{so } c = 1 \text{ but } r = 0. \end{array}$$

(W₂; C) yields a similar picture:

$$T = 20 \text{ minutes} \quad N = 11 \quad n = 8 \quad n_0 = 8;$$

so $r = 0$, whereas in general equality between n and n_0 only occurs with shorter thinking times and with much lower values for n . Many moves pass in review, but the subject never returns to a move previously considered. There is no progressive deepening and broadening of the investigation – which is to be interpreted here as a shortcoming in thought organization (cf. Section 52).

Subject M₂ stands apart from his master colleagues by his high values for N , n , and n_0 . Thus:

TABLE 7: N , n , and n_0 for Subject M₂ compared to \bar{N} , \bar{n} , and \bar{n}_0

	$N(M_2)$	$n(M_2)$	$n_0(M_2)$	\bar{N}	\bar{n}	\bar{n}_0
Position A	20	17	11	8.4	6.2	4.8
Position B ₁	14	12	7	6.3	4.8	3.3
Position B ₂	7	5	4	9.4	7.8	5.2
Position C	21	14	7	13.6	9.4	5.6

Only for position B₂ is subject M₂ below the average; all his other means are much higher. Values such as $N = 21$ and $n_0 = 11$ are unique in the material; the next highest value for n_0 is 9. Apparently subject M₂ figures out more moves than the others; his way is less intuitively selective, he tends to 'try out' more variations than the others (cf. Section 59). Of course, this is only a matter of more or less, only a tendency, not a rigidly applied 'system.' This is evident from the protocol (M₂; ... P-QR₃) where $T = 3$ minutes, $n_0 = 1$, and the formula is $a - a - a$ (see Section 49).

Three protocols from the main series were not included in this statistical description: (C₂; B₂), (M₅; B-R₇ch), and (M₂; K-N₂). The protocol (C₂; B₂) consists of nothing more than the move decision itself ($T = 0$ minutes) and therefore is unable to tell us anything. In the second, subject M₅ simply could not find a possible winning combination and finally, in his own words, felt enmeshed in 'a certain pathological doubt.' This finds its expression in an abnormally high value for r ($r = 13$ while elsewhere the maximum is 7). Concerning (M₂; K-N₂): this position is so exclusively strategic that the thought process develops along lines of more general considerations and plans (solution proposals or solving propositions in the broader sense). Move proposals play such a small part that the formula and the numerical values deduced from it in no way represent the structure of the thought process. Of course there is always some imprecision in the formula representation, but this extreme case is better eliminated entirely.

There now follows for positions A, B₁, B₂, and C a register of the move proposals of each subject. What appears in the chart is the number of times that the subject investigated a particular move, while the move actually played is given in parentheses. A 'o' in the chart means that the subject mentioned the move in the first Phase or a transitional phase but never returned to investigate it.

TABLE 8: Position A: moves examined by 19 subjects

N	3	6	4	7	13	9	20	14	12	E ₁	E ₂	E ₃	E ₄	E ₅	W ₁	W ₂	C ₁	C ₂	C ₃	C ₅	
n	2	6	3	5	7	8	17	8	8	1	3	5	4	2	3	3	9	4	7	5	10
n ₀	2	4	2	5	6	5	11	5	6	1	3	3	3	2	2	9	3	4	4	5	9
Subject Move Symbol	G ₁	G ₂	G ₃	G ₄	G ₅	M ₁	M ₂	M ₃	E ₁	E ₂	E ₃	E ₄	E ₅	W ₁	W ₂	C ₁	C ₂	C ₃	C ₅		
1. B×N/5 = a	(1)	(2)	2	(1)	(3)	(5)		(7)	6			1			(2)	2		(3)	(2)		
N×B = b	2	2	(2)	1	3	1	4	3	1			2	(3)	(2)	1		4	1	1		
N×N = c		1	0		5		2	2	1			(5)	1		1		(3)	1			
B×N/6 = d							2	1	1												
B-R6 = e	0				1	1	0	1	1						1						
P-KR4 = f						1	2	1	2			(2)			1		2		4		
N-K4 = g						1	1								1				1		
KR-K1 = h						1	(2)		0						1				1		
P-QN4 = i		1				1	2								0	(2)			0		
N-N4 = j						1	1								1						
Q-R3 = k															1						
R-B2 = l												1			1		1				
N-B4 = m															1						
B-N1 = n									(1)										1		
Q-N3 = o															1						
KR-Q1 = p															1						
Q-Q2 = q															1						
P-B4 = r															1						
Q-B3 = s															1						
N-K2 = t															1				1		
P-QR4 = u															1				1		
N-R4 = v																			1		

Any number in a cell indicates a move (row) examined by a subject (column). Empty cells: moves not considered.
 Zero: The move was mentioned in the first Phase but not investigated in the main part.

N : Fresh Starts.

n : Solving Propositions.

n₀ : Different Moves Considered.

Number in parentheses: the move that was finally played.

TABLE 9A: Position B1: moves examined by 6 subjects

N	8	14	4	5	4	3
n	6	12	2	4	3	2
n ₀	3	7	2	3	3	2
Subject Move Symbol	G5	M2	M4	W2	C2	C5
1...R-N1 = a	(5)	(5)	(2)	(3)	(1)	
P-KR4 = b	2	2	2		2	
K-B1 = c		2				
B-Q7 = d		1				
P-QR4 = e		1		1		
K-N2 = f	1				(1)	(1)
B-KN4 = g		2				
R-Q1 = h		1				
B-Q2 = i				1		
P-N4 = j						2

See legend, Table 8.

TABLE 9B: Position B2 (after 1...R-N1; 2.P-N3*)

N	16	7	12	3	-	9
n	13	5	9	3	-	9
n ₀	6	4	5	3	-	8
Subject Move Symbol	G5	M2*	M4	W2	C2	C5*
1...R-N4 = a	(4)	(4)	1			Charac-
P-KR4 = b	2	1	(7)			ter of the
K-B1 = c	5		1			position
B-Q7 = d	3	1	2		(1)	is too
P-QR4 = e	1			(1)		different
R-N5 = f		1				after
B-R3 = g						1...K-N2
B-Q3 = h				1		and
R-Q1 = i	1		1			2. P-Q4

* For M2, after 1...R-N1; 2.QR-N1, the position was essentially the same.

For C5, after 1...K-N2; 2.P-Q4, it was not.

See legend, Table 8.

TABLE 10: *Position C: moves examined by 5 subjects*

N	4	16	21	16	11
n	2	11	14	12	8
n ₀	2	4	7	7	8
Subject Move Symbol	G ₅	G ₆	M ₂	M ₄	W ₂
1...N-K ₅ = a	(2)	1	(11)	6	2
P-K ₄ = b				(5)	2
Q-K ₅ = c	2	3	3	1	(1)
P-Q ₄ = d		(5)	1	1	1
R-K ₁ = e		7	1		1
KN-Q ₄ = f			2	1	2
N-R ₄ = g			2	1	
K-R ₁ = h			1		1
N-R _{7ch} = i				1	
P-N ₄ = j					1

See legend, Table 8.

MAIN AND SUBPROBLEMS

A. THE SYSTEMATIC ANALYSIS OF THE PROBLEM STRUCTURE

Section 37: Interpretation of the elements of a protocol

For the following systematic and general analyses we need a method for the interpretation of specific expressions as they occur in the protocols. We shall have to read the protocols with *the organization of the thought process* as our main interest and pay no heed to other possible points of view. The types and the details of the subject's experiences as investigated by the Würzburg psychologists are not of any concern to us now. It would, for that matter, be impossible to pin them down since 'thinking aloud' protocols provide little information in this respect. But we shall pay attention to the *operations*, to the *methods* applied by the subject in his mental activity.

Naturally, it is impossible to draw up strict formal standards for interpretation. For such a rigid approach the meaning of each formulation is too dependent on the entire context. The main problem with which the subject is faced, that is to say the choice of a good move in the given situation, evolves in his mind in the course of his thought process. Thus a particular expression is only understandable in relation to the present state of development of the main problem (and of the detail problems into which it has split itself).

We may, however, enumerate the various possible interpretations of text elements. Which types of statements, formulations, and expressions occur in the protocols and what categories can be used for a psychologically useful classification?

As a starting point to try to answer this question we use what may be called *the assimilative cycle of a detail problem* as a hypothetical unit. Logically and psychologically, such a unit should contain the following *mental processes*:

- establishing the detail problem in question;
- setting, in terms of mental operations, the goal to solve this particular problem (operations-goal);
- carrying out the operations of investigation (calculations);

- d. determining the result and evaluating the outcome¹;
- e. restructuring and integrating result(s) and outcome(s) into the formulation of a new problem.

With this the cycle is indeed closed: we have a new problem.

Thus we can now read the whole protocol as a system of such cycles, either linked in series or overlapping. In this light it should be possible to interpret *the verbal elements of a protocol* according to the following schema:

- a. formulations of a problem;
- b. statements of operations-goals (i.e., goals with regard to the organization of thinking);
- c. reports of the investigatory operations as they proceed (mainly calculations);
- d. formulations of partial results and outcomes.

Category *e* of the series of mental processes – integration into the formulation of a new problem – will appear in the protocol text as a problem formulation again. So for the protocol elements no *e* category is needed.

Indeed it appears to be possible to read the protocols in this way provided the categorization scheme is somewhat further elaborated.

- a. *Direct formulations of detail problems*, of varying order with regard to the main problem, are found in the protocols mostly in the form of a direct or indirect question. For instance in (M2; B): 'The Bishop, can that do anything?' and (after calculating three moves deep): 'Am I doing anything to him then?'

And in other protocols:

(M4; ... Q-K2):

- 9 How to forge ahead with the attack?

(G1; A):

- 32 Who is actually better off?

(C2; B):

- 51 The question is, in fact, must anything concern Black so much that he cannot play 1... P-KR4? On principle I am inclined to play 1... P-KR4.

¹ From here on a distinction will be maintained between 'result' and 'outcome.' 'Result' will be a more general term which can be either qualitative (e.g., a Rook on the seventh rank) or quantitative (e.g., advantage for White). 'Outcome,' however, will be strictly quantitative – a numerical evaluation could be imputed.

In addition, more *implicit problem formulations* appear as well. For example:

(C5; A):

- 81 I have the awkward feeling that if I want to start something I immediately get simplifications and then my attack is gone. So maybe I should think about a positional move, but if I make a positional move, then Black can himself simplify.

Further, the protocols contain a vast number of *contributions towards the formulation of the problem at hand*. They cannot be considered complete problem formulations, but they do contribute to a problem formulation in that they express some feature of the problem at hand. Naturally, they are especially frequent during the first Phase of problem formation. They may occur in many different forms of which the most important are recorded below.

When subject (M2; B) says, 'Difficult: this is my first impression,' we may say that a feature of the problem yet to be investigated is anticipated. In lines 13 and 14 we find, 'then I can't win, but that won't be too easy anyway.' The latter part is also an anticipation, both of the final result that can be attained by best possible play and of the supposed value of the position, still to be determined. Thus, in the protocols *anticipations* occur concerning the difficulty of the problem, its solvability, concerning the strategy, or the direction of an investigation to be followed, and so forth. The concept 'anticipation' is used here to refer to certain kinds of statements or formulations. They are often characterized by the use of such expressions as: 'I feel that...' (cf. above (C5; A)), 'I have the impression...', 'It seems...', etc. Very often, too, words such as 'perhaps' and 'probably' reflect the subject's temporary insecurity about the correctness of his judgment. In general, an 'anticipation' is a statement in which *the subject speaks with a certain restriction about (his expectation of) the nature of the solution of the problem at hand or the problem that is taking shape.*²

A second type of contribution to a problem formulation, again particularly prevalent in the first Phase, is the *survey of relevant*

² An 'anticipation,' as a category of verbal statements or expressions in the protocols, is something very different from Selz's schematic anticipation. The two concepts are related, however. An anticipation statement results from an abstraction process by which some particular feature of the (sub)problem to be solved is brought to the fore. Since a feature of the problem-to-be-solved is a feature of the solving-goal-as-set as well, an anticipation can be said to *express the observation of a feature of the schematic anticipation* – if we accept Selz's statement that setting a goal always implies a schematic anticipation of the goal as attained (the problem as solved).

possibilities and plans. Statements of this type may or may not be immediately followed by elaborative investigations. Such a survey delimits the territory in which the solution will have to be sought. We find a clear example in (M2; B1):

- 12 I might try to block his Pawns; then I can't win, but that won't be too easy anyway. Also seek something to do along the Rook's file: P-N4 - P-KR4 - K-N2 - R-R1.

This is almost a complete, implicit formulation of the typical alternative problem of position B. Even clearer in its brevity is:

(M2; ... N-K1):

- 5 Different ideas are possible: either aim at P-QN4 or P-B4.

A third type of contribution to the formulation of a problem is made when the subject points out or delimits the path that his strategy or tactics might take. For instance, in (M2; B1):

- 18 Exchange of Rooks in general not good; must avoid that.

Such *general directive statements on how to proceed on the board* contribute to the formulation of a problem in that they help to define it. They delimit the problem and circumscribe the sphere of further investigations and considerable methods.

Even more important are explicit formulations of strategic or tactical goals on the chessboard: statements on *board goals*. They reflect much more completely what the problem is all about and what must be striven for in the game. In this respect they are almost equivalent to actual formulations of the problem given in question form (cf. p. 132). Again the protocol (M2; B) yields several clear examples:

(M2; B1):

- 72 The Rook must do something; other moves are a bit too passive in that position.

(M2; B2):

- 9 To hold back the Pawns doesn't work so well. No, I must do something on the King's wing.

Or, taken from another protocol, (C5; A):

- 41 Now to try and open up the King's side a bit more.

The difference between these goal formulations and the general directive statements cited above is neither great nor essential. An explicit board goal formulation does, however, possess a more direct meaning for the organization of the thought process insofar as it controls and largely defines what will happen in the next phase. When subject C5 says that he wants to try and open up the King's

side a bit more, then in practice this means that he is going to investigate if and how this can be brought about. In this respect the formulation of the board goal practically implies a goal-setting with regard to the organization of the thought process. With this we enter the realm of category *b*.

b. Statements of mental operations the player plans to perform, i.e. his operations-goals, figure prominently in the protocols. The formulations are given in relatively uniform and sometimes elliptic expressions. The infinitive 'to look at' is frequently used in the sense of 'looking for.' An example of an elliptic formulation reads:

(M2; B2):

- 16 No, look again:
2...R-N4; 3.P-B4 (investigation follows).

A few examples of other kinds of formulations:

(C5; A):

- 35 First the combinational stuff there with 1.B-R6.

(M1; A):

- 10 Now work out the complications.

(M3; A):

- 2 To search for a combination in connection with KB6 and Q5.
5 To figure out 1.BxN/5...
27 Look for something else...

(G5; C):

- 32 Let's look for a move.

(M2; ... N-K1):

- 16 Yes, we should really make a plan.

These are all operations-goals for searching in a particular direction or exploring a particular move possibility. In the material these indeed constitute the majority (cf., however, Section 41, D; p. 164 ff.). The goal the subject sets is to elaborate and to investigate the usefulness of a specific solving proposition with regard to a more general problem (often the main problem).³

c. The main body of practically all protocols consists of the reports of the analysis. Everything that is calculation comes under this. In (M2;

³ A *solving proposition* becomes a *proposed solution* only in the event that it is followed by such investigations and elaborations, however brief. This distinction will be consistently maintained in the translated text.

B2), for instance, the above mentioned operations-goal (line 16) runs into an elaborative phase, covering 16 lines. We should keep in mind, however, that such an elaborative phase must in its turn be supposed to contain a number of assimilative cycles of subordinate detail problems. This would certainly have become manifest by the appearance of other types of text elements if a technique more refined than 'thinking aloud' could have been used. In a number of cases subcycles and sub-subcycles can be clearly distinguished even in the present protocols.

d. *The report of the (partial) result and, finally, of the outcome of the analysis* is the natural end of each elaborative phase. The formulations may vary somewhat, but they are always easy to recognize. Sometimes they are just objective statements without qualification:

(M2; B1):

41 ...so that is not so good.
54 ...not worth much either.

(M2; B2):

20 ...not so bad.

Sometimes they are objective statements but with some qualification that expresses subjective uncertainty:

(M2; B1):

29 1...P-KR4, I don't like the idea of that very much.
60 Maybe not so crazy.

Sometimes they are subjective statements expressing the player's feelings:

(M2; B1):

32 1...R-N1, then 2.R-K7 is annoying.
48 No, doesn't suit me though.

Such expressions can be found in all the protocols. Sometimes the result of a particular elaboration is immediately *generalized*, to the extent that the subject, after arriving at the solution of the detail problem, formulates a result with regard to a more general problem. For example in (M2; B1):

41 ...so that (continuation) is not so good. I do not stand well after all.

The second part can also be considered a formulation of a result, this one regarding the more general question of how much the position is worth. As in most cases, the generalization of the outcome:

'not so good' is not based solely on the results of the continuation in question but on previously examined, other move possibilities as well (cf. (M2; B1), lines 12 to 67). Reaching the 'not so good' outcome is the last step prior to the integration of partial results into a more general evaluation that leads to the *generalized result statement*.

By this time, however, we have reached a stage where a partial result begins its assimilation into a new detail problem. The statement: 'I do not stand well after all' already represents an important contribution to a re-formulation of the main problem in its present stage. With this the cycle is closed. Indeed, in the protocol, a new contribution to the problem formulation now follows (which is at the same time a generalized 'qualitative result'; cf. Section 46):

42 He can always play P-QB3

immediately followed by a more specific problem (and board goal) formulation leading to a new proposed solution:

43 Can I prevent that?
1...R-N1; 2.QR-N1 and now B-Q7-QB6.

As usual, the solution proposal can be considered to imply the setting of an operations-goal (further investigation) that is not verbally expressed in the protocol.

With the aid of these categories, derived from the assimilative cycle of a detail problem, each expression in the protocols can be put into at least one category. Frequently, however, multiple categorization is possible too. A few examples will elucidate this.

When the subject, within the framework of a more general problem – to attain a certain tactical or strategic goal, to execute a plan, etc. – reaches his solving proposition, which starts with a certain move Z_1 , he often formulates it briefly: ' Z_1 maybe?' This is, of course, primarily a solving proposition, that is, the beginning of an elaboration (c). But as such, it implies a goal-setting with regard to subsequent investigative operations (b). But it may also be conceived as an abbreviated problem formulation (a): Will Z_1 satisfy the demands and fulfill the expectations? These may not have been previously verbalized, but they are undoubtedly present in the subject's mind. Moreover, the expression itself contains an expectation (anticipation), even if it is a weak and vague one: 'Perhaps Z_1 is the right idea; perhaps this move will bring the hoped-for result.'

For another example, see:

(M5; Q-Q1):

10* The question is how to organize my pieces to get them to the King's wing.

We are dealing here with a relatively clear problem formulation (a) that, however, is completely equivalent to the operations-goal (b): to *investigate* how to get the pieces to the King's wing. The formulation implies as well an important board goal of a strategic character: to direct the pieces to the King's wing. Here, too, we meet a number of the cases distinguished above in a single formulation.

Finally, a word about where the problem formulations, goal-settings, elaborations, and results occur in the protocol and about their meaning, for the total interpretation of the thought process.

In Chapter IV, A (especially Section 32) we saw how the phase structure of the thought process as a whole can be defined in the first place by the succession and subordination of the elaborative phases. Indeed, these elaborative phases always form the core of each problem (assimilative) cycle. Therefore, everything that does not fall under elaboration (c) belongs, as a result of our interpretation of the structure, either to the first Phase or to the transitional phases. The operations-goal (b), more particularly the investigation goal, always opens an elaborative phase – even though in the protocol it is sometimes formulated during the elaboration or even afterwards – while the partial result (d) forms the natural end of the phase. Problem formulations and the contributions to them (a) stand somewhat more on their own, however, and consequently, form an important part of the typical contents of the first Phase and the transitional phases.

It is a very rare protocol in which we find all five of the main aspects of one assimilative cycle verbally expressed. Such completeness is hardly to be expected, for that matter, since there always exists a very close bond between problem formulating and goal-setting, for instance; so close, in fact, that when there is a problem formulation in the protocol the subsequent goal-setting may be omitted without loss of clarity in the interpretation. Thus the wording of the problem: 'I wonder if, (what, how, where, etc.)' considered from the viewpoint of the organization of the thought process, is practically equivalent to the goal-setting: 'I intend to investigate if, (what, how, etc.).' Hence it is

* Asterisks indicate line numbers of the Dutch protocol text for untranslated protocols.

possible in general to interpret a problem formulation as a goal-setting. Consequently it strikes one as almost pleonastic when, upon occasion, a subject does express the two separately:

(M3; A):

60 Yet possible perhaps to get material advantage? Search for something.

This case must be carefully distinguished from a more specific solving proposition within the framework of a more general problem, as occurs, for instance, in (M2; B1) line 43-44 (see page 101). Here the two are not equivalent. The problem formulation 'Can I prevent that?' (line 43) would correspond with the unexpressed operations-goal: 'to investigate *if* I can prevent that.' It is only after this goal has been specified: (unexpressed) 'and more especially try out the variant....' that the operations-goal can inaugurate the actually following elaboration: '1... R-N1; 2. QR-N1 and now B-Q7-QB6.'

The practical equivalence of problem formulation and goal-setting and the variations thereof, discussed under *a* and *b* above, stems in part from the experimental conditions. The instruction was that the subject should not regard the position just as a task but rather that he enter into the position as though he were actually playing the game. This plan generally worked well, so that the 'consecutive determination' effect was strong (cf. 56). Most subjects became personally involved in the position on the board. With such an attitude, which was, of course, reinforced by the extraordinary skill and corresponding habit formation of most of the players, it goes without saying that the recognition of a problem immediately evoked a goal-setting (to solve the problem).

Apart from the formal linguistic distinction between (a) and (b) formulations, it is of course possible to maintain theoretically the distinction between the two types of processes, (a) and (b), as well. But then, they are automatically linked: each problem formulation contains a goal-setting.

To a reader of the protocols problem formulations and goal-settings are equivalent, to the extent that they both contribute highly to the clarification of the composition of the thought process as a whole. The protocols of subjects G5, C5 and M2 are extraordinarily clear in this respect; others are sometimes much more difficult to follow for, even though the subjects enumerate the calculated variations, they do not always mention the actual problem from which these calculations result, nor the goals – either operations- or board goals – towards which they are striving. In such cases the interpretation had to be

based on contextual 'interpolation,' i.e., the developmental stage of the main problem was reconstructed from preceding and following passages, in order to cull from these the special goal-setting at a given moment. Needless to say this was not always a simple matter: often there remained some uncertainty, some room for more than one interpretation.⁴

Section 38: Problem and goal-setting

From the close bond between the statement of a problem and the setting of a goal, described above, the following questions arise: how can concepts such as goal, goal awareness, goal-setting, and problem, problem awareness, problem formulation be given a more exact meaning than they have in normal language usage? In this section we shall briefly discuss these questions.

First and foremost it should be expressly stated that the concepts *goal* and *problem* are always used in Chapter V in a subjective psychological sense. For *goal* this is almost self-evident, but for *problem* it is much less so. We may speak of a chess problem, or a positional problem, but also of the food problem in wartime, of scientific, military, and other problems, without having in mind a specific subject for whom such a problem is a psychological reality at a certain moment. The choice-of-move-problem in Chapter I has also been introduced in this objective sense. In the rest of the present text, however, the term *problem* is meant to refer to a subjective, variable, and to that extent 'psychological' *problem as it is experienced by the subject at a certain moment during the thought process*. Unless explicitly stated otherwise the term never stands for a factual (objective) answer to the question of what the problem 'really is' but always for the (subjective) answer to the question of what the subject feels the problem to be at the moment.

The term *main problem*, in particular, will have the meaning of the psychological choice-of-move-problem at a particular moment for a particular subject. Since this can be shown to undergo a contin-

⁴ This uncertainty is not only the consequence of the incomplete reporting of the thought processes in the protocol. It may also result from actual ambiguity and inconstancy of the particular goal-settings of the subject himself.

In general, interpolation and interpretation – with the attending uncertainty – can never be totally avoided, however complete the subject's introspection, since he is generally rather more aware of the elaborative operations themselves, than of their actual function and his underlying goals. This represents one of the greatest difficulties in the applications of *Denkpsychologie* (see further Chapter IX).

uous development during the thought process,⁵ it is meaningful to speak of the *stage of the (main) problem* at a given moment.⁶

The term *goal awareness* is meant to refer to the subject's supposedly being conscious of striving for a certain goal that is explicitly intended (*bedoeld*, see Chapter II, p. 40). In the same way the term *problem awareness* is used for the introspective phenomenon of the subject who, at a certain moment, is conscious of the fact that he is aware of a certain problem (to be solved). Here, as well as in a goal awareness, the *minimum* requirement is that a specific problem be meant or intended (*bedoeld*) by the subject; it need not necessarily be present in his mind in a more elaborated form, however.

There exists a noticeable difference between goal awareness and problem awareness phenomenally, by means of which the two can be clearly distinguished in introspection. We have already seen, however, that this phenomenal difference is of minimum importance in our task of analyzing thought processes so that it is not necessary to pin it down descriptively in more detail.

In the term *goal-setting* (Dutch: *doelstelling*) the verb 'to set' introduces a new element. We must distinguish two different concepts of goal-setting:

First, *goal-setting as a mental activity or process* (setting *yourself* a goal). Every goal-setting as a mental activity obviously implies the existence of a goal awareness, though the reverse is not the case.

It could be said that when a subject *sets* himself a goal, his goal awareness must be 'stronger,' more 'conscious.' For in terms of introspective techniques this means, if nothing more, that an explicit verbal formulation of the goal(-setting) is in a higher state of readiness. This leads to the second concept:

A *goal-setting as a formulation* (establishing, verbally formulating a goal; or the result of this formulating activity: the *goal-as-set*). In connection with the text of the protocols the term goal-setting will be used mostly in the sense of goal-as-set. The term then refers to a more or less adequate verbal formulation by which the subject reports that he is setting (or has set) himself a goal – either an operations-goal or a board goal. In such cases we shall also speak of goal formulations.

In view of the parallelism between the concepts of 'problem' and

⁵ See the discussion in Chapter VI, Section 44.

⁶ Operational definition: Interrupt the subject at a given moment during his thought process and – while he refrains from continuing his analyses – ask him to explain in detail how he now sees the choice-of-move-problem. The verbal record of his exposition defines his *main problem* here and now.

'goal,' analogous distinctions can be made between problem awareness, problem setting (as a mental activity) and problem formulations. In the following text, however, we shall only rarely use the term problem awareness and shall altogether avoid the easily misunderstood term 'problem setting.' This is possible since 'setting (yourself) a problem' is practically equivalent to 'formulating' or rather 'trying to formulate a problem.'

As with both goal-setting and goal formulation, the term *problem formulation* may either refer to the activity or process of formulating (setting) a problem or to the result thereof, e.g., a problem formulation in a protocol.

These distinctions are by no means foolproof. Hopefully, however, they will be sufficient for an understanding of the meaning of the terms in the context of the following sections. In Chapter VI, Section 44, we shall have to discuss these concepts anew, particularly in view of their relationship to Selz's schematic anticipation.

Section 39: The general problem structure of the process of chess thought

The main problem develops through a series of successive *problem transformations*. To a large extent these transformations occur already in the first Phase, as we shall see. In the later stages of the thought process problem transformations occur whenever results of calculations are integrated; that is, they occur typically in the transitional phases.

In principle, we can distinguish three types of such transformations:

1. *purely qualitative transformations;*
2. *partitive transformations;* and
3. *specializing transformations.*

In a purely qualitative transformation a new problem is generated that is 'essentially the same' as the original one; only it is conceived (possibly formulated) in a different way. In a partitive transformation the problem splits up into parts that are to be tackled in turn. In a specializing transformation the problem becomes more specific or specialized, possibly simplified, as a result of the subject's solving a subproblem (cutting off a branch).

An example from the field of mathematics may serve to illustrate the three. In order to prove that $a = b$, the solver of the problem may decide to demonstrate the impossibility of $a \neq b$ (qualitative transformation). Next he realizes that he will have to prove the latter in two parts: first $a > b$ is impossible, and second $a < b$ is impossible (partitive transformation). Then, he may see that the assumption $a > b$ leads immediately to a contradiction, so the problem is again transformed: prove that $a < b$ is impossible (specializing transformation).

In the process of chess thought all three forms of problem transformations must be assumed to occur repeatedly. The purely qualitative (and the specializing) transformations are more difficult to discover in the protocols than the partitive transformations. Only transformations of the last type can be recognized by the complexities in the external structure of phases and subphases to which they lead. The composite structure of the chess thought process is, in fact, primarily due to continual divisions of main problems into subproblems and of goal-settings into subgoal-settings.

In this section a general description will be attempted of *the basic structure of the process of chess thought*, in terms of goals and problems, as this structure is built up through typical problem transformations.

As an introduction to the subject and in order to avoid any misunderstandings of the concepts of goal, goal-setting, etc., we shall begin with a word about the goal structures during the *actual game*.

The final goal for the player is, of course, to conclude the game as favorably as possible – to win or draw, according to the situation. As long as the final goal is not yet in sight, the player concerns himself with intermediate, less remote goal-settings, but even these may often span several moves and sometimes substantial portions of a game. Such subgoal-settings are known to every chess player. They are described in all textbooks on chess strategy: how to realize a certain configuration or plan of attack, how to exploit a certain weakness in the enemy camp, etc. During the game they generally form the essence of the player's goal awareness, whether he is at the board or walking through the room. When it's his turn his move must be in accordance with his plan and contribute to its realization – except when sudden changes occur in the situation as a result of the opponent's last move. While the goal of realizing a plan is a subgoal to the player's final goal, the goal of playing a move is in its turn a subgoal with regard to the plan.

Here, however, we must remind ourselves that we are concerned with goals of the game itself, i.e., *board goals* and not with *operations-goals*. The distinction made in Section 37 between goals concerning the organization of the thought process and goals concerning the strategy and tactics of the game now comes to the fore again in an even more general form. With respect to board goals the move (and the subject's pondering over it) is generally subordinate to the plan and certainly to the final goal of the game. During the process of thinking, however, we may always consider the choice of move to be the operations-goal par excellence.

This certainly holds for the experimental situation, where the natural move-game subordination is lacking. Here *the main goal* is and remains *the playing of a 'good' move*. That is not to say that in the process of thinking the problem cannot expand: the subject may make a plan or begin a combination, not only a single move but several moves in advance. But these broader considerations are significant only in the service of the specific problem of choice of move: one must plan in order to be able to choose a good move with sufficient certainty.

This is clearly seen in protocol (M2; B) – in all B-protocols for that matter (see also Sections 30–34). The alternate consideration of Plans A and B is necessary only for the investigation of the attainable results which are then decisive for the choice of move. That the plans play only an intermediate role becomes apparent from the fact that the struggle between the plans has not yet been decided when the first choice of move (1...R-N1) is made. The struggle is pursued only to the extent that it is necessary for the choice of a move. In (G5; B) the process was exactly the same. And when we read in (M2;...N-K1): 'We should really make a plan,' we may safely fill in 'to come to the choice of a good move.'

Whereas in the course of the game itself the moves form elements of the execution of plans, in the thought process the devising of a plan – whenever it appears – must be regarded as an element (subproblem) of the main problem: to choose a good move.

The following train of thought seems like an appropriate starting point for the consideration of *the general goal- and problem-structure of the thought process*.

The subject searches not only for a move but for a *good* move. He will only play a move when he is satisfied that it is 'good.' This satisfaction must be based on a sort of evaluative, possibly comparative, analysis of the move that is carried out in the thought process. To put it differently: In the subject's general goal awareness 'to play a good move,' one must assume a rudimentary schematic anticipation of an argumentation in favor of the move – of a chess-analytic proof. For the chess player this is already contained in his idea of 'a good move.' The general structure of this kind of reasoning was discussed in Chapter I (Sections 9 and 10).

How does the subject come to such an argumentation? By means of a mental analysis in which the position and the possibilities contained in it are viewed especially from the standpoint of his own in-

terests. Besides being carried out mentally, this investigation also distinguishes itself from an 'analysis of a chess position' in the ordinary chess-technical sense by being aimed at a somewhat different, more subjective goal. The goal is to build up a subjectively convincing argument for a move as yet to be chosen. To a large extent this particular goal determines the structure and methodology of the investigation.

We can thus assert that *in the position in question the subject concerns himself during the thought process primarily with an analysis of the possibilities for his side, and he does this in such a manner and as thoroughly as is necessary in order to arrive at a subjectively satisfying argument for the move to be discovered through his investigation*.

It would seem evident from the above that a distinction can be made between the analysis proper, on the one hand, and restructuring the results of the analysis for the final argumentation of the move, on the other.⁷ This would amount to a division of the main problem into two subproblems, of which the first is the preparation for the second: an *investigative search* functioning within the *striving for a 'proof'* and for rounding off the thought process.

A reasonable division in many respects. With the investigative search already geared to reaching a satisfying argument, however, it is difficult to detect a separate *phase* of argumentation-proof in the protocols. What we find in practice generally amounts to no more than a short check immediately before the move decision, or a brief recapitulation of the preceding, already implicitly completed, argument. Sometimes before the choice there is just a short but distinct pause in the thinking. More than once the chosen move is stated as a conclusion ('So...') and very often as a confirmation ('Yes') arrived at through recapitulation of previously established decisive considerations:

(M5; N-Q2):

13* (Pause). Yes, I should just *castle short* here.

(W2; C):

61 So let's play
1...Q-K5.

⁷ Compare, for instance, the process of proving a geometry theorem: (1) 'finding the solution,' that is, getting to the point where you think you know how to proceed, and (2) ordering your ideas to try to produce an actual proof – and possibly to write it out.

(M4;... N-B3):

52 Yes, Black's position is enormous then. Indeed. I play
1... N-B3.

The following cases are even clearer:

(M1; A):

48 Now to look over everything again. It's becoming clearer.
1. B×N/5 is the move.

(E3; A):

59 Just go over the main points again.
1. N×N.

The reasoning is usually checked briefly before the move is played.

The subject's striving to round off the argumentation – and with it the thought process – may also become apparent from those cases in which his first attempt is abortive. The protocol text then shows that the subject would like to come to a conclusion (final Phase). He is, however, apparently not yet satisfied with the arguments generated by his investigation; so he takes it up again. In (M2; B1) we find, in line 33, the rather final looking formulation: 'I should play 1... K-B1.' Since this is the third time this move has been mentioned, one could expect the decision to follow ('Yes, 1... K-B1'). Instead subject M2 renews his investigation; the actual final Phase comes later.

The occurrence of abortive attempts to round off the argumentation is one reason – next to others – why it is often difficult in a protocol to locate precisely the boundaries between analysis and proof. Even so, transitions to another way of reasoning, corresponding to another subproblem in the process, are often clearly demonstrable in the protocols, in particular near the actual ending of the thought process. We shall maintain from now on the distinction between analytic (or investigative) Phase(s) and a *final Phase of recapitulation and/or checking* (of the 'proof'-qualifications of the argument). It should be noted, however, that the two parts (subproblems) distinguished are greatly unequal in terms of their duration or protocol length.

Is it now possible to make a natural, empirically tenable *subdivision* of the investigative problem?

The only subdivision possible in all thought processes is between a provisional *position investigation* for orientation purposes and a further, deeper *analysis of possibilities*. From a structural standpoint this division corresponds to the one between the *first Phase* and the *main part*, that is, from now on, the rest of the thought process with the exception of the *final Phase*. It is by concentrating on the *subproblem* of 'position

investigation' that the subject during the first Phase effects the characteristic 'problem formation,' namely, the concretization of the *main problem* (cf. Section 31).

Here, too, boundaries are not so easily definable. In the position investigation subjects already look to see 'if there's something to be done' ((M4;... N-B3), line 11); conversely, each investigation of a possibility is at the same time a contribution to the position investigation. The chess technical concept 'position' itself already contains a dynamic element for the initiated: he speaks of a 'good position' when the future possibilities are favorable, and he calls a position 'won' when the game is still to be won (not seldom the most difficult part of the game).

The criterion for the distinction between position investigation and analysis (investigation of possibilities) and thus, generally, for the boundary between first Phase and main part, is *the occurrence of solution proposals* (see also Section 37). The beginning of the main part is characterized by a transition to more directly purposeful and systematic methods of investigation. Prior to that – within the first Phase in the broader sense (cf. Section 31) – possibilities are 'looked at' and lengthy variants may even be 'seen,' but purposefully calculating variants is typical of the analytic part. In closed positions, where there is not much to calculate, solution proposals take the form of plans. From the moment that the subject starts systematically to analyze, evaluate, and possibly compare strategic possibilities or plans, the first Phase is over. To this extent the following goal-settings (problem formulations) are all equivalent:

(E3; A):

12 1. N×N- let's calculate all the variants.

(E1; A):

9 Look for a combination;

(M2;... N-K1):

16 Yes, we should really make a plan.

(M4;... Q-K2):

9 How to forge ahead with the attack?

Though the place of these goal-settings in the protocol does not always correspond with the actual beginning of the analysis – sometimes there is a delay in executing the expressed intention; at other times the formulation lags behind – they do roughly mark the point at which there is a transition to a more systematic approach. Before the

actual formulation, one might mentally insert: 'Now for a deeper investigation, namely...' in the manner of (G5; A):

35 So far a somewhat disorderly preliminary investigation. Now, let's look in some more detail at the possibilities for exchange: $1.N \times B...$

Such a clear transition is rare, however.

The type of investigation that has been dubbed 'position investigation' is not limited to the first Phase. During the transitional phases, which have much in common with the first Phase, the subject often 'looks around' again, weighs possibilities without closer investigations, and uses other approaches typical of position investigation. This happens especially if the subject's first perception of the position yields a promising move possibility which is then immediately taken up as a solving proposition and investigated as such. In that case, the first Phase is very short: espying the move is immediately followed by calculations. If the results are disappointing, however, the interrupted position investigation is resumed. In a sense, then, the first Phase has a sequel. In such cases it may even be difficult to answer the question by distinguishing between a first Phase in a narrower and in a broader sense (see Section 31).

When we disregard such complications, however, we arrive at a basic structure for all protocols, founded on the above, a basic structure of three subproblems corresponding to Phases. These are (I) the provisional position investigation, (II) the analysis: investigation of possibilities, and (III) the 'proof,' check, or recapitulation of the argument.

I. *Provisional position investigation* ('looking around' and 'looking at'). This occurs in the *first Phase*. The attitude of the subject is more receptive than actively organizing; his thinking is less directly purposeful than in II: he does not yet systematically try out and test explicit solution proposals. The position investigation results in the concretization of the main problem: it is, therefore, also the Phase of problem formation.

II. *Analysis* ('looking for,' 'making plans,' 'calculating,' etc.). This occurs in the *main part* of the thought process. The investigations are more and more geared to their function in the fulfillment of the final goal, namely to prepare the final, subjectively convincing argumentation of the chosen move. The analysis gets more and more 'argumentative.'

III. *Check or recapitulation of the argument* ('go over again,' etc.). Although a separate phase of argumentation and proof seldom exists, in all protocols a brief *final Phase* can be distinguished in which the

most important results and part of the argumentation are checked and recapitulated. This then is the last preparation for the move decision.

B. TYPICAL SUBPROBLEMS IN THE THOUGHT PROCESS

Section 40: Subproblems in the position investigation (first Phase)

We have seen that the first Phase is largely devoted to an investigation of the position. In this investigation the main problem takes shape, becomes more concrete and specialized; the subject finds out for himself what is of importance from a chess technical standpoint, and he more or less establishes what can be attained.

Three typical aspects or three *moments* – at the same time three groups of subproblems – can be distinguished within this process:

1. The question of how the position really is. While looking at the position the subject assimilates and retains its most important features, its characteristics, and the functional relations among the pieces (*static moment*).

2. The question of the possibilities for direct action for both sides. There is a process of orientation with regard to such possibilities although no actual solution proposals are yet formulated (*dynamic moment*).

3. The question of how much the position is worth (from one's own side). The subject tries to arrive at some preliminary evaluation, based partly on the results of (1) static characteristics of the position, material situation, room for action, etc. and (2) threats, attacking chances, coming developments, etc. (*evaluative moment*).

In other words: The subject is primarily interested in three things: static features of the position, possibilities for direct action, and value of the position.

It is practically impossible to set up for the first Phase one general tree of the development of the main problem into subproblems and their branches. There is no generally stringent hierarchy which holds good for all of the various goal-settings and problem formulations in the protocols. They can be grouped exhaustively according to the three moments, but even the order of presentation of the moments – 1. static, 2. dynamic, 3. evaluative – cannot be interpreted as a hierarchy. The sequence maintained can only be said to represent the most frequent chronology in the protocols.

1. *Static moment*

It would be quite possible to devote an entire study to the perceptual processes with which every experimental session begins (cf. Chapter VIII, Section 61). The sub-‘problem’ to be solved here can be formulated as follows: ‘How is the position actually? What can be seen by just looking at it?’ To the subject, however, this subproblem is so self-evident that corresponding formulations do not occur in the protocols. On the other hand, formulations of the corresponding operations-goal are quite frequent in the protocols, as for instance in (G1; A):

- 1 First let's have a look at the position.

In most cases, however, the operations-(sub)goal formulation is lacking too; we find only some of the results, reported in the form of *remarks on the characteristics of the position*. These remarks represent, of course, but a fraction of what has, in reality, been perceived. By far the largest part of what the subject ‘sees’ remains unsaid. With the strong players of the G- and M-classes in particular, statements on perceived characteristics are rather rare. They appear to occur solely in the first few moments while the subject reports on his ‘first impressions’ or, if later, while he tries to solve other subproblems of the first Phase. The protocol (G5; A), for instance, starts with:

- 1 First impression: an isolated Pawn; White has more freedom of movement.

Later G5 mentions ‘the hanging position of the Knight at KB6’ (line 10) and the fact that ‘the pieces on KB6 and Q5 are both somewhat tied down’ (line 17); both times in the context of a broad examination of his own possibilities for action (dynamic orientation: see under (2)). In the following example the specification of characteristics serves as a justification of the position-evaluation that has just been given:

(M2; P-B4):

- 1 Count Pawns. Won for White, if only for positional reasons: the bad (Black) Bishop, passed (White) King Rook Pawn, wretched Black King position.

In the experiment (M1; A) subject M1 was requested beforehand not to forget to mention his first impressions. In this example the information the protocol provides is indeed somewhat richer.

(M1; A):

- 1 The first thing that strikes me is the weakness of the Black King's wing, particularly the weakness at KB6. Only after that a general picture of the position.

Finally, the complications in the center are rather striking: possibilities for exchange in connection with the loose Bishop on K7. Still later: my Pawn on QN2 is *en prise*.

So much for master protocols. Characteristically the protocols of less strong players contain more statements of the static type even when no special request was made for first impressions. Their perception of the position and abstraction of features proceed more slowly and require more effort; their first Phase, in its entirety, is of longer duration and consequently provides a better opportunity for details to be reported. In this respect protocols like (W2; B) and (C3; A) are quite instructive. (See Appendix II.)

Are there any specific features for which *all* subjects are apt to look, regardless of the position? Are there any static questions that are hit upon in *every* protocol?

The answer must be in the negative. With strong players, in particular, the perception proceeds unsystematically. Subjects report only what ‘strikes’ them in the situation. Features are striking when they distinguish themselves by their deviation from a (non-specified) *type* of position, one which is more or less well-known and considered ‘normal.’ Space advantage, the pair of Bishops, passed Pawns, isolated Pawns, and even material advantage are, in general, mentioned only if their presence is important and for that reason ‘striking.’ Practically the only exception to this rule was subject M2 who, in many cases, started by ‘counting Pawns,’ that is, with a systematic survey of the material situation. For the rest we find only some indications of this kind of systematic procedure, as for instance in (C3; A), a protocol from the group of less strong players.

With the master perception and abstraction proceed in an automatic way. Only the very first seconds of the thought process can be considered ‘pure perception.’ That does not mean that he does not look at the position (perceive) anymore after those first seconds; rather the perception serves more and more directly to solve other problems. It becomes ‘seeing if...,’ that is, searching for, investigating, analyzing. Thus when we find the words ‘looking’ and ‘seeing’ later on in the protocols, their actual meaning is more that of ‘seeing if...’ Only in the transitional phases does a recurrence of the real orientative inspection of the situation on the board occasionally crop up. For master subjects the looking at the position and the pure perception of its characteristics can be considered as an independent subproblem for just the first few seconds of the thought process.

2. *Dynamic moment*

Broad examination of possibilities for direct action in the first Phase – and sometimes in the transitional phases – differs from the investigation of possibilities in the main part of the thought process, mainly in the following respects:

- A. attention is primarily given to moves of a rather forceful nature;
- B. by definition: the broad examination of possibilities for direct action does not yet involve solution proposals;
- C. the subject looks at the situation from a more open-minded point of view: he considers possibilities for both sides and is less primarily concerned about what he himself can do;
- D. here, more often than later in the process, the examination of possibilities for action serves to build up the evaluation of the position (see under (3)).

In the protocols dynamic orientation can regularly be found in three distinct forms:

- a. consideration of plans and long-term board developments (for both sides);
- b. broad examination of possible threats of the opponent;
- c. broad examination of one's own (short-term) possibilities for direct action.

It could be said that these are three separate subproblems, but then it should be kept in mind that one is likely to have priority or to be particularly stressed, depending on the situation on the board and even on the subject himself. Rare are the protocols in which all three subproblems are clearly expressed.

2a. Explicit searching for and consideration of *long-term possibilities* are predominant mainly in 'strategic' positions such as B, ... N-K1, and K-N2, where there is for the time being 'really very little cooking' tactically ((M2; K-N2), line 22).

In protocol (M2; B) we encountered a clear example (lines 12–16, cf. page 101 ff.); dynamic orientation had begun much earlier, however, namely in line 4:

- 4 I can do a whole lot of things – as usual. Get my Rook into it, at the Pawns. Nowhere for his Rook to stand on the King's file, except on my K2. And that I can always prevent with K-B1.

Even in the 'tactical' position A some examples can be found in spite

of the fact that threats and forceful moves (cf. 2b and 2c) generally demand the most attention. One example:

(E4; A):

- 2 1. N×B looks good in order to exchange the strong Bishop, and next to exchange on Q5, to get a favorable ending. Another possibility is to start a direct King's attack, possibly to be prepared by Q-N3 and N-N4 or something like that. But most promising seems the former system.

2b. As a separate subproblem the skilled chess player only looks for *the opponent's threats* if the position provokes such a practice. In quiet and clearly structured positions such as position B he is likely to see at first sight that there are no immediate dangers. In contrast, position A is quite complex and contains tactical possibilities for both sides; it is not surprising, therefore, that the clearest cases of examination of threats are found in the A-protocols.

But here again there is a difference between weaker and stronger players. Less strong players seem more obliged to search explicitly for possible threats in the position so that fairly often clear-cut problem formulations are found in their protocols:

(C3; A):

- 6* Does Black have some dangerous threat? No. Well, yes, he could take the Pawn on N2. That is one.

(C2; A):

- 4 Let me see. There's no threat, is there? The Pawn on Q4 is attacked but defended. The Pawn on QN2 is attacked.

The stronger player, on the other hand, seems to see the threat as a matter of course; looking for (paying attention to) possible threats is with him an automatic operation or, in other words, part of a self-evident habitual set. If threats are mentioned in the protocol the stress is on a different aspect. The seriousness of the threats is sized up in order to get to a better evaluation of the position and a structuring of the main problem. Compare for instance the three following fragments from grandmaster protocols:

(G4; A):

- 7 Black has few threats. Q×NP is probably impossible. Or is it?

(G5; A):

- 3 Black threatens Q×NP. Is it worthwhile to parry that?

(G2; A):

- 8 Is the Pawn at QN2 really attacked? Or isn't that essential?

2c. The broad examination of one's own (*short-term*) *possibilities for direct action* is in many respects the pendant of the examination of threats. Often the subjects ask and answer the two questions 'Can he do something to me?' and 'Can I do something to him?' immediately after each other, either in this or in the reversed order:

(G5; A): (after a consideration of threats)

16 Can I do something myself? Investigate that first: the pieces on KB6 and Q5 are both somewhat tied down.

(C1; A):

3* Any attacking possibilities on the King's wing? (further on follows the counter-part):

5* Any dangers? Possibly ...B-N4.

In the second example the formulation is already somewhat specific: subject C1 asks particularly about (forceful) possibilities on the King's wing. Such a specification in the formulation occurs rather often, in some cases to a much higher degree. If, however, the position contains just one playable forceful move, Z_1 , no general questions about forceful possibilities are to be expected: only the question 'does Z_1 lead to something tangible?' Unless more explicit investigation of consequences follow, the subject's self-questioning and looking at possibilities for action should still be considered, in spite of their specificity, a part of the orientation to the position (first Phase). *Mutatis mutandis* the same can be said for the examination of threats, as discussed under 2b.

In one respect, however, the examination of one's own possibilities for action takes a special place among the subproblems of the first Phase: it may without further preparation start to function as a direct trying out of moves, that is, as an investigation of solution proposals. It does more than contribute to 'finding out how things stand,' in particular to 'what the position is worth,' for it also contributes directly to 'finding out what to do': to reaching the main goal of finding a good move. As we have seen on p. 148 it is in this way that an early transition from orientation (first Phase) to the investigation of possibilities (main part) can take place. One other example will suffice:

(G1; A):

4 Let's first have a look at what can be taken; are there any immediate attacks?

1. B-R6 and 2. N×BP – not sufficient.

1. N×B maybe? He must take back with the Pawn; with the Rook costs a Pawn, and with the Queen will not be possible either – indeed – so

1. N×B, P×N; 2 B×N/5, BP×B; 3. Q-B3.

Here we see an evolutionary transition from the first Phase to the investigative Phase via the subproblem: examination of one's own possibilities for direct action.

3. Evaluative moment

The question of the value of the position or, in the words of one of the subjects, (G1; A), line 2: 'Who is actually better off?' appears to be a very important subproblem of the position investigation in the first Phase. In the majority of the protocols the subjects somehow express, either directly or indirectly, their (preliminary) judgments on the value question.

Just how difficult it is to arrive at an adequate judgment on the value of the position largely depends on the position itself, of course. There exist cases where obvious material or space advantage is observed at first sight by every player with some experience; such positions, however, were not used in the main series. In the experimental positions it was typically necessary to weigh the advantageous against the disadvantageous characteristics of the position (1). At the same time the value of short- or long-term action possibilities (2) had to be estimated before a judgment could be given. In other words: The subject can only arrive at an integral judgment by taking both the static (1) and the dynamic (2) factors into account; *he must somehow 'integrate' the total position into one functional whole.*

On these grounds it is quite understandable that the subproblems discussed under (1) (characteristics of the position, the material situation) and (2) (threats, possibilities for action, possible plans) serve so frequently as subproblems of the position evaluation. Compare again:

(M2; B1):

1 (S counts the Pawns.) Difficult: this is my first impression. The second is that by actual numbers I should be badly off, but it is a pleasant position.

Here the counting obviously serves to determine the material situation; but the count is then compared with the general 'pleasant' impression (which probably derives from a very broad consideration of possibilities for action). The result of the comparison, that is the final evaluation, is not clearly expressed in the protocol, but it appears indirectly in the later course of events. The result must have been 'somewhat better or at least equal' since later on the subject says:

(M₂; B₁):

41 I do not stand well *after all*.

Furthermore with regard to the evaluation of the position, it is clear that there is a large difference between the masters (plus expert players) and the less strong players (of the W- and C-classes). For the latter group the 'integration' requires much more effort; correspondingly, they remain much more uncertain of their evaluation. They do not express their evaluative judgments as easily as the masters do, in whose protocols the first lines often contain a judgment of the position already:

(M₃; A):

1 White's position is superior in any case.

(M₄; B₁):

2* At the moment I'd say that Black's position is terribly bad.

Often no evaluative statements at all are found in the protocols of the weaker players. If, at the end of the experiment, the experimenter asked their opinion, it sometimes appeared that even then they had not arrived at a definite judgment.

In summary we can say the following about the development of the perceptual and thought processes in the position investigation of the first Phase:

The process starts with a looking at and an abstracting of characteristics from the situation on the board; as we shall see later, this process can itself be split up into (a) recognizing the position as one of a certain general type and (b) noting certain individual, characteristic features of the position. On the basis of the subject's knowledge and experience of the general type of position, on the one hand, and the individual, characteristic features of the position, on the other, the cues for a dynamic orientation are provided. Next comes a rough examination of short- or long-term possibilities corresponding to the general type and the individual features observed. In an analogous way, that is, via the subject's knowledge and experience of the general type and via the observed characteristic features, the subject next ventures an evaluation of the favorableness of his position. Without considering a particular position and, to a certain degree, the personality of the subject, it does not appear possible to determine whether this very first evaluation occurs before or after the (very first) dynamic orientation. Even in a very early stage some degree of

mutual interaction between the results of dynamic orientation and evaluation appears. By means of looking at the position (static moment), examining the possibilities for action (dynamic moment), and, finally, roughly determining how much can be expected (evaluative moment), the subject delimits his further investigations and determines the direction they will follow. He gets to the heart of the matter; generally speaking, he finds out what is 'essential.'

The protocols bring out quite clearly that what is done in the first Phase of position investigation serves to solve the main problem. Even the very first perceptions and abstractions are instrumentally related to the main goal of finding a good move and presenting a subjectively convincing argument that the move is good. The main problem is formed here as appears, for instance, in the implicit formulations of plan alternatives which were discussed above in (M₂;...N-K₁), (M₂; B), and (E₅; A). The originally indefinite task 'to look for a good move' specializes and concretizes during this Phase into 'looking for a move that is effective in leading to (direction)...'; the originally vague 'investigation' here becomes 'investigation in the direction of...'; 'seeing if... can be realized'; 'seeing if so much as ... can be attained.' Obviously the last specialization is dependent on the evaluation, which quantitatively defines what can be expected or what should be aimed at.

From this exposition it should be clear why the first Phase was called the Phase of problem formation (see also Chapter VI, Section 45).

Section 41: Goals and problem formulations in the main part (in the investigation of possibilities)

This section consists of a classification of the goal and problem formulations from the main part of the process. Since the treatment is restricted to the actual formulations as they occur in the protocols, the classification is more of a formal than psychological nature.

Again the interpretative standpoint is the organization of the thought process. Accordingly, problem formulations as well as board goals, strategic considerations, and statements on direction that really do lead to (or accompany) an investigation are all viewed as implicit operations-goals.

The following *classification* is based on *the generality of the subgoal-setting* (respectively the subproblem) where generality refers to the external form of the statement.

- A. Goal-settings with regard to the investigation of possibilities without any indication of the 'direction' that the analysis will take;
- B. goal-settings with regard to the investigation of possibilities with an indication of the direction but without mention of any specific first move;
- C. goal-settings with regard to the investigation of a specific move possibility and its consequences;
- D. specific subgoal-settings and problem formulations either within the investigation of a specific move possibility (C) or in a certain direction (B).

Apparently the series is hierarchical: each problem of a preceding category can be transformed into one or more subproblems of the next category either by a specializing transformation or by a partitive transformation (see Section 39). Thus, for instance, the general goal-setting for analysis (A) can split up into the goal-settings to investigate two plans, that is, in two directions (B, first partitive transformation); each plan investigation may split up into the analysis of a number of possible moves (C); and finally, each move investigation into the analysis of several variations and other subproblems (D). A specializing transformation can begin when the general goal-setting for analysis (A) specializes into the goal-setting to investigate a specific plan, that is, in a certain direction; and so on down the line. Compare the treatment of (M2; B), Sections 32 and 33.

The general goal-settings of type A occur most frequently at the very beginning of the investigation in the main part, although some examples exist in which they occur at the start of later phases of deeper investigation. Groups B and C contain all statements regarding the analysis of solution proposals that are aimed at the main problem, both plan (B) and move proposals (C). Group D contains all further subgoals and subproblems that may occur within the investigation of specific possibilities. It should be noted that the D-subgoals are not necessarily subgoals for analysis of possibilities, but they are always subgoals with regard to the organization of the thought process (operations-subgoals). Group D is divided into three subgroups:

- 1. goal-settings to analyze *specific variations*;
- 2. goal-settings to examine specific positions in the analysis (to be called *envisaged positions*);
- 3. goal-settings that are aimed neither at the analysis of a variation nor at a position: *other goal-settings*.

The fact that the 'rest group,' (3), has by far the least members shows

again that chess thinking is primarily investigating. The thought process is, so to say, predominantly empirical: it consists largely of examining, analyzing by trying out, sifting on the basis of outcomes, etc.

There follows a descriptive elaboration and illustration of this classification system:

A. General goal-setting for analysis without indication of direction

The transition from the first Phase to the main part, or sometimes from one investigative phase to another within the main part itself, is quite often introduced by a general goal-setting or problem formulation. For instance:

(M2; A):

13 Now let us calculate some:

(G2; A):

11 Which moves are worth considering?

It should be pointed out that here, too, the fact that nothing is stated about the direction of the ensuing investigation cannot, of course, be taken to mean that the subject had nothing more specific in mind (cf. Section 23). This general principle holds for the following cases as well.

B. Goal-settings to investigate in a certain direction but without mention of a first move

Much more frequent than the neutral formulations of group A are the verbalized goal and problem formulations which, to some degree, reflect the schematic anticipation. Various kinds of goal formulations can be descriptively distinguished according to the degree of precision and the manner in which the operations-goal is indicated. Each of the following types of formulation (italicized in the text) occurred at least five times in the main series of 43 protocols.

First, the *level of quantitative expectancy* or the *aspiration level* is often indicated. The statements generally refer to the estimated value of the positions that one hopes to arrive at.

(G3; A):

11 One has to see first if there is anything decisive in the position.

In other cases the subject searches for 'something positive' (E3; A) or a 'winning liquidation' (M5; N-N6), etc.

A second and frequent form of indicating what direction the

investigation is to take is by *considering and juxtaposing different plans*, often in the form of an alternative:

(M₄; B):

34* Either try to open up his King position or play B-Q7 and try to hold him back.

Similar formulations of the same alternative problem are found in practically all of the B-protocols; for instance (M₂; B₁), lines 12–16.

It is not always easy to distinguish the cases just mentioned from those in which the direction to be taken in the coming investigations is determined by formulations that *plot the board strategy*.

(C₅; A):

58 The whole thing is a matter of maintaining the pressure.

(C₁; A):

38* Something forceful must be done because, among other things, QN₂ is attacked.

In still other cases the direction is negatively determined, namely, by a statement that the subject is going to *try something different* (from what he has so far investigated). He searches for and is going to investigate subsidiary solution proposals. E.g.: ‘Now let us tackle it from another angle’ (C₂; A), line 25, or he just searches for ‘other moves’ (M₂; A) different from the direction (plan) thus far pursued. Exactly what and how much is rejected, that is, just how specific the goal ‘other moves’ does in fact become, depends, of course, on the extensiveness and the depth of the preceding analysis.

Still another form of indicating direction is found in ‘lumping’ or grouping formulations in which some *specific class of moves* is designated for investigation. For instance, the subject turns his attention to ‘Knight moves’ or to ‘Queen moves’ (C₃; A), or he poses a question like:

(M₂; B₁):

68 The Bishop, can that do anything?

At least in the master protocols, it should be noted, formulations of this type can hardly ever be assumed to represent a systematic scanning of what the various pieces can do; rather the formulations generally set apart classes of moves that have the same or a similar function. Thus subjects look at (and next start to investigate) ‘exchanges’ (G₅; A), (C₂; A), or ‘direct combinations first’ (M₄;...Q-K₂) or ‘a calm move’ (M₅; R-Q₁); etc.

When the subject states that he is looking for a *combination in some*

specific direction, both the board- and operations-goal are already much more precisely prescribed. He would, for instance, ‘really like to annoy his Knight on B₃’ (M₂; C), line 43, or he searches for ‘a combination in connection with KB₆ and Q₅’ (M₃; A), line 2. A very clear case is:

(C₂; A):

42 Is there really no feasible plan for an attack? The Bishop (on R₂) stands on the diagonal – it reminds me of a game of my own where sacrifices on K₆ were possible. Aren’t there any possibilities for sacrifices on KN₆ or K₆?

Finally, there occur formulations that explicitly set aside for investigation *moves* (and continuations) *that aid in carrying out a certain plan*. One example came up in the discussion of (M₂; B₂) in line 11 when plan A (King’s side attack) was definitely chosen: ‘No, I must do something on the King’s wing.’ In (M₂; K-N₂) the subject concludes a rather elaborate positional analysis with the words:

62 So let’s play the King away [to the Queen’s side] and then bring the Rook to the KR file.

In the succeeding protocol lines we find the corresponding elaboration (with investigations) of this plan. In general, the subjects explicitly search for a method that will lead to ... (the realization of a specific plan); that is, the protocol statement of the subgoal is followed by an investigation of how it can be worked out – and whether it works. The following goal statement is quite clear:

(M₅; R-Q₁):

115 The search is for a possibility to avoid the exchange of the White Knight, so that after N-B₅, it doesn’t have to be withdrawn. So I am looking either for a good square for the Knight or a threat such that after N-B₅ I can leave the Knight because he cannot afford to exchange.

C. Goal-settings to investigate a specific move possibility and its consequences
The frequencies in the protocol materials of the various cases that will be distinguished under this heading differ a good deal. The minimal frequency for a subcase to be mentioned – again indicated by italics in the following text – has been set here at ten. C-statements although erratic in appearance, are generally more frequent than B-formulations; an average of 4 to 5 C-formulations appear per protocol.

By far the largest subgroup – averaging two C-statements per protocol – represents the trivial case of *isolated goal-setting*, without any specifying comments, e.g., ‘1.B×N/5.’ The fact that the subject intends to investigate the move then appears only a posteriori, namely at that point when an analysis of the move actually begins.

Hardly more informative are the goal-settings which *solely state the intention to investigate* a specific move; for instance, the typical formulations of a solving proposition: 'Let's calculate Z1,' 'Look at Z1,' 'Z1 maybe?' 'Z1 deserves consideration,' and the like. To give specific examples of these simple and omnipresent formulations seems superfluous.

Somewhat more interesting are those goal and problem formulations that by their very wording betray something of what the subject has in mind, what he hopes to achieve, what he expects. Psychologically speaking, such statements partially, at least, bespeak the subject's schematic anticipation of the current subgoal. The following distinctions among various ways of specifying operations-goals should not be taken too strictly. Many ambiguous statements creep in that can be classified in more than one way, and, moreover, some formulations explicitly specify more than one goal.

First, as in the B-group, there are indications of the subject's *quantitative expectancies and/or aspirations*. The goal or problem formulation implies how much he hopes for or will try to achieve by the move he intends to investigate. Such an expectancy indicator is often found at the very start, but it may also occur somewhat later as a separate statement in between calculations.

A few examples:

(G3; A):

8 1.B×N/5 looks good. Let's calculate it: Does it give a decisive advantage?

(G1; A):

15 Immediately
1.B×N/5 maybe? Nothing special. Let's calculate it though.

(M2;... P-R5):

16 1...N-N5 doesn't appeal to me either – let's get a closer look: 1...N-N5; ...

To guarantee a clear notion of the subject's expectancy – often expressible as a numerical rating (cf. Section 8) – the wording of the formulations should be interpreted in the context of the complete protocol, of course.

A second type of specification of the operations-goal is found in formulations that indicate the subject's non-quantitative board goals, either strategic or tactical. Highly frequent are *indirect indications of board goals* that accompany the move to be investigated. They specify the board goal by referring to previously considered possibilities for action; that is, the move in question is intended as a modification of

or a preparation for an already analyzed – or at least mentioned – plan of action:

(M4;... Q-K2):

27 Maybe
1...B-B1 to prepare for N-N4—

(G3; A):

15 (After an investigation of 1.B×N/5). No direct decision.
Maybe first neutralize the Bishop at QB6?
1.N×B, then 1...P×N.

Since strategical positions in particular require careful preparation for any actions, it is not surprising that the protocol (M2; K-N2) provides a host of such preparatory goal statements (see Appendix II).

The counterpart to 'first prepare' is 'try immediately' (skip preparations):

(G1; A):

15 (After an investigation of 1.N×B, P×N; and on the second move 2.B×N/5, etc.): Immediately 1.B×N/5 maybe?

Within the third group, *direct indications of board goals*, two subgroups can be distinguished. The board goal may be *simple* so that it can be adequately expressed in a few words, or it may be *complex* and require a more involved formulation. Since complex goals are apt to be formulated incompletely or in an abridged fashion, some chess judgment and context interpretation are often needed to distinguish between the entries to the two subgroups.

Clear-cut cases of the first subgroup occur when simple defensive moves are proffered against direct threats, or when waiting moves, or moves that aim at the exchange of one of the opponent's dangerous pieces, or simple developing moves are suggested – provided that the aims are explicitly stated. 'Normal' attacking moves also belong to this subgroup, that is, moves that prepare an obvious plan of attack. For instance:

(C1; A):

40* 1.P-QN4 maybe, in order to have a moment's rest.

(M3; A):

31 There is no decisive combination. Then maybe an ordinary attacking move:
1.P-KR4 for instance.

In the second subgroup we find, from a chess point of view, very heterogeneous board goal formulations. A move may serve in 'building

up the attack,' to 'keep hold of the Pawns and the pressure' (C5; A), or to unfurl a 'masked attack' (G6; C), and the like. Such statements succinctly refer to complicated strategic or tactical maneuvers (goal-settings). More expanded descriptions of the ideas are rare. (C5; A) is a notable exception:

(C5; A):

62 ... quite another idea:

1. Q-R3, to start with a kind of pin of the Pawn on K6 – after Pawn takes on Q5 then the diagonal to QB8 opens up. 1. Q-R3 to be followed by taking on Q5; then after Pawn retakes on Q5, the diagonal's open.⁸

D. Subproblems and subgoals within the analysis of solution proposals (C and B)

The types of goal-settings listed under this heading are mainly subgoals within the investigation of specific moves (C). Normally the subgoals within plan investigations (B) are themselves the goals to investigate specific moves (C); it is relatively rare when the intermediate C-goal is missing. The few cases of (non-C)formulations of subgoals for B-investigations will be treated here together with the subgoals for C-investigations. Except for their different hierarchical positions in the system, there is no reason to distinguish between the two.

D1. *Goal-settings to investigate specific continuations (analysis of variations).* The investigation takes off from an envisaged position – not from the position on the board as in A, B, and C – at which the subject has arrived in the course of his analysis of possibilities. The question arises how to continue now. In such a situation he has to solve, so to say, a new choice-of-move-problem: he must either find a suitable continuation for his own game or for his opponent's. In principle, one could argue, such a situation occurs after every (White or Black) move in every sequence of calculations. The least we require, however, is that the protocol text provides some explicit indication that the subject is tackling a new choice problem or sets himself a new operations-subgoal. Minimally, in addition to the analysis itself, the text should contain such questions as 'What then?' or 'Maybe now Z1?' Cases with at least some goal specification occur on the average two to three times per protocol.

⁸ It should be noted that the inadequacy of C5's 'complex idea' – inadequate from a chess point of view – does not diminish its value as an example. The same obtains for some of the other examples. Often the weaker players are more explicit in their formulations.

In an envisaged position the *subject himself* can be *on move* (a); in that case, A, B, and C types of goal-settings can again be distinguished. The only difference from the ordinary A, B, and C goal-settings is that the position from which the operations start is not perceived materially but, rather, mentally – at least in part. Or (b) *the opponent* can be *on move*. The one additional difference is that the subject must consider the situation from his opponent's point of view. In any case, two more or less equivalent subgroups, D1a and D1b, can be distinguished; each has its own subdivision into A-, B-, and C-problems (goal-settings) with all their descriptive variety of cases. It hardly seems worthwhile, however, to go into such taxonomic detail.

Only a few examples will be worked out.

(E5; A):

(After a brief investigation of 1. B×N/6, there follows):

10 Or taking on Q5?

1. N×N. Then two, actually even three ways to retake. 1... N×N; then 2. B×N. Then he plays 2... B×B/N. How does that go on? Hold on: 3. P-B4, etc.

The problem structure of this fragment is already quite complicated. It illustrates clearly the specialization and splitting up of subproblems and goals (specializing and partitive transformations).

The process starts with 'taking on Q5?': a B-problem since this 'exchange' indicates a class of moves (either the Knight or the Bishop can take); see p. 160. In the next few words this is specialized as a C-goal-setting (without specifying comments, see p. 161), namely, to investigate 1. N×N. The next subgoal is to investigate the possible replies: the first D1-goal, obviously of the D1b-type (opponent to move). The result of the operation is all that is mentioned: 'Then two, actually even three ways to retake.' Since the second half of the formulation suggests that only 'ways to retake' were sought for – quite an understandable restriction from a chess point of view – the subgoal-setting parallels the B-goal-setting of considering a class of moves.

Next, the word 'then' in the protocol text (between '1... N×N' and '2. B×N') suggests a reasoning of the 'if... then' variety, corresponding to a partitive transformation: *First* the consequences of 1... N×N are to be investigated (a D1b-type analogous to a C-goal-setting: to investigate a specific move – for Black). Two second moves follow, a White one and a Black one: 2. B×N and 2... B×B/N. From a chess point of view these moves are natural continuations that do no more than follow up the idea underlying the initial exchange. White continues exchanging at Q5; Black tries to interfere by taking

the other Bishop. But then a new subproblem arises: 'How does that go on?' Since the subject himself is to move, we have a D1a-problem. The formulation is neutral, so this particular D1a-problem formulation (and implicit setting of an operations-goal) is analogous to the A-group.

In those few protocol lines we seem to find a series of specializing and partitive transformations which lead to a marked specialization of the problem. The implication, of course, is that the results obtained can have no more than a partial bearing on the original (B-)problem: 'taking on Q5?' Other variations must and indeed do complete the picture.

Most of the D1-subgoal formulations tend to be neutral, as in the (E5; A) fragment. Indeed, in the texts in general, the specificity of a subproblem appears to be inversely related to the likelihood of appearance of explicit indications of the subject's schematic anticipation.⁹

More explicit indications can sometimes be found in the goal-settings to analyze variations in a certain direction, that is, D1-problems analogous to class B. For instance, G5's question with regard to position C after 1... Q-K5; 2. Q×Q, N×Q; 3. N×BP, R×N; (G5; C): 'Can I then demolish him?' (a D1a-problem, analogous to group B). The drastic formulation means that only sharp, forceful continuations are sought and that the subject's quantitative expectations are rather high. The goal-setting is highly specific.

The following fragment is interesting in many respects:

(M2; ... P-QR3):

12 1... P-QR3; 2. R×N, P×N; 3. Q-B2 - 3... Q-B3 must be stopped, can that be done in another way? No - 3. Q-B2, R-B1; and 4... Q-B3; so that won't wash.

Within the analysis of the specific move 1... P-QR3, M2 investigates the possibility of countermoves other than 3. Q-B2 - a D1b-problem analogous to B (particularly, a goal-setting to search for and to investigate subsidiary possibilities; cf. p. 160). But the tactical goal for White is also mentioned: '3... Q-B3 must be stopped' (compare p. 162). Interestingly enough, the whole subproblem is liquidated here, as a prelude to the calculation of the main variation. Since the search

⁹ This certainly does not mean that serious interpretation problems arise with regard to the meaning of variations. On the contrary, in a master protocol the ideas underlying the investigation of a specific variation, that is, the board goals aimed at, are either self-evident for the chess expert or can be derived by specification from preceding, more general goal statements.

for 'another way' is fruitless, a continued, more specific investigation is dispensed with.

D2. *Goal-settings to investigate an (envisaged) position reached through analysis and the subgoals thereof* ('position investigation,' see p. 146).

This subgroup of goal formulations is characterized by the subject's more or less explicitly stated intention to investigate some position resulting from (mental) analysis. Here, as in the position investigation of the first Phase, the three moments discussed in Section 40 can be distinguished. Clear-cut goal statements with regard to the investigation of an envisaged position - as a subproblem of the investigation of possibilities - are relatively rare: on the average, one per two protocols. This type of subproblem is, however, highly important for the dynamics of the thought process in two respects: first, an investigation and evaluation of *the final envisaged position of a variation* is generally needed to arrive at a 'partial result.' Second, the investigation of an envisaged position may serve the same purpose as in the first Phase, namely, *to prepare the investigation of possibilities*. In the first respect the evaluative moment is primarily involved; in the second, the dynamic moment: a renewed dynamic orientation.

In both respects the subject's activity can be adequately described as a drawing up of the balance sheet, both materially and positionally - albeit to two different ends. Typical formulations of such subproblems occur after the calculation of a few moves at some point of relative quiescence (cf. Section 8, p. 24):

(G5; B1):

86 what have I actually got then? (meant both evaluatively and dynamically)

13 Now let us count Pawns (static moment; here a subproblem to the evaluation of the position)

(C5; B):

50* Is there still enough power in the position then? (dynamic and evaluative moment)

(M5; R-Q1):

109 Let's see how it stands then; maybe worthwhile. (static and evaluative moment)

(M2; A):

50 How is the material situation now? (evaluative moment)

The function which a balance sheet actually serves partially depends, it seems, on its entries. If, for instance, the position falls short of the subject's expectancy, a previously intended continuation may be omitted in the analysis. Or the other way round: An evaluation had

been intended, but now it seems more worthwhile to continue the analysis of variations. Again, no absolute distinction can be made.

D3. Goal-settings and problem formulations reflecting subordinate operations that are not reducible to investigative goals.

Finally, we come to the separate thought operations that might be expected in protocols using systematic introspection as an experimental technique. In the thinking aloud protocols, however, these kinds of goal and problem statements are rather rare: hardly ten can be found in the 43 protocols.

Most important are the cases of *immediate checking operations* pertaining to a partial result just reached. The subject quickly goes over some critical points, without entering upon an explicit re-investigation, however.¹⁰ The following fragment contains two examples:

(G5; B2):

102 2...R-N4; 3.P-B4; R-N4, 4.P-B3. Is that forced? Of course. 4...P-KR4, 5.P×P, R×Pch; 6.K-N1, B-K4, 7.QR-Q1; B-Q5ch. Well, that's a drawing combination; that's nice. *Is that so?* No, 3. P-B4 is not necessary to begin with.

The fact that checking operations appear relatively rarely in the protocols should, of course, not lead us to believe that 'determined checking operations' (cf. Section 18, p. 57) are less frequent in chess than they are, according to available evidence, in other types of directed thinking. Their scarcity might well be ascribed to the thinking aloud technique. In point of fact, the subject must generally be supposed to have no interest in nor time for reporting on detail processes, such as routine checkings. Indirect evidence in the protocols is sometimes found immediately after the statement of a partial result: another remark follows that casts doubt on the validity of the same partial result. In certain problem situations 'methodical doubt' even appears to be a distinct mental operation, indeed, a method of primary importance (to be treated separately in Chapter VII, Section 55).

Besides checking operations we occasionally find explicit *retention operations*, that is, statements setting a goal to retain some partial result:

(C3; A):

30* Not bad. Hold onto that.

or to retain some specific tactical possibility:

(G5; A):

81 That is to say, we will have to look out for [the possibility] 2...B-B5.

No other types of subgoal statements reflecting subordinate thought operations were found in the protocols. In particular, no subject ever explicitly sets himself the goal to imagine some position or variation in the analysis although it is obvious that this is automatically done all of the time. Most routine operations are never reported. To repeat: Any analysis of the organization of the thought process must of necessity remain macroscopic.

Section 42: Re-investigation of specific possibilities

In Section 36 repeated investigation, or *re-investigation* of the same move possibility (solving proposition), was shown to be a characteristic phenomenon in chess thinking. In the present section different types of re-investigation are analyzed and classified, particularly with regard to various goal formulations in the protocols. Besides, this analysis gives us an opportunity to comment on the function and the significance of a phenomenon that is somewhat surprising at first sight, namely, that the chess player frequently takes up 'the same thing' for a second (third, even fourth) time in his thought process. A few introductory remarks are first necessary, however.

First, in the statement above, 'the same thing' does not refer solely to move propositions as it did in the analysis of the formula of solving propositions in Section 36. It is true that re-investigations of the same *move* are the easiest to pinpoint in the protocols and generally preferable for purposes of illustrating the phenomenon of re-investigation. The phenomenon itself, however, is found to occur with regard to *plans*, *variants* and (envisaged) *positions* just as well.

Second, *both immediate and non-immediate re-investigation* are examined (cf. Section 36: both c- and r-cases). As to the former, we shall stick to those clear-cut cases which are not just (pre-determined) continuations of the investigation: not just preconsidered 'next steps,' perhaps introduced by no more than taking up the thread from where it was left off. The criterion for a case of *re-investigation* was, actually, that the protocol text and/or the following calculative branchings provide unequivocal evidence for a *change in the subject's problem- or goal conception*. Either his operations-(sub)goal, his board (sub)goal, or his expectations must have changed noticeably. True enough, most of the following examples are of non-immediate re-investigation since, again,

¹⁰ As a matter of course re-investigations after checking do occur, but then the operation (and its goal-setting) is again investigative and hence is excluded from the present category.

they are easier to pinpoint with certainty (straight from the formula) and are apt to be more spectacular – or at least more interesting.

Third, it seems in order to stress once again the *importance of the phenomenon of re-investigation*. In particular, the phenomenon cannot be interpreted as somehow reflecting a weakness of the human mind: 'indecisiveness' or 'weakness of memory.' If that were true the weaker players especially would produce many 'repetitions,' but this is by no means the case. In Section 36 the phenomenon was shown to be practically a general one; non-immediate re-investigation alone, for instance, was found in more than half of the protocols of the main series. A few illustrations:

(M3; A):	a - b - b - b - c		f - e - a - a		a - a - a - c - a - a
	orientation		investigation		proof ¹¹
	and exploration				

(E1; A):	b - c - a		a - a - d - a - a - f - f - a		n - n
	orientation		investigation:		proof
	and		a versus non-a		
	exploration				

A rather extreme example – although less extreme than those listed in Table 5, p. 123 – is Dr. Tartakower's treatment of position C:

(G6; C):	d - a - e - c		c - e - c - e		d - d - e - e - e		d - d - e - d
	↓		↓		↓		↓
	↑		↑		↑		↑

Move *d* (1...P-Q4) is (non-immediately) reconsidered three times before it is finally chosen, while move *e* (1...R-K1) even occurs five times: once, like *d*, in the exploratory Phase, and four times as a serious solving proposition.

Absence of non-immediate re-investigations rarely appears when the following three conditions are fulfilled: (1) the subject is a *strong player*; (2) he makes a *serious effort*; (3) he has to solve a *difficult problem*. Protocols in which there is hardly any re-investigation, like (W2; A) and (W2; C), are highly exceptional:

(W2; A):	c - e - l - k - r - i - s - g - b - b - b
(W2; C):	b - b - c - a - a - e - j - h - f - f - d - c

While conditions (2) and (3) were certainly fulfilled in both experiments, condition (1) is in some doubt. In any case, to explain the

phenomenon of re-investigation as a result of some mental weakness is out of the question.

The goal and problem formulations of re-investigation can, of course, be divided into groups A, B, C, and D as in Section 41. In A we are concerned with (the goal-setting to) a new, deeper investigation *in general*; in B with re-investigation in a certain direction (*plan*); in C with re-investigation of a certain *move*; in D with re-investigation of a certain *variant* or an already envisaged *position*. Such a classification, however, yields nothing new about the function and meaning of the phenomenon. The relevant question is rather: What is the subject's aim when he takes up an investigation anew? And what is the relationship between the goal-setting to re-investigate, on the one hand, and what went before, particularly the results of the first investigation, on the other?

All forms of re-investigation are, by definition, some sort of revision, some sort of deepening of the investigation. In the case of non-immediate re-investigation this need not be expressed in protocol formulations, however. Indeed, cases of (non-immediate) re-investigation *without any comment* are highly frequent. Just as uninformative and almost as frequent are re-investigations that are introduced by a *simple goal statement only* (compare group A in Section 41) such as 'look again...' (G5; B), 'Now look at that for the second time' (G6; C), 'Once more' (in many protocols).

More interesting are goal statements that express a striving for *extending* the previous analysis and/or for greater *precision, detail, certainty*, such as:

(C1; A):

19* 1.N×N; now a somewhat more serious look.

(G5; B2):

54 2...K-B1... (Subject has difficulty with the continuation)

57 So we have to go about it more carefully.

Generally such goal formulations bear upon the investigation of a specific move or a specific plan. Sometimes, however, they simultaneously introduce *an entire, new phase of deeper investigation* of the same or of largely the same possibilities, analogous to the transition from the first Phase to the main part. The re-investigation then provides an instance of the method of 'progressive deepening' (cf. Section 32, p. 106).

Can we hypothesize that non-immediate re-investigations in

¹¹ The reader is referred to Section 32 for the meaning of the letters; to Section 52 for a discussion of the terms orientation, exploration, investigation, proof; to Appendix II for the complete protocols.

general serve this purpose of overall deepening, that is, that they are always part of a more or less methodical, phase-wise progression of the analysis? This hypothesis appears to be tenable (cf. Chapter VII) provided we leave some room for qualifications and admit some borderline cases.

As to qualifications: The function of non-immediate re-investigations is, of course, not exhaustively characterized if we state that it serves the purpose of 'deepening the analysis.' In one instance the subject may do this because he has detected an error in his previous calculations – e.g. in position C, after 1... N-K5 it is not possible to play 2... N-B6; see (M2; C) and (M4; C). In another because his general expectations have dropped, he is forced to get back to and to reconsider what he had before. In still other instances he may be inspired in advance by some new idea for strengthening or reinforcing the old plan (see below); etc. Even so, in all these instances, the re-investigation can be said to serve the purpose of 'deepening the (old) analysis' as well.

As to the borderline cases: Sometimes a (non-immediate) re-investigation yields hardly anything new. Thus in the last part of (M5; B-R7ch) the subject in his despair practically begins to repeat himself; the same may happen from fatigue, particularly in actual chess games where fatigue is known to play a role. In a 'sound' thought process of a strong player such anomalies can be disregarded, however. Re-investigations generally yield new variants, new points of view, new evaluations; the analysis is, in fact, deepened.

On the basis of the deepening hypothesis for non-immediate re-investigations, the structure of progressive deepening phases can often be read directly from the formula of successive solving propositions. For instance, the 'rhythm' in a protocol like (M1; A), with the formula:

$$(M1; A): g - a - e - a - a - k - a \quad i - a - a$$

where move a (1. B×N/5) is evidently the favorite from the beginning, suggests a phase structure of progressive deepening as follows:

$$(g - a - e) - (a - a - k) - (a - i) - a - a.$$

The repetitions of a are clear guideposts. Such an interpretation can and must, of course, be checked by studying the complete protocol text.¹²

¹² The reader is invited to do this for himself. Apart from (M1; A), (G5; B1) and (M2; C) provide good examples.

Only rarely do immediate re-investigations have such a general, deepening function. They usually serve only to clarify the results just obtained. Sometimes the goal-setting even takes on the special character of a striving for proof of a partial result – thus approaching the cases discussed in Section 41, D3.

Progressive deepening of an investigation is not always a voluntary nor premeditated process. If we consider it a 'method' (see Chapter VII) it is clearly a subsidiary method since the subject hopes that the first (or second) investigation will provide sufficient certainty for him to make the choice, so that a deeper investigation is superfluous. Such an optimistic attitude is even found in the first Phase: the subject spies a move which he immediately calculates, hoping to end the thought process then and there (see Section 39, p. 148). He may, in fact, succeed like Subject M2 in (M2; ... P-QR3). Indeed, the importance of the method of progressive deepening lies in its very economy of thought.

What a linking such as (M1; A): (g - a - e) - a - etc., amounts to is that the subject takes up the investigation of a move that he has previously, albeit temporarily, rejected. Had he immediately come to the conclusion that move a was a winning move, then he certainly would not have taken the trouble to investigate move e . The decision is only made of necessity: other moves (plans) are unsatisfactory too. The difficult and unfavorable position B is a case in point. Most of the subjects were quite frank about their difficulties. Only after a somewhat painful problem analysis in a lengthy transitional phase do they decide to re-investigate (See (M2; B2), lines 8 12 and protocol (G5; B)).

One of the additional functions that a deepening re-investigation may have (see p. 172) appears quite often in the goal statements, namely, the *striving for strengthening*, reinforcements for the previously considered move or plan. In case of non-immediate re-investigation the subject then returns to an earlier rejected idea, move, or variant in the explicit hope that an amended approach will lead to better results:

$$(G5; B1):$$

44 Let me look again at the Rook file combination, if there's really nothing positive in it.

$$(C2; A):$$

54 Let's start over with:

1.B-R6. Another little plan: 1... R goes (etc.).

In case of immediate re-investigation the subject is not forced by necessity to reconsider an old thought, but rather ventures an immediate revision on the basis of the results just obtained. The results are not yet quite clear and do not come up to expectations; the subject feels there is some reason to hope for an improvement (anticipation) and sets himself the goal to search for a reinforcement. (G5; B) yields a good example:

(G5; B2) (after figuring out a variant):

92 We have had that before. Then he has five Pawns against my piece. But maybe I can improve that; or win something back immediately after? Then, he stands not so well after all.

The most frequent cases of descriptively discernible goal formulations, in case of re-investigation, have now been treated. For each italicized type there occur at least ten cases in the protocols of the main series.

As could be expected, re-investigations are frequent near the end of the thought process. There they often acquire a special character, however, stemming from the fact that they become more and more instrumental to the striving for probative reasoning, for rounding off the thought process. No longer is the analysis solely directed towards clarifying, deepening, strengthening, etc. Slowly the transition to the final Phase comes about. The cases of re-investigation that have this special character will be discussed in the next section.

Section 43: *The convergence to and the recapitulation of the argument (final Phase)*

In Section 39 we established that the investigations in the main part serve the goal of producing a subjectively convincing argument for a particular move. To the extent that the subject gets nearer to his goal this striving becomes more and more explicit. Towards the end of the thought process the analysis converges to an argumentation with a characteristic dual structure (positive and negative parts; cf. Section 9). This process can be described as a *convergence of the usually multiple choice-of-move-problem to an alternative with the general, logical structure: choice between 'a' and 'non-a,' 'a' being the favorite*. The subject's striving is increasingly directed towards proving the superiority of *a*.

The most frequent binary choice in the final Phase is between a favorite move Z_1 , and all other considerable moves Z_i : Z_1-Z_i . If the 'other moves' are not specified, the alternative problem can be symbolically represented as: $Z_1\text{-non-}Z_1$. There may be only one other considerable move, so that the final choice is: Z_1-Z_2 . In other cases

there may be two favorites, Z_1 and Z_2 , which embody the same basic thought and are together balanced against other possibilities Z_i ; therefore: $(Z_1 \text{ or } Z_2)\text{-}Z_i$. In such a case, if the favorites win out, there remains a final alternative, $Z_1\text{-}Z_2$, to be solved. The same obtains when the process converges to a plan alternative: $P_1\text{-}P_i$ or $P_1\text{-}P_2$ (as in many B-protocols); after the choice of P_1 , say, there remains the choice between the moves within P_1 – generally again in (a convergence to) binary form. The various types of structures will be discussed in Section 49; at the moment we shall deal with the characteristics of the convergence phenomenon itself.

Different subprocesses and operations ('methods,' see Section 54) can be distinguished in the convergence process.

The first is *clustering, whereby several move possibilities make up a group*. Although we do not find corresponding goal formulations in the protocols – thus grouping is not a demonstrably conscious part of the subject's organization of his thinking – the process itself and the subject's striving in this direction are generally clear enough. Sufficient proof would seem to be forthcoming from Section 41, B, where goal-settings to investigate 'in a certain direction' are recorded. As an investigative technique *all* of the subjects are apt to combine move possibilities into plans, or along certain ideas and strategies, or into apparently more formal groups such as 'Bishop moves,' 'calm moves,' etc. The frequent group formulation, 'other moves' (see p. 160), is especially significant since it is characteristic of the forming of an alternative problem: $Z_1\text{-}Z_i$. Through clustering into groups of possibilities, the number of choices is narrowed down; the multiple choice problem comes closer to an alternative problem.

A second important operation that narrows down the choice is the *elimination of certain move possibilities or groups of move possibilities*. Quite obviously this operation goes on all the time in chess thinking. When the subject concludes from his analysis that a possibility leads only to bad results, he of course discards it. Even apart from elimination or negative result *statements* a glance at the formula of successive solving propositions often shows clearly how moves are eliminated towards the end of the thought process. In (G6; C), for example, finally only two moves (*d* and *e*) are left so that an alternative problem of the type $Z_1\text{-}Z_2$ has arisen. If $1\dots N\text{-}K5=a$, $Q\text{-}K5=c$, $P\text{-}Q4=d$, and $R\text{-}K1=e$ (see Table 10, p. 130) the formula for (G6; C) is:

$d\text{-}a\text{-}e\text{-}c\text{-}c\text{-}e\text{-}c\text{-}e\text{-}d\text{-}d\text{-}e\text{-}e\text{-}e\text{-}d\text{-}d\text{-}e\text{-}d$.

More important than the mere occurrence of elimination is the fact that the striving for elimination can be found in many protocol texts and checked in the structure of the calculations of variations. Thus we may certainly speak of a *method* which serves the convergence of the choice problem. In (M₂; C) the approach to the final elimination of the move 1...Q-K₅ is as follows:

(M₂; C):

60 1...Q-K₅ perhaps; (the first investigation of 1...Q-K₅ follows)

64 No,

1...N-K₅ is much more attractive. The Q-K₅ variations don't appeal to me at all.

1...Q-K₅; 2.KR-K1... (re-investigation of 1...Q-K₅ follows).

The unfavorable judgment of the Q-K₅ variations is immediately followed by a rather extensive re-investigation of this move, which can only be accounted for if this investigation is aimed at eliminating the move. This interpretation is supported by the structure of the following calculation which has nothing but own-branchings, as it should be in a negative proof. The elimination of 1...Q-K₅ pushes up the stock of 1...N-K₅ which more and more develops into the favorite.

Favorite forming is the third method. By this we mean the (striving for an) increasing preference, an increasing positive-negative asymmetry in the gestating alternative problem. The tell-tale signs are easy to read. Most readily detected are the favorable expectations (anticipations) such as the above: '1...N-K₅ is much more attractive.' The signs come in gradations in the protocols: from 'a nice little move,' (G₅; C) and 'rather suits me,' (M₄; ... Q-K₂) to 'Indeed that probably is the best,' (M₂; K-N₂) even to 'I really don't know what could prevent me from playing...' (M₄; ... N-B₃). When such expressions are lacking, a gradual favorite forming process is at least suggested whenever the protocol shows a series of non-immediate re-investigations of the same move, in particular if the scope of the calculations increases and the favored move is finally played. This is, for instance, the case in (M₁; A), with the formula:

$$(M_1; A): \begin{array}{ccccccc} & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ g & - & a & - & e & - & a & - & a & - & k & - & a & - & e & - & a & - & a. \end{array}$$

The structure of the calculations (sequence of linkings) is again an important criterion. The clearest cases are those in which an originally explorative investigation becomes gradually more and more positively directed. At first, attempts at strengthening are found (own-branchings) and attempts at broadening (counter branchings); but the latter

in his favorite increases. Eventually he gets to the positive part of the proof (counter branchings exclusively).

The process of favorite forming is obviously the counterpart of elimination. While elimination proceeds from an originally rather neutral, explorative way of investigating to a negatively directed one, here the development proceeds from neutral to positive.

Like grouping (p. 175) favorite forming appears to be such a natural process for the subject that we find little evidence in terms of explicit goal-settings to strengthen a given preference (strengthening up to a point of justifiable proof). It is clear from the protocols that purposeful favorite forming occurs very frequently though. Whenever for the purpose of trying out one possibility is selected from several, favorite forming can, in fact, be said to begin since the most 'promising' possibility is tried out first. Thus a weak alternative grouping is formed – which may be compared to a working hypothesis. As soon as the subject shifts to another 'promising' possibility, the original favorite is abandoned, however. If a lasting alternative problem develops in which the subject becomes more and more involved, the working hypothesis changes into a preferred 'theory.' This analogy may shed some additional light on what is meant by the development of and convergence to an alternative problem.

We have just seen how the subject in the course of his thinking turns more and more from 'trying if' to the goal-setting 'to prove that.' We have now arrived at another point in the discussion: the (converged) alternative problem must become an argument. But for this to happen the favorite move Z₁ must first quantitatively dominate all other moves, Z_j; and second, the argumentation must become relatively complete. Only when these two conditions are (subjectively) satisfied does the argument have subjective proving power. Indeed we often find clear symptoms of the subject's striving to satisfy these two conditions towards the end of the thought process and especially in the final Phase.

The striving for quantitative dominance is primarily expressed in *mutual comparisons of move possibilities* and their results. By means of this method the subjective preponderance of the favorite is tested, checked, and evaluated. Evidence for this process is found both in the protocol texts and, sometimes, in the formula. For example, the sequence of moves in (G₆; C) hints at a comparison between the last two alternatively considered moves, *d* and *e* (see the formula on p. 175). That this is indeed the case appears from a sentence like

the following, said just before the move decision (line 60): '1... R-K1 is playable, but I prefer 1... P-Q4.' This clear comparative weighing of possibilities seldom occurs in the main part of the thought process. It belongs to the typical methods employed in rounding off the argumentation and the thought process in the final Phase.

The striving for quantitative dominance of the favorite is even more acutely expressed in a *search for a decisive strengthening of the favored continuation*, which is another characteristic of the final Phase. Just before the decision subjects often look for a final argument in support of the favorite. The following examples speak for themselves:

(M3; A):

(during the investigation of 1.B×N/5):

45 Yes, 1.B×N/5 is the move. With that White gets the advantage. 1...P×B is forced, and then a favorable position is reached. Play 2.P-B4 for instance. Maybe we can get even more out of it.

1.B×N/5, P×B; 2.N×B in order to win a Pawn possibly? No, that doesn't work. Let's look at 2.Q-B3.

(G5; A):

97 On positional grounds one could already decide on 1.B×N/5. Is there some immediate gain?

1.B×N/5, P×B; etc.

In these two cases (both concerning the same alternative problem: 1.B×N/5 versus other moves) there is a last minute re-investigation aimed at strengthening the argumentation by strengthening the favored move.

Finally, we find more than once in the final Phase the pendant of these attempts at strengthening: a *search for a decisive argument to the detriment of 'other moves' (non-a)*. Explicit goal and problem formulations are rare, again, as the striving for a negative part of the proof is, in general, less clearly expressed. But the search for 'rejective' arguments can be inferred from the frequent appearance of rejective statements. Immediately before the move decision we find such a rejective statement in (M2; B1):

72 The Rook must do something; other moves are a bit too passive in that position.

In (C5; B2) the choice problem has boiled down to: 2... P-KR4 or 2... P-N4: Z₁-Z₂. After one last re-investigation of 2... P-N4, Subject C5 concludes: 'So I just *have to* play 2...P-KR4.' That the final linking here is subsidiary (see page 110) is another indication that the last step is negative (and that the argument is indirect).

In addition to the quantitative dominance of the favorite, we

pointed out that a relative (subjective) completeness of the argument is required. Striving for completeness is seen in the characteristic *broadening* of the consideration and calculation of possibilities towards the end of the thought process. In a formal proof (as was explained in Chapter I) *all* possible counter branchings in the positive part and *all* own-branchings in the negative part should be dealt with. Such 'completeness' is, of course, never achieved in practice. We may, however, speak of a proof-directed broadening of the argumentation when at a given moment the subject considers and tries to dispose of new countermoves and -branches in the positive part; and/or new own-moves and -branches in the negative part. This 'disposal' is not absolute, but relative: the purpose of the subject is to convince himself¹³ that any new countermove (positive part) or own-move (negative part) is *no improvement* over already considered possibilities. Thus checking and broadening of both the positive and the negative part of the argument often go hand in hand. The following example of a final Phase demonstrates this very clearly:

(M2; C)

129 Is it possible to do something else?

1...K-R1,

1...KN-Q4, or

1...N-R4? Or

1...Defend the King Pawn (1...R-K1); I really have even less faith in all that. Everything is going badly.

Yes, I play

1...N-K5. In case he does nothing, then I play 2...N-B6 nonetheless. Then:

3.B×Pch, B×B; 4.Q×Bch, K-R1; and then on 5.N-N5 I probably play

5...QR-K1. Yes, I play

1...N-K5.

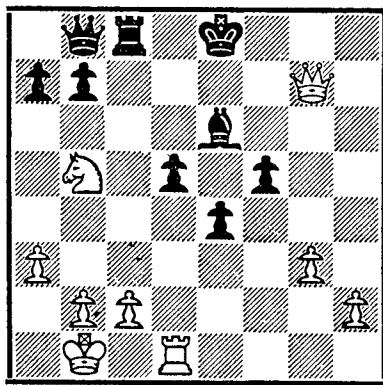
In line 129 the subject, for the *third* time, asks about other moves. Practically from the beginning 1... N-K5 was the favorite. This time the purpose is hardly to find other playable possibilities but rather to make sure that 1... N-K5 is the right move after all: so to check the negative part of the argument. At the same time the negative part is broadened: other own-moves as yet unmentioned (1... K-R1 and 'defend the King Pawn,' i.e., 1... R-K1) are expressly eliminated. This is immediately followed by a checking and final broadening of the

¹³ It would appear that quite some 'rationalization' (or 'justification') in the psychoanalytic sense occurs in the final Phase – as, for that matter, in last moments before decisions in general. The question as to how far the subject cheats himself in building up his subjective choice certainly does not concern us, however. Only the phenomena of decision and preparatory reasoning are registered here.

positive part of the argumentation. Broadening in the sense that a new counter branching is investigated, namely: 'in case he does nothing.'

The broadening here clearly serves the purpose of checking the argumentation for completeness: the subject checks to see if the scope of his reasoning was perhaps too narrow, if he missed something, be it own possibilities, Z_i instead of Z_1 , or counter possibilities after Z_1 .¹⁴

The final broadenings as well as the decisive strengthening of the argumentation often supplement the *recapitulation* part of the checking process. Finally, an example of this:

POSITION P-B₄

White on move

(M₂; P-B₄):

(after investigation of 1. P-KR₄, 1. N-Q₄, 1. Q-B₆, and 1. P-B₄, respectively; the last move has developed into the favorite)

- 28 1. P-KR₄ is a little bit too stormy.
 1. N-Q₄ blocks the Rook's line; too slow.
 1. P-B₄ looks rather nice; do you threaten still more? Yes. 2. P-B₅ is threatened.
 1. P-B₄, P-R₃; 2. P × P, P × N; 3. P × B pretty well does it. Yes,
 1. P-B₄.

In this fragment we find the (subject) summing up the relatively poor results of the first two investigated moves: 1. P-KR₄ and 1. N-Q₄ (the move 1. Q-B₆ had already been definitely eliminated). This obviously belongs to the negative part of the argumentation. But

then the favorite reappears: '1. P-B₄ looks rather nice.' The subject looks for another decisive point, however, in favor of the move: 'do you threaten still more?' (besides 2. P × P) – an attempt at strengthening its supremacy. The fact that there are indeed more threats forces the opponent to take immediate action, for instance by attacking the White Knight (1... P-QR₃). And now, as the final strengthening of the positive part, the so far uninvestigated countermove 1... P-QR₃ is looked into – with decisive results.

In analyzing the phenomena described in this section, the interpretation was based on several groups of data: the literal text of the protocols, the formula of successive solving propositions, and the structure of the calculations. This interpretive method already resembles the complex procedures that will be used for the analysis of data in the following chapters. For the present section, this way of interpreting has meant that the tabulation of 'clear cases' of broadening, elimination, etc., was more involved, for example, than the tabulation of the purely descriptive variations in the goal and problem formulations of Section 41. Therefore, when we state again that each italicized phenomenon appears at least five times in the protocols of the main series, it is not so easy to support this contention objectively – without any debatable interpretation – from the protocols. But then, it is not so important to think of the number 'five' as the result of an exact count anyway; it is nothing more than a rough measure of the generality of the phenomena. We are in fact concerned with what can be considered 'typical.' From this point of view the main point is that the occurrence of the phenomena described here is most certainly not confined to the protocols of one subject nor to one experimental position. For the methodology of chess thinking the importance of the phenomena can be taken for granted.

¹⁴ Among chess players of every class this is often a deliberately acquired habit: looking around to see if something has been missed, just before the move decision.

CHAPTER VI

THE DEVELOPMENT OF THE PROBLEM

Section 44: The solution process as development of the problem

According to the classical *Denkpsychologie* of Otto Selz any productive thought process may be regarded as a series of specific reactions introduced by a schematic anticipation of the goal to be attained. It is true that Selz developed this conception mainly via his study of relatively simple and brief processes of experimental problem solving (see examples in Chapter II), but he was convinced that it was just as applicable to more complex and/or long-term processes of, e.g., invention and mental creativity.

Later investigators, however, like Selz's pupil Julius Bahle and the Gestalt psychologist Karl Duncker, felt the need to introduce a few modifications. First, for the description of creative and productive processes, both of them used a less strict theoretical model: they did not insist that a productive thought process be a linear series of specific and even 'reflexoidal' reactions. Correspondingly, their language usage was somewhat looser. Second, and more important, both thinkers contributed to a more adequate conception of the dynamics of the productive process by drawing attention to previously unnoticed aspects and by introducing new descriptive concepts. In the following paragraphs some of these innovations will be briefly discussed.

In BAHLE's work (1936 and 1939) the creative process is described as a 'methodical activity structure' (*methodische Tätigkeitsstruktur*) instead of as a 'system of specific reactions' (SELZ 1924). The main theoretical intention – viz., to stress the high degree of explicit organization, contrary to romantic views of mental creation – remains the same, but the expression is less 'mechanistic,' indeed. Furthermore, Bahle includes as important new notions the creative pauses (*schöpferische Pausen*) and what he calls the principle of creative form-making (*das schöpferische Gestaltungsprinzip*).

Pauses in the creative activity must be considered regular parts of the process as a whole; they form 'meaningful links in the methodical activity structure.' They are also phases of a different character,

however, which deserve a place of their own in a theory of thought.

The principle of creative form-making is defined by the interaction of whole and part, that is, in the present context, of the problem as stated and the finding of means (*dem Wechselspiel von Ganzem und Teil, von Problemstellung und Mittelfindung*, BAHLE 1939, p. 312). The principle stands for the reciprocal and alternating influence of the total 'work problem' at a certain stage and the specific outcomes of its implementation (cf. Section 21). As a result of this reciprocal influence the process cannot be adequately described as a step by step implementation of one and the same problem: the problem itself undergoes a development. Along with the work problem, the schematic anticipation of the final goal also develops. Minimally it will take on a more elaborated and specialized form than it had in the beginning of the thought process.

This way of considering a *problem development* is also taken up by Karl Duncker. In Chapter I, Section 8 of *On Problem Solving* (DUNCKER 1945, pp. 7–8) he writes: 'We can ... describe a process of solution either as development of the solution or as development of the problem.' The idea or principle of a solution proposal becomes 'successively more and more concrete' and simultaneously entails a successive sharpening and specializing of the problem setting. As the solving proposition progresses, the problem-as-set becomes more concrete, more specific, and more productive, too. (Cf. *Ibid.*, p. 9: 'It is meaningful to say that what is really done in any solution of problems consists in formulating the problem more productively.')

As to chess thinking, the idea of a development of the (psychological) problem has been repeatedly expressed in the previous pages. It seems indispensable, indeed, for a proper grasp of the course of the thought process. The question remains, however, how this idea relates to Selz's much more precise theoretical conception. In this respect the innovations made by Bahle and Duncker must be formulated more explicitly. In the following paragraphs an attempt is made to clarify the issue.

1. For chess thinking as well as thinking in general, the creative pauses and the principle of creative form-making should be considered as mutually related.

In chess thinking we have found 'pauses in the thought process,' too. Although in general they were of a relatively short duration, they can be considered analogous to Bahle's creative pauses. They occurred typically in the *transitional phases*. These we have already

come to know as phases in which the subject returns from the special subproblems of the earlier elaborations to higher-order problems, in the most pronounced cases to the main problem. In the transitional phases the results already achieved are integrated, and it is here primarily that what we have called 'problem transformations' occur. After the first Phase of problem formation it is by means of these transformations that the problem develops – as discussed above. Therefore the interplay within the principle of creative form-making (which will be further investigated for chess thought in Section 50) corresponds with the alternation of phases of active elaboration, on the one hand, and phases of integration, 'looking around,' apparent pauses, on the other. During an elaborative phase the problem in its present state of development determines the course of the thought process via the schematic anticipation contained in the goal awareness; conversely, in the transitional phases the results of the elaborations cause a more or less drastic transformation of the problem.

The distinct pauses in a creative process or an ordinary thought process enjoy, in fact to a very high degree, this character of a phase of integration, of returning to more general problems, of renewed receptivity in an enlarged field. It is often during these very pauses that the most important problem transformations appear: the subject takes a 'fresh look' at the entire problem. The characteristic 'looking around' in the situation – for the chess player: at the board; for the composer: in life – which is accompanied by a negative abstraction of all details of the situation to the advantage of the most general, but still influential, schematic anticipation, is a natural continuation and extension of the return to more general problems demanded by the preceding 'integration of results.' The distinct pauses must be viewed as special cases of transitional phases that almost inevitably occur in complex thought processes (see Section 34).

2. The development of the problem actually takes place in successive transitional phases. It is true that not all of these phases, let alone the details of the corresponding problem transformations, leave their traces in the protocols. But we assume that there is an undercurrent: a practically continuous development of the main problem. In a protocol, this assumed continuous development then shows up only in those formulations that actually do get expressed in the more pronounced transitional phases.

This conception implies that at any moment in the thought process the main problem, together with the subproblems into which

it has been split and transformed, 'exists' in a certain state of development independent of whether or not the subject is experiencing a problem awareness at that moment. Note that we are not getting too far afield from the concrete phenomena since it is always possible to interrupt the subject at any moment to ask for a comprehensive report on the current state of the development of the problem. To this end such questions as 'What is the difficulty?' 'What is it really about?' 'What are you driving at?' and 'How far have you gotten?' are well suited. So, in principle, the 'state of development of the problem' is a concept that can be operationally defined – provided the subject is intelligent enough to report adequately on 'how far he has gotten.'

3. When we speak of the state of development of the (main) problem we shall mean, from here on, the problem together with the entire compounded substructure of all subproblems 'existing' at that moment. It is important to establish this because another psychological concept of problem is often used which does not include the substructure. We may say, for instance: 'The problem for the subject throughout the thought process is to find his next move.' In this sentence the word 'problem' refers to (a formulation of) the main problem without subproblems. Or when we say that the subject at a certain moment 'is aware of the main problem,' it is not necessary for him to have taken the entire structure into account. Certainly the vague forms in which, according to protocols, problem and goal awarenesses appear in the thought pauses preclude the presence of the entire structure.

4. A further consequence of this conception of a continuous problem development (see point 2) is that the original problem does not cease to 'exist' with the occurrence of a transformation, not even with the occurrence of a partitive transformation. Whenever the problem 'splits' into two subproblems, for instance, then it merely acquires a dual structure. The same holds for the corresponding operations-goal. At a particular moment in the subject's thought process several overlapping problems and goals (both meant in the psychological sense) exist, one of which may predominate as the problem or goal he is aware of at that moment.¹

This and the following sections will be devoted to an analysis of the development of the main goal and main problem (see Section 39, p. 147) which, following point 3, includes the substructure. For this

¹ 'Existing' (subproblems) means again, operationally speaking: subproblems that would be explicitly mentioned by the subject in case of interruption and questioning on 'the state of the problem.'

analysis every formulation (or contribution to a formulation) of a *subproblem* is of some value, since – again as a consequence of point 3 – it contributes to (the developmental state of) the main problem too.

5. If we assume a continuous problem development during the entire thought process, then there is no longer any reason to differentiate between the problem formation in the first Phase and the further development in the later Phases. Indeed, the distinction between first Phase and main part was made on structural grounds. From the point of view of problem development there is no essential difference. Nor is it possible to indicate a definite point in the thought process where the general instruction and the given chess position fuse into one problem, that is, a point where the solution process can be said to 'begin.'² There is no determinable point at which the problem can be said to take on a 'definite shape.'

6. In connection with the plural interpretation of the concept 'problem,' as seen in Section 38 and point 3 above, it is desirable to introduce a new term here. We may consider the development of the problem to be reflected in a *total goal conception* or *total problem conception* which is itself steadily developing. Into this total conception are incorporated all aspects of goal and problem which, at a given point in the thought process, are essential to the subject. So it contains even more than the state of the main problem with its substructure. Besides the problem structure, all anticipations concerning solvability, difficulty, methods that are possibly applicable, etc., belong to it. So do all mobilized (*in Bereitschaft versetzte*) dispositions as well as all nuances of intuitive and emotional preference which are based on experience and vague insight (favorite forming), etc. All of these aspects of the state of the problem at a given moment should be thought of as one whole, indeed as a total problem conception.

The state of development of the total goal (or problem) conception, like that

² This was possible for Selz to do because (1) his tasks were much simpler, and (2) he used systematic introspection and not thinking aloud. In a chess protocol the moment when, by a combinative process, the unified total task (*einheitliche Gesamtaufgabe*) comes into being cannot be pinpointed. Selz's criterion for considering the preparatory process terminated, namely, at that moment when the subject 'understood the task set to him,' is not applicable. When can the chess player say that he has 'understood' the position in front of him? Maybe when he 'knows how things stand,' when he knows the position – or, in more operational terms, at the moment he would, on interruption, be able to reproduce the position? From a point of view of *perception* this appears tenable (cf. Section 61), but we are considering the thought process. In any case, thinking aloud protocols do not provide much help here.

of the main problem contained in it, may be approached by the operations of interruption and questioning at any random moment in the thought process. The interrogation must be more thorough, however; the subject must be requested to render an inclusive report (artificially evoked by the questioning) of all moments of the problem as he sees it at the point of interruption. The completeness and exactitude of the report will, of course, depend upon the skill of the subject in introspection. He must especially be able to discontinue thinking of the problem itself: the introduction of new components must be avoided. A really exact and complete description of the total goal conception, one that includes all half-conscious overtones, vaguely echoing complexes, multiple interpretations, nuances, etc., is, of course, hard to get hold of with any subject – but it can be approximated.

In the case of long-term creative thought it is often possible to help concretize the state of development of the total goal conception through work already accomplished, or preparatory outlines, notes, and sketches. Obsolete materials must, of course, be excluded, but whatever is still instrumental to the subject in making his report on the present state can be said to belong to the goal conception. Thus in an actual game of chess the conditions that previously prevailed on the board may still be meaningful in making the subject's total goal conception explicit; in the experimental situation, however, this component was lacking.

7. The concept total goal conception is related to the *schematic anticipation* of classical *Denkpsychologie*. In particular, the two have the common feature of incompleteness, the problem character that Selz called the schema (with a gap). Since it seems that the two have been frequently confused in the older literature, it is necessary to make a clear distinction here (already begun in Section 21).

In the first place, the total goal conception like the problem undergoes a continuous development in the thought process; it 'exists,' which means that it can be elicited *at any moment*. This does not hold for the schematic anticipation of the total goal which depends on the subject's supposed awareness of the total goal at a given moment of the thought process. It is only possible to 'catch' the schematic anticipation of the total goal during its active moments, when it introduces the beginning of a thought process or a relatively independent part of one.

In the second place, the total goal conception lacks the functional, dynamic significance which is attributed to the schematic anticipation.

The latter is supposed to induce thought operations, which explains the importance of the concept in Selz's theory.

In the third place, the contents of the total goal conception are more inclusive and generally more detailed – certainly in complex productive thought processes – than a possibly emerging schematic anticipation. Only this does not hold for the very first schematic anticipation in the thought process; at this stage differentiation between the two concepts is practically meaningless. When, however, in the sequence of mental processes an awareness of the main goal (main problem) appears in the thought process, the implied schematic anticipation may be more or less detailed and worked out but will always be less complete than the total goal conception that the subject is able to produce on interruption and interrogation. In the thought pauses, the difference between the two concepts is particularly clear. We know that a vague goal awareness incorporating a likewise general and sparsely detailed schematic anticipation, may abide throughout the pause; whereas the total goal conception may already be highly developed. Thus at one and the same state of development of the total problem conception it is theoretically possible for schematic anticipations of varying degrees of detail to occur.

The relation between the two concepts may perhaps be most clearly expressed by a visual metaphor: each occurring schematic anticipation represents a certain 'aspect' of the total goal conception in which certain 'facets' are accentuated and others omitted. All of the elements and features of the total goal conception *may* appear (become active) in some schematic anticipation but hardly ever are they all simultaneously active. The latter situation only occurs in relatively short and simple thought processes and, as has been said, in the very first stage of problem formation in more complex thought processes. Only in these cases might one just as well speak of a continuous development of the schematic anticipation – as has sometimes been done in previous sections. Because Selz's experimental *Denkpsychologie* has been mainly concerned either with schematic anticipations within less complex processes, or with schematic anticipations at the very beginning of a process, it has been possible to neglect the difference between the two concepts.

A similar relationship can be postulated to exist between the total conception of a subgoal and a schematic anticipation included in a subgoal-setting. The latter also represents, so to say, an active 'aspect' of the total conception – i.e., not of the main goal (problem) but of a subgoal (subproblem).

In summary: A *problem* can generally be succinctly worded in a few phrases – although a composite subproblem structure may sometimes cause difficulties. A *total goal* (or problem) *conception* includes a clear and complete formulation of the problem; but it is a much more exhaustive exposition in which the subject's present anticipations and considerations are included as well. It is 'all there is to be told now' about the state of the subject's problem. A *schematic anticipation* presupposes an activated goal awareness at some point in the natural course of a thought process. It is not a descriptive nor operationally specifiable concept, but rather a hypothetical construct, used for a theoretical explanation of the dynamics of a thought process.

Section 45: Problem formation during the first Phase

Selz defined an '*Aufgabe*-transformation' as an 'exchange of the original goal for a more specific one' (*Vertauschung des ursprünglichen Ziels mit einem spezielleren*; SELZ 1913, p. 87) or as 'the substitution for the task of another task, through whose solution the original problem is also to be solved' (*die Ersetzung der Aufgabe durch eine andere Aufgabe, durch deren Lösung die ursprüngliche Aufgabe mitgelöst werden soll*; SELZ 1922, p. 41). If we accept an equally general, analogous definition for '*problem transformation*,' the development of the problem can be said to begin even earlier than the revealing of the position. The specialization of the main problem, as described in Section 39, p. 147, may be thought of as the first part of the process of problem development. In the skilled player this type of specialization (problem transformation) is automatically triggered off when he hears 'find and play a good move.' To the subject, this means that he will concern himself 'during the thought process primarily with an analysis of the possibilities for his side,' and he will do this 'in such a manner and as thoroughly as is necessary in order to arrive at a subjectively satisfying argument for the move to be discovered through his investigation' (p. 145.). We have to do here with an automatic transformation of the operations-goal. After the transformation the operations-goal can be defined as (the construction of) a satisfying argumentation for some move. The proper – for that matter, absolutely necessary – and implicitly intended means, both for finding the move and for consummating the argumentation, is the investigation of the position and its possibilities. Also, the possible necessity for more refined substructuring of the main problem is a priori included in the subject's total goal conception; he has a general idea of what he is aiming at.

As a result of the subject's skill in innumerable similar chess situations, an alarm is set off for a whole *system of established thought patterns* as soon as the task is accepted.³ Almost naturally the subject is set to look to see 'how things stand' (static moment) and 'what is going on on the board' (threats, immediate possibilities for action, and/or long-term strategic possibilities: dynamic moment) and then provisionally 'draw up a balance sheet' (evaluative moment). The manner of tackling such situations, the general method of investigation and working out an argumentation, is well known to the skilled player and is schematically anticipated in the general goal-setting.

The total goal conception acquires more concrete features only once the position is revealed. This is self-evident. The question is, however, how and to what extent can it develop in the first Phase? In other words: What characteristics of the goal-as-attained or of the problem solution ('solution' encompassing both process and result) can be schematically anticipated in the first Phase?⁴ Our main interest now lies in the main goal: the final argumentation. So the question is, in particular, to what extent does the total goal conception at the end of the first Phase already contain anticipated specific characteristics of the final argumentation and how do these characteristics arise from the position investigation?

1. *Pertinent moves: selectivity, board planning, and expectancy*

By the end of the first Phase the subject is certain to have found out that but a fraction of the legally possible moves, K , are worth considering. This appears from the fact that the argumentation after the first Phase is based on the analysis of just a few moves. The subject's 'selectivity' is high, since n_0 is small in relation to K (cf. Section 36, Table 4). The selectivity in terms of pertinent moves appears even more directly from the protocol text: in the great majority of cases, the subjects clearly search in a certain 'direction,' even immediately after the first Phase (compare Section 41).

Introduction of the notion 'pertinent moves'⁵ has already put us in a position to give a more concrete meaning to the expression 'searching

in a certain direction.' A certain group of possibilities for continuation that will play a special role in the further process of investigation and argumentation can be set apart. This grouping represents an important aspect of the total goal conception as it develops in the first Phase.

The very first taking in of the situation on the board (*static moment*) leads to concretization and specification of the problem. Through the observation of the characteristics or properties of the position – e.g., stage of the game, material balance, space control, strong and weak points in the Pawn structure, advantages in material or space on one wing or the other, exposed position of the King or some other piece, possession of the two Bishops, etc. – the subject classifies the position as one belonging to a certain *type*. Each type then corresponds to certain methods of play (just see textbooks on strategy, e.g., EUWE 1937; LASKER 1932). Such methods more or less define the 'direction' of the coming investigation of possibilities and, therefore, the group of *pertinent moves*.⁶

The typical first Phase examination (orientation) of possibilities for action (*dynamic moment*) leads to an even stronger specialization. If the subject notices a threat by the opponent, it must be parried, either directly or indirectly. Because of this only a limited number of moves require consideration; the direction of the ensuing investigation is prescribed. If he notices some possibility for his own direct action this may also lead to a limitation, via a positional evaluation of its 'promise.' If some forceful move appears to be bad at first sight, it may be discarded; if, on the other hand, it promises success, then the very large group of calm moves may be tentatively

not the move has already been 'seen' by the subject. In principle he must be able to give to the question: 'According to your judgment should Z_1 be considered?' an answer that is in accord with the state of development of the problem at the moment the question is posed. The subject is supposed to deliver his judgment immediately, with respect to the 'direction' in which at that moment he believes the investigation should go. A practical difficulty arises in forbidding him to continue thinking, a necessity in order to prevent the falsification of the current stage of development.

6 The concept 'type' must be taken broadly. We must not so much think of a rigid theoretical division into types, each having its own name, but rather of a highly differentiated but unwritten system which is in part only individually valid. The evidence for the existence of such a system is primarily found in the obviously immediate reactions of master players, reactions that must be based on *experiential linkings*: type linked to method of play. That is, (knowledge and experience of applicable) specific methods of play are immediately realizable – and, in fact, immediately realized – upon seeing the position.

3 In machine terminology things sound much simpler: a detailed and specific program is prepared for action.

4 See the explanation of the relationship between total goal conception and schematic anticipation, especially p. 187.

5 Operational definition of 'pertinent moves': moves which, on questioning *at a certain moment* in the process, are stated by the subject to be worth considering. Thus, whether or not a move deserves consideration has nothing to do with whether or

abandoned. In any case there occurs a subgrouping of the pertinent moves into 'forceful' and 'calm' moves, one of which tends to be the favorite (see points 2 and 3 below). Finally, via the preliminary examination of long-term possibilities the subject regularly engages in devising and juxtaposing plans applicable to this type of position. These plans, separately or together, give direction to the investigation and more restrictively define the group of moves to be taken into consideration.

In other words, by means of dynamic orientation the subject establishes what may be called the *dynamic core problem on the board*.⁷ In doing so he renders another and at least as great a contribution to the concretizing of the problem as is rendered by the classification into static *types*. Naturally, these two concepts are not wholly independent of each other and cannot be easily disengaged.

The temporary establishment of the estimated value of the position in the first Phase (*evaluative moment*) has a less direct influence on the group of pertinent moves. Still it does play a role: whenever the subject feels that he has an advantage, then he will, for instance, avoid all maneuvers leading to a mere exchange of material or an evening out of the position. *Noblesse oblige*: because of the high expectation certain methods of play do not come into consideration. The direction of search and the practicable freedom of choice is further delimited. This does, of course, not happen independently of the establishing of type and core problem; for that matter, we have already seen in Section 40 that the valuation is based on the static and dynamic orientation of the position.

We may summarize the specialization of the main goal-setting as follows:

The subject searches for a move that

- a) fits into a general method of play corresponding with the (static) *type* of position;
- b) contributes something to the solution of the (dynamic) *core problem*;
- c) guarantees positions and possibilities for continuations that are quantitatively in keeping with the *estimated value*.

2. Subgroupings of pertinent moves

In Section 43 we saw how the mental operation of converging toward a binary choice is based on a grouping of move possibilities within

⁷ This may be of a tactical or strategic nature. The psychological distinction made between static and dynamic (moment) does *not* correspond with strategic and tactical.

plans or along certain ideas and strategic lines. The first indication of such subgroupings of pertinent moves often appears as early as the first Phase of problem formation. From the protocols it would seem that they may arise relatively automatically as well as through conscious organizational action by the subject. From the data on various types of problem development that will be taken up in Section 49, it can be deduced that such a subgrouping had, in fact, already occurred in the first Phase in about 50% of the material – while in the remaining cases it can only be said that nothing showed up in the protocols. These early subgroupings, too, foreshadow the structure of the solution process and the argumentation to follow; they form important anticipatory elements of the total goal conception. Plan alternatives anticipate a groupwise aggregation as part of the argumentation and/or prepare an elimination of groups in the course of the solution; move alternatives, such as the type Z_1-Z_i , directly anticipate the duality of the final argument (positive and negative part).

How do such groupings come into existence in the first Phase? They are mostly of a generic nature, often automatically generated by the establishment of the type and core problem. Certain groups of alternative playing methods belong with certain situations. Here, too, we find a system of experiential linkings in the subject. As we know from numerous examples, the presence of possibilities for direct action leads to the distinction between 'forceful' and 'calm' moves. A hovering threat may be parried directly or indirectly; one can, for example,

(G6; C):

- 12 Simply cover the Pawn, *or*
- 18 pull off a masterly stunt, *in other words*
- 19 Search for combinative finesse.

In position B just two plans to combat the eventual threatening advance of the White Queen side Pawns come in for the master's consideration: try to hold them in check (blockading) *or* begin a counter-attack on the King's wing.

Evidently the dynamic moment is of special interest here: the orientation to possibilities often leads quickly and easily to natural subgroupings. The tactics and strategy of chess confront the player time and again with such alternatives. In more precise terms: Certain typical situations with regard to forcible action on the board actualize certain alternative groups of pertinent possibilities by means of specific problem transformations of the general type: 'one can either

do A or B.' E.g., exchange or let be, liquidate or maintain the pressure, retreat or protect, counterattack or defend, retake or keep going (so-called desperado combinations), open the position or keep it closed, etc. Sometimes there are three groups. E.g., the choice between strategic possibilities (in various openings): push, exchange, or leave a Pawn. We have become acquainted with the majority of these 'solution methods' from the problem formulations in the protocols; all subjects apply them. However, the master's system of linkings is quite obviously much more differentiated than that of the less skilled player. As a result, he is faster and surer in his subgroupings, while his alternatives are more adequately tailored to the objective situation on the board (see Chapter VIII).

In some cases there is a quantitative difference between the goal-settings of the two groups of move possibilities. These cases deserve special attention. When the subjects begin the investigation of possibilities by searching for a 'decisive combination' (as in (G₃; A) and in (M₃; A)) this, too, implies a subgrouping of pertinent moves; namely, forcing, combinative moves versus other (pertinent) moves. Furthermore, the difference in value determines the sequence of the investigation:

(G₃; A):

11 One has to first see if there is anything decisive in the position.

Only after that are other moves taken up – if necessary. These quantitatively inspired groupings are typical, conditionally partitive, problem transformations that attend certain types of favorable and tactically promising positions. Official chess theory has not until recently dealt with descriptions and classifications of 'tactical characteristics' of positions, from which probable combinations can be inferred – besides Lasker and Spielmann, Dr. Max Euwe has made his special contribution (Euwe 1937). But for centuries chessmasters have had an unwritten but highly differentiated system of fitting experiential linkings at their disposal. The old masters would not have found their combinations if, at the right moment in the game, they had not first looked 'to see if there is anything decisive in the position.'

3. Formation of favorites

When a move or plan is the favorite this means, in terms of problem development, that an alternative problem Z_1-Z_i or P_1-P_i exists which anticipates the positive-negative structure of the final argument. At the very first appearance of groups in the first Phase one of them may enjoy certain overtones of the favorite. Some preference for one or

the other approach to the board problem is practically always evident from the protocol text. It is, however, not so easy to detect how this preference comes about during the position-investigation.

In a great many cases, at any rate with masters, we may ascribe early favorite formation to an automatic activity of experiential linkings. First to come up for investigation in a given position is the special method of play which attends its 'type' and its (type of) core problem. If there is only one move that can achieve something positive in the direction defined by type, core problem, and estimated value, then this move automatically becomes the favorite. If there are more moves that correspond to the direction (board goal), often while scanning the possibilities the subject 'intuitively' chooses a tentative favorite. Rarely does completely pure, comparative examining of different pertinent moves or plans occur without a trace of emotional preference for one of them. In Section 43 it was noted, for that matter, that every examination of a move could in itself be conceived of as a temporary choice of a favorite, in the sense of a working hypothesis. In any case the shades of favoritism in the subgroupings, a regular occurrence in the first Phase, also contribute to the concretizing of the problem.

4. The quantitatively attainable

The estimated value of the position is not only of significance as a contributing factor to the three previously discussed aspects of the total goal conception; it also forms the foundation for the *quantitative expectancy* – the subject's specific aspiration level – with regard to which the outcomes of investigations are evaluated.

At first sight it may seem that the estimated value is at the same time the 'expectancy (value),' that is, a quantitative anticipation of the results obtainable from the position. The estimated value roughly sets a level for the rating of end results of the essential variations in the argumentation; to a certain degree it determines the minimum of the positive part and the maximum of the negative part (cf. Section 9, p. 28).

A complication presents itself here, however; the chess player (even the subject!), involved in his own cause, generally sets his hopes and expectations as high as possible, that is, in general, *higher* than the estimated value of the position at the moment. That is why the estimated value does not itself play a decisive role in his goal awareness; rather the role is played by its derivative, a subjective quantity, which will be called the *maximal expectancy*. The latter is roughly

expressable by a number (cf. Section 8) that is always larger than or equal to the estimated value. The maximal expectancy at the end of the first Phase must be regarded as an active anticipation derived from the established estimated value of the position. In representing (anticipating) the quantitatively attainable the maximal expectancy is of great importance in the final argumentation in regulating the main part of the solution process – as we shall see in more detail in Section 50.

The chief difference between the maximal expectancy and the estimated value is that the former contains an element of hope and is not independent of the will to win. The latter is more nearly objective: it is the rating that the subject would assign to the position at a particular stage in the development of the problem *if asked to evaluate, as objectively as possible, the position on the board*. The maximal expectancy is the rating that he would give if asked how much he could hope to attain in the position, given ‘good’ play by the opponent. If, for instance, the subject is of the opinion that he stands badly yet is not resigned to losing, then one could say: the estimated value of the position equals 3, while the maximal expectancy equals 5.

The numerical *difference* between the maximal expectancy and the estimated value appears, above all, to be dependent on the subjective certainty of the evaluation. The margin of uncertainty surrounding the estimated value depends in its turn on the character of the position. Note the difference between a dead drawn position that can be exactly assessed and one containing many complicated combinations. Further, it depends on the stage of the thought process. In general the difference between the maximal expectancy and the estimated value (both variable) contracts as the thought process progresses as a result of the increasing objectivity and certainty of opinion of one’s chances – a quantitative echo of trying to fit the desirable to the possible. Last but not least, the certainty of the subject’s assignation of value is dependent on the skill of the player (cf. Section 40, p. 156). Thus the master’s maximal expectancy generally remains closer to the estimated value – which in its turn is closer to the objective value of the position – than does the less skilled player’s. In general, it is also more nearly stable.⁸

8 There exist, however, remarkable differences in temperament. For instance, there are definite optimists: they try to maintain a maximal expectancy of 10 even in equal or somewhat inferior positions. They answer, when asked how they stand, with their maximal expectancy rather than with their estimated value. Others of a more cautious nature not only place the maximal expectancy less high but also

5. *The view of the objective board problem*

In the four preceding points the main concern has been to describe the more formal and structural aspects of the problem development in the first Phase. Most of what has been said so far is in all likelihood just as valid for the thought process of, say, the checker player or for the solver of other problems of rational choice. We shall now briefly discuss the problem formation of the first Phase from a point of view of content: the subject’s view of the strategical and tactical problems on the board.

Understandably this is the most important aspect from the standpoint of chess. In fact, goal formulations and especially problem formulations are generally conceived in terms of chess strategy and tactics (see Section 41). To the chess player the type, core problem, estimated value, and the pertinent moves together with their subgroupings and streaks of favoritism are especially important as anticipations of the solution of the *board* problems that the position poses. Moreover, the subject, as a chess player who wants to play an objectively good move, is primarily interested in their *objective* solution. That is, the subject generally considers his view of the board problems, not as an anticipation of what *his* solution will be but rather as an approach to what an objective solution (analysis) would show. The latter is a fresh complication which to avoid confusion will be briefly discussed here.

The state of development of the problem (total goal conception) can be said to anticipate the solution of the problem not in two (p. 190) but even in *three senses of the word ‘solution.’* In the first place the ‘solution’ is anticipated as a process (functional meaning of the anticipations); in the second place as a result, as the product of the subject’s act of solving (*his* reasoning, argumentation); in the third place as the ‘right,’ objective solution to the choice of move and analysis problems such as an ideal analysis of the position would render.⁹ The chess player himself considers his provisional (‘at first sight’) view of the objective board problem, as contained in the total goal- or problem conception by the end of the first Phase, almost always as an ‘anticipation’ of the solution in the third sense of the word,

answer the question with the estimated value, which itself may have been reduced. Among chess players in general no clear distinction between these values is made, which sometimes leads to misunderstandings between players and outsiders.

9 If the position is objectively solvable, then in practice an ideal analysis is fully realizable; if it is not objectively solvable, the ideal analysis can only be approached. It is easier here to introduce distinctions than to maintain them consistently!

thus as an approach to the 'true nature' of the problem. In this sense the definition of the operations-goal could be revised to read as follows: 'The construction of an *objectively* demonstrable argumentation...' (see p. 145). Indeed that is the ideal: the player (subject) strives for objectivity and completeness in his argumentation (see also Section 43). But, on the other hand, he knows quite well that this ideal can but seldom be attained – hence the formulation 'subjectively convincing.'

Value judgments of the board problem are, of course, only possible by viewing it objectively. Here the crucial question is whether the initial problem conception comports with the objective analysis and not whether it comports with the subject's final argumentation. All the moments of the total goal conception described above may also be considered as parts of the development of *the subject's view of the objective board problem*. Corresponding to the subjective concepts already introduced: type, core problem, estimated value, considerable moves and plans, favorite formation, there are objective concepts; the former then are approaches to the latter. To subjective subgroupings of pertinent moves correspond subgroupings in an objective analysis; the favorite move is the best move 'in all likelihood,' etc.

Moreover, in the first Phase all sorts of *special anticipations* are formed in connection with specific features of the objective nature of the position, the choice of move problem, and the analysis. Perhaps many of these anticipations are subsumable under the above named groups, but because of the extraordinary frequency of such formulations in the protocols they merit a brief discussion.¹⁰

Among the anticipations found in the first Phase are those concerning the general nature of the objective choice of move problem (difficulty, complexity, objective solvability, etc.); the nature of the (board) problems in the position itself (strategic, tactical, etc.); the presence of combinative possibilities; the practicability, playability, desirability, and necessity – as well as the respective opposites – of certain methods of play, plans, maneuvers, combinations, parries, moves; the objective, qualitative and quantitative results of certain variations or ways of playing; the objective valuation of envisaged positions; the objective value of certain plans, ideas, moves, and variants. In the protocols the formulations of these are usually very simple. They present the thought with a single word or elliptical

¹⁰ In Section 37 the anticipations were introduced as a separate type of content element.

phrase so that they can only be contextually interpreted. Typical expressions are 'I have the feeling (impression, idea) that...', 'it looks as if...', 'it may...(be like that, have to happen).' Very frequently 'probably' is inserted in the verdict. Also the words 'perhaps' and 'maybe' – occurring seven, eight, or even more times in some protocols! – are generally indicative of the presence of an anticipation. A few examples:

(E₁; A):

- 1 White has more room. Look further. The Pawn at QN₂ is attacked. Can it be taken? Quite probably; a threat. 1.P-QN₄ is to be considered. Does it threaten anything? Hardly. KB₆ is weak, and White has more room; he is probably somewhat better off. 1.P-QN₄ simplifies too much; not so strong.
Look for a combination; *maybe* something there.

(M₅; N-Q₂):

- 2* Is there really much of a problem here? *Presumably* you could make several moves here.

(C₅; A):

(interrupted after 15 seconds):

- 9 Now I'll have to start thinking of a combination, but *I am afraid* that there are no immediate winning possibilities.

(M₂; A):

- 1 White's position is good, but for the endgame not so good. Will *probably* have to try an attack on the King.

Sometimes an emotional formulation hides an anticipation: thus we must often read '... I don't *like* it...' as '... *is probably* no good.' In colloquial language distinctions between intuitive and emotional preferences and arguments are generally unclear. When the subject in (G₅; C) says '1... N-K₅ is at first sight a nice little move,' then he means little more than '1... N-K₅ looks good and is (probably) to be considered.' When the subject in (M₁; A) says about a certain possibility, 'I don't *feel* quite safe,' he means 'it will *presumably* be necessary to exercise caution during the execution of the coming continuations' or 'the analysis will probably confirm that against less strong play the opponent can get dangerous counterchances.'¹¹

One could say that 'intuition' and a player's 'feeling' for a position are what actually gives rise to such special anticipations. Such statements are in order provided that we maintain that this intuition is

¹¹ Sometimes one finds a real, purely emotional preference or dislike, which is independent of objective or intuitive judgment. E.g., subject M₂ states in (M₂; A): 'White's position is good,' but, nevertheless, he later finds it 'an annoying sort of position,' while it is clear from the context that he still finds White's position 'good.'

based on experience. The special anticipations, as with the anticipations treated in points 1-4, stem mainly from the highly differentiated system of experiential linkings which the master has at his disposal. On this basis the characteristics of a situation on the chessboard – of which every position has many – are immediately taken account of and correctly rated. Functionally, the special anticipations like any other anticipation serve to help in cuing off adequate methods. We shall return to the great importance of these purely reproductive mental operations later on.

From the preceding enumeration (which has certainly been incomplete) of the elements in the total problem conception by the end of the first Phase, it can be seen that the problem has already specialized and concretized itself to a high degree at the very beginning of the thought process. Justifiably the first Phase can be called the Phase of problem formation.

From a comparison of protocols of different subjects it appears that the process of specializing and concretizing is carried out much more quickly, to a higher degree, and more adequately – i.e., compared with the objective solution – by the master than by the less skilled player. This was particularly evident from the experiments in which the subject was interrupted after ten or 15 seconds of looking and thinking. When asked to give as good a recount as possible, the masters often appeared to be completely 'in the know' after this short exposure time. Not only did they remember the position, perhaps with the exception of some unimportant detail, but they also knew quite often what was at stake and in which direction to search, which methods of play and/or moves were pertinent, what was within reach, etc. The following is an impressive example:

(G5; B1):

(Blind recount after ten seconds of looking at the position. After that the normal thinking aloud procedure.)

5 *Materially it is not too good, two Bishops against a Rook and three Pawns if I'm not mistaken. It is not too bad, though, since the pair of Bishops can often perform miracles. I have already been planning how to get my Rook into action. Something like: Rook to the left, then advance, then again to the left – but that may be difficult to carry out because of the White Pawns. Another possibility is to*

proceed along the Rook file; maybe K-N2 and P-KR4 or something like that. I must take care, however, not to carry on the analysis blindly!

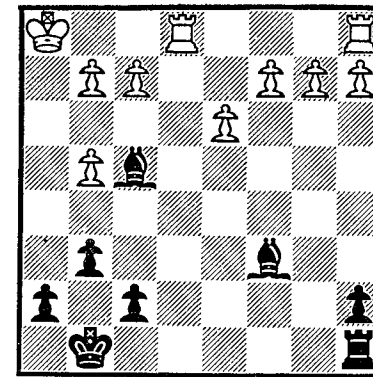
(E: Indeed, that should be avoided.)

I have the impression that I must proceed actively; not because I am exceptionally aggressive by nature, but because the position should be handled that way. If you wait and see, you will probably lose against all those Pawns.

10

15

20



POSITION B

Black on move

(E asks if any moves have possibly been seen.)

Yes, I thought of 1... R-N1 - to attack the Pawn at N7 - then 2. P-N3.

25 (E asks for the position. S dictates

everything correctly, except for some uncertainty about the Bishops – KB5 and Q4? Finally, only about the Bishop on Q4 (the Bishop is on QB3) and about the position of the White King – 'R1 or KN1? Probably R1.') 30

Such a recount tells us concisely what the state of development of the total goal (problem) conception is after ten seconds of thought. All of the factors mentioned in points 1-5 are found here.

The position is, first of all, almost completely grasped. The report begins by drawing up a balance sheet; the estimated value is implied by 'not too good' and later 'not too bad, though.' Let us say a '5'; even the maximal expectancy appears to be hardly higher than: equal play. The tentative estimations are clearly based on knowledge and experience: e.g., material balance, what a pair of Bishops can perform.

In line 6 the core problem is hit upon: how to activate the Rook. Two possible methods are mentioned (both are objectively pertinent). Line 15 contains a special anticipation (to proceed actively is probably necessary) which, again, contributes to specializing the problem and delimiting the range of considerable continuations. By now the latter are rather sharply defined: proceed actively, get Rook into action, either left, up, left; or along the Rook file. A slightly favored first move already exists: 1... R-N1.

G5 has a more nearly complete grasp of the objective problems of position B1 than do subjects C2, C5, W2, and even M4 after an entire thought process of ten minutes or more! In its entirety the

retrospection of (G5; B1) is indeed a very successful approach to an objective solution (analysis) and, at the same time, quite a complete total anticipation of the subject's final argumentation: the necessity of proceeding actively, the choice of the plan (attack on the King), how to execute it (Rook first to the left, etc.), and the first move (1... R-N1) are all represented. The only thing not brought out is the possibility of a blockade. During the course of the normal thinking aloud G5 spends most of his time on the blockading plan before he definitely rejects it (see protocol, p. 422).

From this example we see that for a master an important part of the problem formation takes place during the first few seconds of the thought process. A strongly specialized schematic anticipation which steers the subject's thinking in a sharply defined direction may be contained even in the first stage of his goal awareness.

Section 46: The completion and enrichment of the total goal conception after the first Phase

In the previous section we have seen how the total problem (or goal) conception obtains sharp, concrete aspects as early as the first Phase and even in the first few seconds of the thought process. In particular with masters and grandmasters the larger part of the problem development takes place before the more systematic investigation begins. From the standpoint of problem development the master begins his systematic (mental) trying out and reasoning at a point which a weaker player reaches only after a laborious analysis, if at all. The difference between a master and a strong class player is nowhere as great as here (see also Chapter VIII).

Naturally, however, the process keeps going for the master too: his appreciation of the board problems is still subject to improvement. The subjective move decision is not yet ripe.

Concerning this further course, we may distinguish between *two kinds of problem development, namely, with and without drastic structural changes in the total goal conception*. Both kinds evolve in a series of problem transformations, but only in the first category do we find pronounced restructurings (*Umstrukturierungen* in the Gestaltist terminology, cf. WERTHEIMER 1925) in the outline of reasoning. In the second category the problem transformations merely give rise to additions, elaborations, and concretizings in the total goal conception. In the present section only the development *without* structural changes will be discussed.

An important part of this process, namely, the convergence to a probative reasoning, to a conclusive argument, as it comes out in the final Phase especially, has already been treated in other contexts (cf. Section 43). Attention has been directed to the fact that although the convergence is most clearly seen toward the end of the thought process – in the text of the protocol (the goal-settings and problem formulations especially), in the succession of solving propositions, and in the pattern of choosing positive or negative continuations – it does, in fact, cover the entire thought process. In the preceding section it has been shown that an important part of the converging takes place in the first Phase already, to the extent that the pertinent moves are more or less pinpointed, and certain subgroups, favorites, and quantitative expectations appear. If we take these features of the total goal conception together, they can be said to include a schematic anticipation of the final argument. The question therefore remains: How do the process and the results of the investigation of possibilities in the main part, between the first and final Phases, complete and concretize the total goal conception?

First and foremost, it must be established that each partial investigation, each calculation or inspection, each trying out of a move proposal has a *double function*: (1) a direct attempt at solution and (2) an orientation to the (objective) problem situation. In particular, the trying out of pertinent moves right after the first Phase, as it occurs in numerous thought processes such as (M2; A) and (G4; A), is more than a direct search for a good move. It is, at the same time, a further orientation into the possibilities, the results of which contribute to the development of the problem. The subject reconnoiters the dynamics of the position; he obtains a more concrete impression of the possibilities the pieces have for action. On that basis he is able to arrive at certain ascertainments and expectations concerning what can and cannot be attained on the board, what the dynamic range and function, e.g., defensive or attacking value, of each piece is, etc. These findings may be classed together under the name of *qualitative results* of the investigation. Certain dynamic *properties* of the position can be abstracted from one variant or generalized from several. The subject's appreciation of the board problems is sharpened and clarified by calculating a few illustrative variations.

As to the *quantitative results*, that is, the ratings given to end positions of variations, these either directly furnish criteria for the strength of the move itself, or of a counter move or variation thereof – in case the variation that follows is more or less forced – or are at least important

in illustrating existing chances. Although unforced variants and incomplete calculations do indeed *prove* nothing, they may be meaningful as *sample variations*. Their quantitative results 'illustrate' the worth of the variation, of the move in question, of the plan in which the move fits, or even of the position itself. When, for example, in an even position the quantitative results of three successively calculated variants (own-branches) of 1. Z_1 are all negative (a lost game), then the likelihood of 1. Z_1 's being bad appears to be greater than of its being good though, of course, no proof has yet been delivered.

Thus each partial investigation contributes to the completion, enrichment, and specification of the total goal conception.

Now, however, this question will be approached from the other side. An attempt will be made to describe the manner in which specific aspects and parts of the total goal conception develop as a consequence of partial results already achieved.

First we shall consider the more formal and structural aspects of the goal conception with, as important determining elements, the group of pertinent moves with their subgroupings and features (cf. Section 45, points 1-4).

The total development may be roughly described as follows. An originally large but only vaguely delimited group of pertinent moves becomes smaller and more and more definite. At first the subject seldom has a definitive answer available to the question: 'Which moves are pertinent?'; but by the end of the thought process he usually has. In the final stage an exact answer can frequently be derived from the protocol text, in particular in those cases where the pertinent moves are simply mentioned by name in the subject's recapitulation. Towards the end of the thought process these moves become more and more clearly grouped into alternatives (converging toward an alternative-problem, cf. Section 43). Finally this ends up in a move alternative which anticipates the attainment of the final goal; namely, the subjectively convincing argument with its positive and negative part: Z_1 - Z_i or Z_1 -non- Z_i , with a steadily increasing preference for Z_1 .

How does this ever increasing specialization emerge from the results of the investigation?

This has been partially discussed in the preceding pages and is so obvious anyhow that just a few remarks will suffice. The discussion will be limited to a few interesting types of problem transformations which have not as yet been elucidated.

The *formation of plans* in strategic positions can substantially delimit the group of pertinent moves – for example, when the subject (maybe provisionally) accepts a certain plan. It can also promote the transformation of a multiple choice problem into an alternative-problem (plan-alternatives P_1 - P_i or P_1 - P_2 , for instance). This process, which belongs to the explicit methodology of chess thinking – every skilled player knows that in certain situations he 'should really make a plan' – is, however, not restricted to the first Phase. The same holds good for those other conglomerations of moves or possible continuations which do not really deserve to be called 'plans' (cf. Section 41, sub B). The advancing investigation continually gives rise to the introduction of new, 'natural' groupings of investigated and to-be-investigated moves, variants, methods of play, etc.¹² All of these groupings and subgroupings can be considered as structural anticipations of the final argument.

Whenever a plan is chosen after the first Phase, the choice is generally preceded by a convergence process (P_1 - P_i , P_1 - P_2 , or P_1 -non- P_i) analogous to the one already encountered for move alternatives (Z_1 - Z_i , etc.). The group of pertinent moves then is substantially reduced at the moment the struggle of the plans is decided. Since a plan is not played on the board in the same way as a move, the moment of decision is not always clearly discernible. A plan is and remains only an aid for the move decision even though it may occupy an important place in the goal awareness of the player. Sometimes in a choice-of-move process the choice between two alternative plans turns out to be superfluous for the time being, namely, if a move can be made that serves both purposes.

In (M2; B) for example, the decision in favor of plan A (King side attack) occurs only in the second part ((M2; B2), line 11). From then on M2 is confronted solely with solving a highly specialized and noticeably simplified problem: the choice of a move *within* plan A.

Similar examples are found in protocols of other strategic positions where the method of plan formation is applicable. Compare for instance (G5; B), (M2; K-N2), and (M2; ... N-K1). In the last one, the subject distinguishes in the first Phase between 'different ideas... : either aim at P-QN4 or P-B4.' The main part then begins as follows:

12 'Natural' means here: groupings which result immediately from the structure of the problem situation because they are linked to the type of situation through the subject's experience in analogous cases.

part of the task give a feeling of satisfaction which sometimes has an effect on the general expectancy level. Here, too, the problem becomes more specialized though the pertinent moves need not yet be affected. They are affected, however, when important counterplay possibilities are eliminated. In (M₂; A), for example, the discovery that after a calm move the countermove 1... Q×NP fails to 2. N-B₄ has the result that from here on only calm preliminary moves come in for consideration. Before the discovery the subject felt he had to choose between a number of forceful and calm moves, none of which were particularly appealing. Now the impasse is suddenly overcome. There is no reason for rushing the attack: the Pawn on QN₂ does not have to be defended, directly or indirectly, because it already is indirectly defended. From this moment on the problem specializes to the finding of a suitable calm move; only 1. KR-Q₁ and 1. KR-K₁ are looked into and weighed against each other.

Finally, *favorite forming* belongs to the processes which enrich the total goal conception and help to complete the implied schematic anticipation of the goal: the subjectively convincing argument. The subject's preference for a certain move or plan often arises from or is at least strengthened by the results of his calculations. (We just saw an example of this on p. 207 when discussing the problem transformation due to an unexpected favorable result.) Next to the reproductively determined favorite forming based on experiential linkings, which we know from the first Phase (see Section 45, p. 194) 'empirically' founded favorite forming occupies an important place indeed. Typically, such an unforeseen preference, based on findings in the investigation of possibilities, turns up by the end of a phase of trying out.

Let us now consider the other aspect of the development of the total goal conception: the subject's grasp of the board problems (cf. Section 45, point 5). He gets more and more familiar with them; all of the partial results of the advancing investigations, quantitative and qualitative, become 'anticipations' of his idea of the final, objective solution (analysis).

This repeatedly mentioned process may be described in various ways. Here special attention will be paid to the way in which the goal conception, qua chess-technical content, is steadily sharpened and completed during the progress of the thought process. As stated before, the final goal – which will frequently be called 'solution' from now on – encompasses more than the move to be found. According to Section 39, p. 145, the final goal is to construct a subjectively con-

vincing argument – which is itself an approach to an objective, chess-analytic proof (cf. Section 45, p. 198).

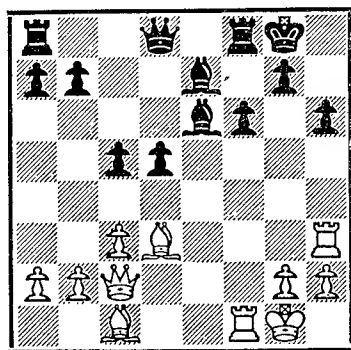
This aspect of the problem development shows up quite clearly in the text of the protocol, that is, in the goal-settings, in the problem formulations (and contributions thereto), in the anticipations, and in the formulations of qualitative results. All such expressions may be considered as *solution attributes* which have been noted or anticipated by the subject, i.e., *attributes of the final argument or of the objective analysis*. In the course of the thought process the subject continually makes comments about the board problem and its solution; obviously more and more solution attributes become conscious. The corresponding formulations in the protocols reflect an increasing definiteness and detail in the subject's conception of the board problem(s). In the examples in Section 41 we already noticed greatly varying 'degrees' of definiteness; the sharpest formulations (e.g., in (M₅; R-Q₁), see p. 161) are always to be found in the later phases of the thought process.

The richer a protocol is in contributions to problem formulations the easier it is to read the process from it. Extremely striking in this respect is (G₅; B), which will be considered in the next section. From the protocol (M₂; B) in Section 30, we can clearly see how the solution attribute of active play is a must, becomes more and more explicit. In the beginning passive moves are investigated in good faith (1... K-B₁, 1... B-KN₄). Towards the end of the first part, however, 1... B-Q₇ is already discarded as being 'a little slow,' and also: '1... P-QR₄ – but that is nonsense. The Rook must do something; other moves are a bit too passive in that position.' Only in the second part does a clear formulation appear:

(M₂; B₂):

- 8 If I do nothing, I have the idea that I am gradually going to lose. To hold back the Pawns doesn't work so well. No, I must do something on the King's wing.

By means of successively found solution attributes the nature of a solution is determined in detail before the solution itself is hit upon. Protocol (M₅; B-R₇ch) provides an excellent example: the subject doggedly searches for a mating combination which he rightly supposes to be present. He does not find it. Little by little each part and aspect of the combination are anticipated. It is literally only the 'combination' of these attributes which does not succeed. The relevant protocol fragments follow:



POSITION B-R7CH

White on move

(M₅; B-R7ch):

- 10 What to do? It must be possible to checkmate the Black King. If 1. B-R7ch, then 1... K-R1.
- 17 Wait a moment, a long variation – first go over it:
 1. B×P, B×R; 2. B-R7ch, K-R1 – doesn't have to; I thought 2... K-R1 was forced, but he can get away now – 2... K-R1; 3. B×Pch, K×B; 4. Q-N6ch, K-R1; and then, for example, 5. Q-R6....

So far we have: (1) the expectation of the possibility of a mating combination, (2) the ascertainment of the forced reply 1... K-R1 after the move 1. B-R7ch (the first move of the combination in fact), and (3) the calculation of the first four moves(!) except for the permutation of the first and second. The 5. Q-R6 continuation is not the strongest, however; the correct move is 5. B-N8. Further on, this detail, too, is mentioned (4) as a result of finding that the Queen should go to KN6 with check:

(M₅; B-R7ch):

- 108 I'm all the time trying to get Q-N6 in with check; the plan is clear to me, but I don't see how to realize it. Getting the Queen in play will lead to a quick checkmate. B-N8 is a possibility, threatening mate at KR7 – gets the Bishop out of the way –; I have already broadly looked into this possibility several times.

Only not at the right place (which is the fifth move)! Also the nature of the rest of the combination is anticipated: the decisive penetration by the Rook on KB1 (5):

(M₅; B-R7ch):

- 143 1. B-R7ch, K-R1; 2. ? Yes. I've also looked several times into the possibility of opening the King Rook file with some sort of sacrifice and then play R-B4-KR4.

Indeed the winning combination runs approximately that way. Actually: 1. B-R7ch, K-R1; 2. B×P, B×R; 3. B×Pch, K×B;

4. Q-N6ch, K-R1; 5. B-N8, R-B2; 6. B×R, Q-KB1; 7. Q-R5ch (also an immediate 7. R-B3 wins), K-N2; 8. R-B3 (along the third rank instead of the fourth, but the principle is the same), Q×B (8... B-N5; then 9. Q-N6ch and 10. Q×B); 9. R-N3ch, K-B1; 10. Q-R8ch, Q-N1; 11. Q×Q mate. The precise calculation of the last five moves of the combination is not necessary to arrive at the choice of 1. B-R7ch, since after 6... Q-KB1 White could regain his material if he wanted to; there is in any case nothing to fear any more and he maintains a dangerous attack. If one has seen the exact results up to and including the sixth move and has a rough idea of the possibilities that can follow, that is sufficient to play the combination.

So practically nothing was lacking in (M₅; B-R7ch); it is almost incomprehensible that M₅ did not find the combination. The exact causes of this failure are, however, not relevant in the present context. The main point is that *with such a highly specialized anticipation of the nature of the combination, it would certainly not have been accidental had the subject indeed found it.*

How does the subject come to establish or anticipate such solution attributes?

In the first place there are, again, experiential linkings. In the case described above certain methods of play typical for such combinations are cued off by the observation and abstraction of certain features of the situation on the board – during the position-investigation in the first Phase (dynamic orientation) and the trying out of variations after the first Phase. *Indeed all parts of this combination, and in essence of every combination, characteristically denote the skilled player.* The possibility of B-N8, the opening of the King Rook file with some sort of sacrifice, followed by the decisive Rook maneuver; 'to get Q-N6 in with check,' so as to bring the Queen into play with a gain of tempo which 'will lead to a quick checkmate': these are all tactical maneuvers which must be applied in many similar situations and which are familiar to the master by experience. Such familiarity does *not* have to consist of an explicit *knowing that* a certain method (which can be put into words) has to be applied in situations of a certain character (which can be described). In many cases the process is rather one of immediate routine actualization of the method of play itself without assistance from any readily verbalizable knowing.

In addition to the functioning of experiential linkings we have here to do with an effect of what we call the mental operation of *excogitation*: a mental operation which is always methodically applied in laboriously progressing thought processes. When efforts to attain an im-

portant board goal that appears to be within reach (checkmate in position B-R7ch) fail one by one, then the attempts to integrate the qualitative results during the ensuing transitional phase necessarily lead to a *conflict analysis* (cf. DUNCKER 1945, p. 20ff.). The subject's goal-setting, the means applied, their failure and the causes of the conflict become more and more explicit. The subject especially tries to answer the question: which *position attribute* (problem attribute, solution attribute) is blocking the realization of my goal? It is in this way that some of the most important insights and anticipations of solution attributes come about.

The protocol (M5; B-R7ch) provides, again, a clear example. After a number of vain attempts to enforce checkmate, the subject first realizes the necessity 'to get Q-N6 in with check': a generalized qualitative result arrived at during a transitional phase. But what is stopping the Queen from rushing into play? The Bishop at Q3 is blocking the Queen. In line 137 we see emphatically expressed: 'The Bishop at Q3 is bothering me. Curious. Isn't there any forcing move to get rid of it?' – again an anticipation of a solution attribute, for that matter (6). It is in the context of this conflict analysis that the possibility of B-N8 (in order to 'get the Bishop out of the way'), a possibility 'already broadly looked into' several times, is now explicitly mentioned.

In fine: Via a process of conflict analysis in a transitional phase qualitative results of the investigation are first integrated and next transformed into abstracted anticipations of solution attributes.

The more effort it costs the subject to solve the problem, the more consciously and explicitly the solution process progresses – this has been more or less established as a general law in *Denkpsychologie*. We are in a position now to describe the underlying processes in some more detail. From the protocols it is evident that in thought processes where it is difficult for the subject to come to a decision, the longest and most detailed transitional phases occur; in these phases the subject obviously tries to make the existing difficulties and conflict elements as explicit as possible. Thus it is the difficulties encountered during the investigation of possibilities which lead to the actualization of the mental operation of excogitation in the ensuing transitional phases. This method serves to make both the subject's thought operations and his methods of play that have been and still have to be applied explicit and manipulable.

What is meant here by 'difficulties'? 'Difficulties' exist whenever the results of the trying out of solution proposals do not come up

to expectation. The more unbridgeable the gap between results and expectations seems to be, the more serious they are. In chess we must think here primarily of the ultimately decisive *quantitative* results and expectations. Whenever the subject's quantitative results keep failing to approach his maximal expectancy, he will finally have to temper his expectations somewhat. But this is an unpleasant decision which the subject may often be inclined to resist strongly. In (M5; B-R7ch) the resistance resulted from 'a distinct feeling that it's somehow or other over,' that is, a strong anticipation of solvability. In other cases there is a strong resistance to recognizing that the position is poor or, in other words: against dropping the maximal expectancy below 5 – e.g., (G5; B). The 'difficulties' may then be described as a *crisis of quantitative expectations*; the expectancy should really be lowered, but the subject (player) cannot or will not yet accept it.

We have now learned to recognize the characteristic phenomena of such a crisis: lengthy transitional phases in which takes place an extensive and penetrating conflict analysis through which problem elements and solution attributes are abstracted and made more and more explicit. All this certainly contributes to an increasing definiteness of the final goal. The schematic anticipation implied in the total goal conception is gradually completed – although an important gap obviously remains to be filled. In case it is filled out, after all – e.g. the mating combination is found, the searched for drawing possibility is discovered – the 'crisis' is favorably resolved. If it is not, the crisis of expectations generally introduces a radical problem transformation of a type to be discussed in the next section.

We shall conclude the present section with some further comments on the mechanics of the quantitative aspect. As a matter of course, along with the concretizing of the total goal conception and the process of converging towards the final argument the quantitative goal (estimate and expectancy) becomes more sharply defined. The margin of uncertainty surrounding the estimated value becomes steadily smaller. In particular the maximal expectancy and estimated value approach each other (i.e., the desired, the striven for, approaches what is judged to be possible; compare p. 195).¹³

In the beginning of the thought process the maximal expectancy is generally placed quite high. This obtains particularly in tactical

¹³ In case of a crisis of expectations this 'normal' adjustment doesn't succeed too well, as we have seen (cf. Section 47).

positions that are difficult to size up; the margin of uncertainty around the estimated value leaves room to expect quite much at first. Position A is an example; see e.g.:

(G₃; A):

11 One must first see if there is anything decisive in the position.

If during the investigation of possibilities these hopes prove to be unfounded, the maximal expectancy drops, adjusting itself to the now more certain estimated value. This *empirical pruning of provisionally highly held expectations* is one way in which the results of the investigations can sharpen the quantitative goals.

Another way was mentioned when we discussed what effect the finding of a minimum solution has on the problem development. Through the *establishment of a minimal expectancy*, the margin of uncertainty within the estimated value is bounded from below. But sometimes, in addition, the estimated value is heightened: *provisionally* if the minimum found is lower than the original estimated value but *definitely* if the minimum found is higher than the original estimated value.

In the event that the minimum found should exceed the maximal expectancy as well (i.e., the subject finds an unexpectedly favorable possibility), the result will be both a rise in the estimated value and a (temporary) disappearance of the margin of uncertainty between the maximal expectancy and the estimated value as they both rise to the level of the newly found minimum. If after finding the minimum the subject tries anew to improve it, there is a difference again between estimate and expectancy.

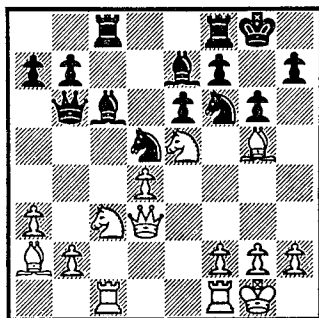
In general, it may be said that every definitive investigation result contributes to the concretizing and sharpening of the quantitative expectations. Even the calculation of sample variations – which are not definitive – contributes to the concretizing of both quantitative and qualitative expectations. The estimated value and the maximal expectancy take part in the entire problem development – *which in turn is largely reflected in their fluctuations*. One can understand a great deal just by tracing the formal progress of a thought process,¹⁴ ignoring the qualitative content; that is, by keeping track of (1) successive solution proposals, (2) the progress of the estimated value, and (3) the maximal expectancy. Protocol (G₅; A) will now be treated in this way; the complete protocol is also presented in full.

¹⁴ The mechanics of solution proposals, with results weighed against a (variable) maximal, and for counter variations minimal expectancy, as described here, provide a pliable skeleton for machine simulation of chess thinking (cf. p. 396 ff).

(G₅; A) S: MAX EUWE E: DE GROOT T = 15 MINUTES DECEMBER 15, 1938
(Italics are for textual clarity and do not represent G₅'s emphases. For diagram see p. 216)

- First impression: an isolated Pawn; White has more freedom of movement. Black threatens Q×NP. Is it worthwhile to parry that? It probably is; if he takes, then QR₃ is also attacked. Can White then take advantage of the open file? Does not look like it. Still again: 2.N×B and then by exchange the Pawn at QR₃ is defended by the Queen. Indirectly in connection with the hanging position of the Knight at KB₆ and possibly because of the overburdening of the Bishop at K₇. But wait a moment: no, Q×NP is rather unpleasant after all because the Bishop at R₂ is undefended. Can I do something myself? Investigate that first: the pieces on KB₆ and Q₅ are both somewhat tied down.
- Let us look at the consequences of some specific moves.
- 1.N×N, possibly preceded by 1.N×B. Then 1...R×N is probably impossible because of taking on Q₅. Black has a number of forced moves, there may be a possibility to take advantage of that.
- Let us look at other attacks:
- 1.B-R₆ in connection with KB₇ – but I don't really see how to get at it.
- 1.P-QN₄ in order to parry the threat – but then exchange on QB₃ will give some difficulties in connection with 2...B-N₄ – oh, no, that is not correct, one can take back with the Queen.
- So far a somewhat disorderly preliminary investigation. Now, let's look in some more detail at the possibilities for exchange: 1.N×B or 1.N×N or maybe 1.B×N/5 or maybe first 1.B×N/6.
- 1.N×B, R×N; 2.take on Q₅; for instance 2.N×N, P×N; wins a Pawn, but there may be compensation for Black on QN₂. But better is 2...N×N; then 3.B×N, R×R is nearly forced, no, it is not, he can play 3...B×B as well. I see no immediate advantage. 1...P×N is not forced therefore; and even if it were forced you couldn't be quite sure of winning. It's happened before that such a position proved less favorable than it seemed to be. The point Q₅ is reinforced by it, that is a disadvantage. 1.taking on Q₅. 1.N×B at any rate gives the pair of Bishops; if I don't find anything better, I can always do this.
- 1.N×N, B×N; is that possible? Q₇ is free then. 2.B×N, B×B/3; 3.N-Q₇, Q-Q₁ can then be done.
- 1.N×N, B×N; 2.B×N, B×B/3 will probably yield something. 1...N×N is also possible; maybe better. Then 2.B×N, B×B/N and now there are the possibilities to take on QB₆, or to play something like P-B₄; once again:
- 1.N×N, N×N; 2.B×N, B×B/N – no, nothing then, 3.R×B does not help any; it is a cute move but at the end of it all everything remains hanging. Something else: 2.B×B – he just takes back. 1...P×N is very favorable; he won't do that, it needn't be investigated.
- 1.N×N, N×N remains. 2.B×N, B×B/N; 3.B×B, B×R is then possible. No, can find no way to make anything out of this. 1...N×N; 2.B-R₆, KR-Q₁; 3.Q-KB₃ with some threats; if Black now has to play his Bishop back to K₁, then one gets a good position.
- 1.B×N/5; this must be looked into. Does that make any difference? 1.B×N/5, B×B is again impossible because of 2.N-Q₇. That is to say, we will have to look out for 2...B-B₅, but that we can probably cope with: the worst that can happen to me is that he regains the exchange, but then I have in any case some gain of time. 1.B×N/5, N×B; 2.Same difficulties as just before. No, that is now impossible: 2.N×N wins a piece.
- 1.B×N/5, B×B; 2.B×N, B×B; 3.N-Q₇, Q-Q₁. Let's have a closer look at that: 4.N×B/5, P×N and I'm an exchange to the good: very strong. 1.B×N/5, P×B is therefore forced. But

- 95 *that's good for White. The Knight on KB6 is weak, the Bishop at K7 hangs – and the Bishop on QB3 stands badly. On positional grounds one could already decide on 1. B×N/5. Is there some immediate gain?*
- 100 *1. B×N/5, P×B; it looks bad for Black. Probably some more accidents will soon happen. Much is still up in the air. One plays, for instance, 2. Q-B3. Defending the Knight on KB6 is not so easy; 2... K-N2 looks very unpleasant. Yes, I play 1. B×N/5.*



POSITION A

White on move

In the first Phase of (G5; A) no clear judgment is expressed on the value of the position. Indirectly, however, the subject seems to think the position favorable (estimated value = 6 to 7). For the time being he does not exclude the possibility that the position is won (wide margin of uncertainty; maximal expectancy = 10): *'Black has a number of forced moves, there may be a possibility to take advantage of that.'*

These variants (with the forced moves) will be the first ones to be looked into during the investigation of possibilities; the subject sets about it methodically and expresses his intentions: *'Now, let's look in some more detail at the possibilities for exchange: 1. N×B or 1. N×N or maybe 1. B×N/5 or maybe first 1. B×N/6.'* He begins with 1. N×B, and hopes to win a Pawn if Black answers with 1... R×N, or hopes for a positional advantage in case of 1... P×N. G5 no longer seems to expect an easy win: meanwhile the maximal expectancy may have dropped to about 9. The investigation itself does not yield much: *'1... P×N is not forced therefore; and even if it were forced you couldn't be quite sure of winning. It's happened before that such a position proved less favorable than it seemed to be.'* Under the influence of this negative result the expectations presumably drop somewhat; the maximal expectancy is now around 8 or 9. No longer is the result far below the maximal expectancy: *'1. N×B at any rate gives the pair of Bishops; if I don't find anything better, I can always do this.'* With this a minimal expectancy is established which might be put at about 6. The bounds of the margin of uncertainty already approach each other.

Then follows an extensive investigation of the consequences of 1. N×N. First the variants of 1. (N×N), B×N are refuted – perhaps that stimulates the expectations of success once again. Then follows the calculation of 1. (N×N), N×N. After that 1. (N×N), P×N is eliminated in passing: *'1... P×N is very favorable; he won't do that, it needn't be investigated.'* The end result of 1. N×N becomes: *'No, can find no way to make anything out of this.'* The result (5 or 6) is too low in any case.

It is evident from G5's rather uninspired approach to 1. B×N/5 that the results of 1. N×N had an unfavorable influence on his general expectations of success. Specifically the maximal expectancy apparently drops to, say, 7 or 8. *'1. B×N/5; this must be looked into. Does that make any difference? 1. B×N/5, B×B is again impossible because of 2. N-Q7.'* A little bit later: *'1. B×N/5, N×B; 2. same difficulties as just before. No, that is now impossible: 2. N×N wins a piece.'* The win of a piece is an unexpectedly favorable result (= 10). It occurs in the only variant which just before (after 1. N×N) had been favorable for Black. As a consequence 1. B×N/5 immediately becomes the favorite, the striving for proof sets in, the quantitative expectations skyrocket. The maximal expectancy goes up to about 9, the minimal expectancy to about 8; the distance between maximum and minimum shrinks. The uncertainty that still exists can be largely removed by calculations. Calculations for the positive part follow: 1... B×B and 1... P×B. The first leads, like 1... N×B, to a loss for Black (win of exchange = 10); the second is thus forced: *'But that's good for White. The Knight on KB6 is weak, the Bishop at K7 hangs – and the Bishop at QB3 stands badly. On positional grounds one could already decide on 1. B×N/5.'* The result of the move is about 8 and its supremacy over the other moves, especially over the minimum solution 1. N×B (value = 6), suffices for the move decision. A further negative part of the proof is superfluous; one could certainly not reach an 8 with a calm move. However, the result after 1. B×N/5, P×B still is no more than a minimum result, based only on positional grounds. Subject G5 still wants to know: *'Is there some immediate gain?'* This means that the maximal expectancy again rises to 10 for a moment. The subject's looking for some immediate gain is simultaneously a quest for a decisive argument in support of the favorite move. The result of this last investigation: *'1. B×N/5, P×B; it looks bad for Black. Probably some more accidents will soon happen'* (expectancy evidently 9 or 10). With the expectations running high the move 1. B×N/5 is finally played. (The value of this move really is 10; an analysis shows that it leads to a forced win – see p. 89).

Section 47: Structural transformations in the argumentation

In a relatively large number of protocols there occur problem transformations which entail *structural changes in the total goal conception* (cf. p. 202). Apart from the step by step, result by result, concretizing of the total goal (or problem) conception and the gradual converging towards the final argument, drastic structural changes are sometimes found in the process. At such a point the entire line of reasoning is overhauled; the problem development shows an apparent discontinuity, it takes an unexpected turn.

These phenomena are of theoretical importance. The influence of partial results on the total goal conception is apparently very strong here, so that this type of problem transformation lends credence to the principle of creative form-making (cf. Sections 21 and 44).

In spite of the importance of such basic changes in the goal conception – and therewith in the implied schematic anticipation – their incidence has been rather neglected in the literature. Even Bahle himself failed to emphasize duly the insufficiency of the idea of an anticipatory (total) schema which is basically constant throughout the process. Especially in creative thinking is such a conception inadequate. For instance, the writer of a book begins with an outline, a plan. He may make a provisional division into chapters, but during the course of his work he may feel compelled to reconsider his outline: it may be necessary to restructure his original goal. A part that at first seemed quite important may be relegated to the background; or the sequence of chapters – surely an aspect of his total goal conception – may be radically changed; or what was meant to be Chapter I grows into a book of its own. Another example is the mathematician who wants to prove a certain theorem but discovers in the course of his work that such a proof is impossible. He may therefore try to prove that the theorem is faulty or solely valid in specific cases or even in fact unprovable. His work continues but with a radically changed goal. In much the same way the chess player may feel compelled by a series of unfavorable partial results in his analysis to abandon his old line of reasoning – that of trying to prove the adequacy of a certain move or plan – and to start anew with a restructured goal conception.

Changes in the basic structure of the total goal conception are characteristic not only of creative thought proper but of *all forms of thought in which the subject sets himself a goal which he is not sure is realizable*. Such changes do not occur in the great majority of tasks

set for subjects in laboratory experiments and certainly not in the solution of examination problems and chess problems (White to play and mate in three, etc.). The composer of the problem then assures achievability of the goal.¹⁵

In this respect there is an important difference between the thinking in (1) a task situation where what is asked for is the solution of a problem that is known to be objectively solvable and in (2) what can be called ‘truly’ productive thinking. In the second case the subject not only has to find and apply the means himself, but he has also to set the goal. The goal-setting thus remains tentative, revocable until the end.

How drastic a structural transformation in the outline of reasoning is depends, of course, on how advanced the converging process is at the time of discontinuation. The stronger the subject’s faith in his grouping, the more sweeping the transformation. This is, of course, a relative matter. It is even impossible to draw a sharp line between the ‘structural’ transformations treated here and some of the (problem) transformations that were described as completions and enrichments of the total goal conception in Section 46. Since, in principle, every trying out may be thought of as a tentative favorite formation (cf. Section 43, p. 176) the subject’s shifting from one move to another (*per se* a problem transformation) might be considered a change of favorite, i.e., a ‘structural’ change in the outline of reasoning.

Other less pronounced cases are those where the subject finds a minimum solution or an unexpectedly favorable possibility (see the previous section). A certain convergence may have taken place before the discovery, a convergence which is now suddenly broken off and replaced by a new problem conception centered around the new favorite. Here, too, we have a structural change (more than just elaboration or sharpening of the problem) in the outline of reasoning. These less impressive cases shall be ignored in this section, however. The important thing is not completeness in registering all possible cases but the description of the phenomenon of structural change itself as it appears in its most pronounced form. The description will be based on some material from the main series.

¹⁵ At least he should do this. However, unsolvable examination problems do occur now and then. They provide gifted students with the opportunity to demonstrate ‘real’ productivity: for example, they can prove that the assignment is incorrect or can even first correct and then complete the task.

In the first lines of (M₄; C) we find a clear converging of the type Z₁-Z_i. The favorite move Z₁ is 1...N-K5. Even in the first Phase we find:

4 Yes, the first thing you look at is the possibility, 1...N-K5 to QB6.

(At least subject M₄ thinks that 2...N-B6 is a threat; in reality, the Knight on QB6 can simply be taken.) Further on, subject M₄ again declares his preference for this move, which is increasingly becoming the favorite:

9 1...N-K5 looks like a very good move to me;

29 1...N-K5 looks very strong. Yes, a good move. I think 1...N-K5 is a good move. 1...P-Q4, then comes 2.N-K5. But 1...N-K5 is good – I really don't have to look into a passive move. How can White answer 1...N-K5?

The first part of the formula of successive solving propositions (cf. Section 36) clearly reflects the converging process; the formula for (M₄; C) is:

a - b - a - a - d - a - a - a - c - i - b - b - g - b - f - b - b

Until line 56, the point at which move *c* is considered, the investigation proceeds with the favorite *a* alternating with various other moves. Even the progressive deepening of the investigation comes out in the formula, namely, through the increasing number of solving propositions (fresh starts) within the three phases of favorite calculation. After that, however, a similar converging process follows for move *b* (1...P-K4 = Z₂); the process may now be symbolically represented as Z_a-Z_k.

What has happened? Subject M₄ has suddenly discovered the flaw in his calculations:

57 Oh! What nonsense (discovers White Knight on QN5) 2...N-B6 is no threat after 1...N-K5! Now, of course, other moves require consideration.

The thought process begins, so to speak, all over again. First 1...N-K5 is liquidated.¹⁶

60 So after 1...N-K5, the only threat is 2...N×BP. 2. R-B1 and then that threat isn't so strong anymore either.

After this there follows another trying out of moves, new ones:

64 Let's look at

1...N-R7ch.

68 If I play

1...P-K4, just look.

and further on:

76 1...N-R4 – is no move. Then he takes on K3.

Because of the results of this phase of trying out (i - b - b - g in the formula), the subject's preference for 1...P-K4 gradually emerges:

78 1...P-K4 is a relatively simple move.

and further on:

90 So just play 1...P-K4.

Finally 1...P-K4 is indeed played:

95 Yes, I play

1...P-K4.

¹⁶ From spite, it would seem. Objectively, the grounds on which the move is liquidated are not very sound. The whole thought process, for that matter, contains a good many calculational mistakes and omissions. The structure is more interesting than the content. Subject M₄ was evidently in poor form.

The cause for rejecting the already narrowly converged alternative Z₁-Z_i is apparently the discovery of an error in the favorite calculations. The positive part of the argumentation which consists of the considerations and calculations devoted to 1...N-K5 now becomes invalid. The supremacy of the favorite over the other moves – never very great – now completely disappears: 'So there *are* objections – that counterattack is strong' (line 54). Most of the quantitative Z₁-results achieved so far become worthless; the subject must begin all over again to build up his argument.

Seen in the light of Section 46, meanwhile, it is clear that the subject must have gained a more mature and true appreciation of the board problems and that nonetheless the group of pertinent moves will have become smaller and more sharply defined than they were immediately after the first Phase. In brief: From the point of problem development the investigation up to that point has certainly not been completely fruitless. The considerations and calculations devoted to 'other moves' (line 59), for instance, retain their orientative and quantitative value. As to the calculations of the now rejected move, 1...N-K5, they become 'dead branches' for the renewed goal conception (see p. 32), but even here subject M₄ has probably achieved some qualitative results: a better understanding of the dynamic functions of the various pieces. Furthermore, the one variant of 1...N-K5 which led to rejection of the move already forms an element of the negative part of the proof while, finally, the group of pertinent moves is reduced by at least one. Indeed in all of these aspects there is progress and continuity in the development of the subject's conception of the problem, in spite of the structural change.

The fact that a fraction of the originally positively intended calculations – the small part that induces M₄ to reject 1...N-K5; namely: 1...N-K5; 2. R-B1, N-B6; 3. N×N – is to play a role in the negative part of the proof is characteristic of all such revolutionary (not evolutionary) turns in the problem development. This is another special aspect of the general law that attempted solutions which fail always benefit to some degree the development of the subject's conception of the problem.

The counterpart of discovering a flaw in the positive part of the argument is the discovery of a flaw in the negative part: the finding of an unexpected good move among the 'other moves' that were destined to be eliminated. This find, much more pleasant for the subject, likewise leads immediately to an overthrow of the outline of reasoning.

Where there had been a convergence in progress $[Z_1-Z_i]$ there is now an immediate successor $[Z_1-Z_i] \rightarrow [Z_2-Z_k]$, where Z_2 is the newly discovered move. Z_k can be expected to consist of fewer moves than Z_i because the doubtful moves included in Z_k are now spurned in the light of the raised quantitative expectations. Another possibility is that Z_1 will temporarily maintain itself as the main alternative: $[Z_1-Z_i] \rightarrow [Z_2-Z_1]$. Or the move decision may only need a checking of Z_2 itself so that the other moves fade completely into the background: $[Z_1-Z_i] \rightarrow [Z_2-(\text{non-}Z_2)]$. A great many possibilities – in which plan alternatives can also play a role – can be found in the protocols of the main series.

In the cases considered so far, the striving for a convincing argument and the rounding off of the thought process failed because of the sudden discovery of an error or omission. It may also fail, however, without such specific causes; namely, when *the subject does not reach the anticipated quantitative results through his favorite calculations*, i.e., does not achieve the desired supremacy of the favorite over the other moves. Sometimes faith in and preference for a certain move (plan, possibility) are so strong that for a considerable time it remains the favorite despite several negative results. If the results of the calculations stay negative, however, the favorite cannot maintain its privileged position; a reconsideration of the outline of reasoning has to take place. The stronger the existing anticipation of solvability, the longer the lifespan of the favorite, the more serious the *crisis of expectations*, and the more drastic the finally inevitable problem transformation.

An extreme example was seen in protocol (M5; B-R7ch). The anticipation ('a distinct feeling that it's somehow or other over') is extraordinarily strong. $T = 35$ minutes and only at the very last does the subject decide on a calm move, even then reluctantly: 1. R-N3. This move had been a second choice for some time, the favorite among the nonviolent possibilities. We are concerned with a problem development which can be symbolized as follows:

$[P_1-Z_2] \rightarrow [Z_2-P_1] \rightarrow Z_2$, where P_1 are the drastic possibilities. The second converging process is of very short duration here. Indeed, the subject's choice appears to stem more from a desire to end the experiment than from conviction.

Such happenings occur in more normal thought processes as well, where the decision is made *with conviction*, as in (E1; A):

Here, too, the favorite 1. B×N/5 has a long life. The formula of solution proposals reflects the problem development fairly clearly:

(E1; A): b - c - a - | a - a - d - a - a - f - f - a - | n - n; $T = 22$ minutes.

After a short trying out period 1. B×N/5 (= a) becomes the favorite (first bar) so that an alternative Z_1-Z_i is created which gradually converges. Subject E1 does not find any convincing results (actually 1. B×N/5 is the best move, see Section 26), but he remains true to his intuitive preference: – 'I don't know why I cling so to 1. B×N/5' though he has previously said – 'It is a pity to waste the King's Bishop' (which is exchanged for a Knight). Later the move is eliminated (the last a in the formula) and with it the struggle for an immediate advantage; thus, the quantitative expectations drop. Among the calmer moves, subject E1 quickly finds a suitable move uninvestigated until now (n = 1. B-N1), which is played after a short process of converging and checking.

(E1; A) E: DE GROOT $T = 22$ MINUTES DECEMBER 6, 1938

- | | | |
|---|--|---|
| <p>White has more room. Look further. The Pawn at QN2 is attacked. Can it be taken? Quite probably; a threat. 1. P-QN4 is to be considered. Does it threaten anything? Hardly. KB6 is weak, and White has more room; he is probably somewhat better off. 1. P-QN4 simplifies too much; not so strong.</p> <p>Look for a combination; maybe something there.</p> <p>1. N×B, R×N; 2. B×N/6 and 3. P-Q5 maybe. How to make use of the weakness of KB6, and then to weaken K6? Thus 3. P-Q5, takes; takes, takes: general exchange, without advantage, rather disadvantage.</p> <p>1. N×N does not lead to anything either.</p> <p>1. B×N/5; is that something? To take back with the Bishop is impossible; 1... N×B doesn't work either, so 1... P×B is forced. And then maybe 2. B×N, B×B; 3. R×B (sic) and 4. N-Q7. But then comes simply 4... Q-Q1 – no advantage.</p> <p>1. B×N/5 may still be good. Let's look at 1... P×B; 2. Q-B3. No, then comes 2... Q-Q1. It is a pity to waste the King's Bishop if nothing comes of it.</p> <p>Let's again calculate the variant with 3. R×B. So 1. B×N/5, P×B; 2. B×N, B×B; 3. R×B, takes back; 4. N-Q7, Q-Q1. Does this lead to anything? Apparently doesn't go any further.</p> | <p>1. B×N/5, P×B; 2. B×N, B×B; 3. R×B (sic), P×R; 4. N-Q7, Q-Q1. Get back the exchange, then probably nothing more: 5. N×R, K×N. What then? Pawn on Q5 is covered, Pawn on Q4 undefended. Is there anything to be done then? 6. Q-N5 for example? Then the Pawn on Q4 hangs – nothing special. Let's look at new variants. These are not clear. White stands better though, so there should be something. And 1. B×N/5 looks like a strong move – I don't know why I cling so to 1. B×N/5. 1. B×N/6 – I don't believe in that.</p> <p>1. B×N/5 and 2. B×N I keep coming back to 1. B×N/5 and 2. Q-B3 maybe. There must be something to that. But no, that is no good either. There follows 2... K-N2 or 2... Q-Q1. The Pawn at QN2 is attacked. Is that a threat I should be worried about?</p> <p>1. B×N/5. 2. B×N and 3. P-KR4 perhaps, to assault the King's wing? Or maybe:</p> <p>1. P-KR4. What to play after 1... Q×NP – on an indifferent move, 2. P-R5 follows. But wait! 2. N-B4 costs a Queen. Yes, so</p> <p>1. P-KR4 now deserves consideration. What will Black play? Probably prevent P-R5. If 1... N×N, then 2. P×N again threatening P-R5. Maybe 2... KR-K1 or something, to cover the Bishop on K7. He cannot play 2... B×RP because the</p> | <p>35</p> <p>40</p> <p>45</p> <p>50</p> <p>55</p> <p>60</p> <p>65</p> |
|---|--|---|

- 70 Knight on B6 is hanging. Is there another move for Black? Once again 3. P-R5 is threatened; is this simple to parry?
 Go on calculating
 1. B×N/5 again.
 75 1. B-N1 also comes into consideration.
- (E: Now you're gradually beginning to get into time pressure).
 Still look quickly at a few things; weigh things. Yes,
 1. B-N1 is my move, later to be followed by P-KR4. 80 81

Because this type of changeover is prepared by a clear and rather long crisis, it is, psychologically speaking, less revolutionary in character than the cases described earlier. This fact does not alter the import of the phenomenon, however; here, too, the problem transformation entails a sweeping structural change in the outline of reasoning.

The interesting and instructive protocol (G5;B) contains very drastic problem transformations of a somewhat different type. A short discussion of the complete problem development follows.

A clear preference to 'proceed actively' especially on the King's wing (plan $A = P_1$) had already been expressed in the introspective report of the first ten seconds (presented in its entirety in Section 45, p. 200). Correspondingly almost the entire first part can be read as a converging of the type $P_1-(P_2)$. The blockade (plan $B = P_2$) temporarily remains in the background. Subject G5 first tries out several different moves in the framework of plan A , namely, $1... R-N1 = a$, $1... P-KR4 = b$, and $1... K-N2 = f$; subsequently a certain preference for $1... R-N1$ develops, and this move is compared only with $1... P-KR4$ (convergence Z_1-Z_2 , where $Z_1 = R-N1$ and $Z_2 = P-KR4$).

One might now expect a quick move decision were it not for the fact that neither continuation satisfies the operative maximal expectancy (approximately 5). This dissatisfaction undermines the supremacy of P_1 over P_2 , the foundation for his outline of reasoning. Subject G5 gradually begins to doubt the validity of his preference: the converging process $P_1-(P_2)$ fails. After the second calculation of $1... P-KR4$ (the second b in the formula below) there follows a clear transitional phase in which the integration of this difficulty begins, or is at least prepared. One last attempt is made to rescue the plan by re-examining $1... R-N1$. This time $2... P-KR4$ follows as a second move: a kind of combination of the two solving propositions. But when this doesn't yield any overwhelming results either, the crisis is unavoidable. In the subsequent lengthy transitional phase some qualitative results of the investigation are in fact formulated, while the conflict elements

of the board situation are made explicit and the pros and cons of both plans are weighed.

For all that, the subject finds himself in the happy circumstance of not yet having to choose a plan: $1... R-N1$ is playable in both plans A and B and does not spoil anything. There is a way out of the dilemma: the alternative $P_1-(P_2)$ goes into Z_1-Z_i , in which $Z_1 = a = 1... R-N1$ need not tangle with the adversaries P_1-P_2 . Subject G5 had in fact tried to get off the hook quite early, as evidenced from the question he asked the experimenter in line 37: 'I have only to make one move, is that right?' But he doesn't get off until he has, on principle, sided in favor of plan B .

The move decision was ripe even earlier: 'Were I in time pressure here, I would already have played $1... R-N1$. But let's examine the consequences somewhat better' (line 81). Moreover, by choosing sides he prepares his next move (part B2).

What follows is the formula for successive solving propositions with the line numbers in which they occurred underneath. The text itself is presented from line 44 to the end of the first part.

(G5; B1): $a - b - f - a - a - b - a - a - a$; ($T = 15$ minutes).

lines: 4 7 22 30 35 48 53 85 101

where $a = 1... R-N1$, $b = 1... P-KR4$, and $f = 1... K-N2$.

- Well, he will be obliged to play P-KB3 sometime. Then one Bishop is eliminated but the other becomes stronger. Let me look again at the Rook file combination, if there's really nothing positive in it. Yes, I have an idea: 1... P-KR4; 2... K-N2... but he is not obliged to take on N3. Yes, I have serious doubts whether that will work out. On the other hand my pieces come into play very quickly. (Pause)
- 45

After this first transitional phase follows the last attempt to save the favorite, plan A . The last attempt runs into the second transitional phase.

- 1... R-N1 and then the Pawn sacrifice. 2... P-KR4 and 3... R-N4. Then can I go to the KN file? 1... R-N1; 2. P-N3, P-KR4; 3. P×P, R-N4; 4. P×P, R-N4 - and possibly he can first take on B2; so for instance 5. P×Pch and 6. P-KB3. Then it would be wonderful if I could get the King Bishop on the diagonal; otherwise there's nothing more than a check. I have the feeling if you don't proceed actively, you will slowly lose. Materially it is not so good. I have seen games, though, where you could not make any progress. But there the passed Pawns were restrained, here they aren't. 1... R-N1 will do no harm in any case; you do not lose a tempo. (Pause)
- 55
- The move P-KB3 cannot be prevented in the long run, and then you can hardly organize a mating attack. After P-KB3, perhaps play B-B2 and to go to N3. Yes, but there are also other possibilities (after P-KB3): B-R7ch; K-B1, B-N6 and forces the Rook to
- 60 65 70 75

- remain on the first rank. (Pause) If only I could force him to go to KB1, but after B-R7ch comes K-B2. That can never be prevented.
- 80 Were I in time pressure here, I would already have played 1...R-N1. But let's examine the consequences somewhat better.
- 85 1...R-N1, then 2...R-N4; 3.P-B4, R-N4 – what have I actually got then? No, that's not it. But wait a moment: I think I see something else though. Put the Bishop on QB6 and then try something with P-QR4. I begin to feel that I shall have to set up a defensive position. The King Rook file may then be tackled later. (Pause)
- (E: What do you think now?)
- I don't think anything – that happens sometimes. (Short interruption for a discussion on the meaning of this phenomenon.)
- The Rooks can do nothing, that's nice. I play in any case
- 1...R-N1.
- 90
- 95
- 100

The trend is clear. The alternative P_1 -(P_2), with P_1 as the favorite, cannot maintain itself as an anticipatory schema for the final argument. It is liquidated and makes room for Z_1 - Z_2 , a move alternative which is unrelated to the P_1 - P_2 opposition; 1...R-N1 is at last played. The struggle between plans is not resolved, but the emphasis has shifted: P_1 -(P_2) becomes P_2 -(P_1). The latter schema dominates the first 22 lines of (G5; B2) as an anticipation of the argumentation.

After the first lines of (G5; B2) there is another shift in the outline of reasoning, even more sweeping than the first one. The direction is reversed – from plan *B* back to *A* – but for the rest history repeats itself: none of the moves investigated satisfies the quantitative expectations. There is again a sharp convergence (within plan *B*) of the type Z_1 - Z_2 ; reading the protocol text one would simply expect a decision. But the alternative (2...K-B1 = c = Z_1 or 2...P-QR4 = e = Z_2) is quite soon discarded as being too narrow. The next move to be calculated (2...B-Q7 = d) is less passive than the two others but still fits into plan *B*, so that the structural change up to this point can be schematized as follows: [Z_1 - Z_2] → [Z_3 -(non- Z_3)] – within plan *B*.

But now come fairly extensive re-investigations of the two moves 2...K-B1 and 2...B-Q7, taken in turn in a pattern of progressive deepening and broadening: [Z_3 - Z_1]; investigations which, in spite of all the efforts, do not produce anything satisfactory. The following transitional phase clearly reflects the crisis of expectations and the sudden switch back to plan *A* (= P_1):

(G5; B2):

- 65 2...K-B1 I don't believe in anymore. Yes, but what then? If immediately:
- 68 2...B-Q7, then he plays 3.R-K7, threatening R-B7 and R × RP. Yes, I foresee a grim future.

- 71 2...R-Q1 immediately prevents P-Q4 easily enough. But then the Rook gets in on the seventh rank. (Pause) I guess I am taking a little rest again to collect some ideas. It is a rotten position. Under these circumstances I am almost ready to look again to see if I can mate him.
- 78 2...R-N4. ...Yes, now I have an idea.
- 79 2...R-N4. Suppose he doesn't want that Rook. I see indeed a possibility to win the exchange. Against the loss of a Pawn, it is true, but still an improvement.

With this a new and deeper investigation begins within the now favorite plan P_1 (= *A*). The moves 2...R-N4 = a and 2...P-KR4 = b are examined in turn and compared with each other. 2...R-N4 gradually ripens into the favorite (convergence of the Z_1 - Z_2 type). With some 'nice' results of 2...R-N4, P_1 becomes stronger with respect to P_2 . Precisely where the definite decision in favor of P_1 is made cannot be said (compare the discussion of (M2; ...N-K1) on page 206); for that matter it is not important any more for the crucial question has become 2...R-N4 or not (Z_1 -non- Z_1). The decision with regard to this alternative coincides with the move decision.

Protocol (G5; B2) shows very clearly that a choice of plan, seemingly definitive from the protocol text, may yet be revoked. It gives a beautiful example of a sweeping structural change in the outline of reasoning; namely, a *change of favorite* with regard to a *plan alternative*. For some formulas representing the whole thought process, refer to Section 49.

Section 48: Transitional phases as phases of problem transformation

In the preceding pages the transitional phases have already been analyzed from various points of view. First, in Section 34 they were descriptively distinguished from the phases of elaboration by certain trademarks: the occurrence of problem analysis and pauses, and on the part of the subject a somewhat broader, less strictly focussed set, a somewhat more receptive rather than actively organizing attitude on his part. Functionally, the transitional phases were characterized as phases of *integration* as opposed to phases of (differentiation by) *elaboration*.

Second, in Section 37 it was decided that all types of content elements of a protocol, except for the subject's reporting of elaborations (calculations mostly), belong to the transitional phases. In particular, all problem formulations and contributions thereto, such as anticipations, strategic direction and board goal statements, must

be considered to stem from transitional phases. These formulations were systematically registered and classified in Section 41.

Finally, all that was said in this chapter on the process of problem development was largely based on transitional phase formulations of the same type. All the phenomena described thus far belong to the transitional phase: the gradual enrichment and concretization of the total goal conception (formation of alternatives, favorite formation, convergence towards the final argument: Section 46) as well as the phenomena accompanying the more radical structural transformations (conflict analysis, abstraction of problem- and solution attributes, crisis of expectations, etc.: Section 47). In brief: All the phenomena of the problem transformations by means of which the total goal conception develops are transitional phase phenomena.

Transitional phases are, therefore, typically phases of problem transformation. We shall now go somewhat deeper into the mental operations by means of which the problem – or the total goal conception – is, in fact, transformed.

By definition a transitional phase constitutes the link between the end of one assimilative cycle (see Section 37) and the beginning of the next one, insofar as assimilation is elaboration, i.e., purposeful investigation of possibilities for action.¹⁷ A subproblem is finished; so the direction of the investigation can be expected to change. But, like a continuous undercurrent, the more general subproblems and certainly the main problem go on developing; and the subject's determination perseveres too. The main problem (the total goal conception) undergoes changes in its structure, but continuity does exist in many aspects, even throughout the most radical transformations. In addition to the most general goal, such constancy is also seen in many qualitative moments and comparatively self-contained parts of the total conception – e.g., parts of the analysis of the position which are likely to prove of later use. This is even true in those cases where the basic structure of the outline of reasoning is affected (Section 47).

¹⁷ Note that assimilation of a subproblem (in an assimilative cycle) is a process in which the restructuring and integration of results is but the last step.

¹⁸ It appears that 'chance' factors, external or internal, may play a role, e.g., fatigue, the sudden discovery (or abstraction) of a new aspect on the board, a small distraction or disturbance from outside, etc.

When the subject discards one of the counter variants of 1. Z_1 , the general determination towards the positive investigation of 1. Z_1 remains in the transition to the next variant. If 1. Z_1 is the favorite and the subject now turns to the negative part of the proof, his striving for an argument for 1. Z_1 remains operative during this transition. If the investigation of a solving proposition yields too little, the general 'direction' (e.g., plan) remains largely the same as the subject shifts to the next solving proposition; the general goal (e.g., how to get the pieces to the King's wing, to find a winning combination, and such; compare Section 41) remains in effect. Even when the subject runs into a blind alley – necessitating a drastic problem transformation – the principal operations-goal of finding a convincing argument is still effective. Likewise in the curious pauses in the thinking – during which G_5 and M_4 declared that they did not think at all – the most general operations-goal, just to make a move, holds sway although stripped of its entire substructure.

So there is always a more general problem that remains effective. The subject *must* revert to some more general problem, for that matter, in order to be able to integrate a partial result.

Indeed we have already seen that the problem formulations and the contributions thereto in the transitional phases are nearly always expressions of attributes of problems more general than those of the preceding elaborative phase. Even solutions attributes of specific subproblems are viewed by the subject as contributions to the main problem. In fact, every particular result, whether qualitative or quantitative, reappears in a transitional phase in a generalized form, if it shows up at all. The integration of a result into the total goal conception demands a broadening of the subject's current appreciation of the total goal conception (cf. Section 44, p. 186). He must be cognizant at least of the next higher order problem into which the result is to be integrated. At least, indeed. A return to the most general aspects of the main problem may also take place even after the completion of a minor detail problem.¹⁸

The *generalization* of problem awareness and goal awareness, i.e., *integrating what has preceded* by an expansion of the scope of vision, is, however, only half of the composite process that takes place in a transitional phase. Next to and generally after this, the *preparation for what follows* demands a new *specialization*.

After the subject has become aware of the problem, the transition to a specially directed investigation requires a freshly set, specialized subgoal. The specialization, in turn, generally requires a partitive transformation of the more general problem operative during the transitional phase (the problem to which a return was made). Yet sometimes the subject must first seek an applicable transformation; hence the transitional phases (like the first Phase) enjoy the character

of phases of means finding (*Mittelfindung*) as opposed to the elaborative phases of means applying (*Mittelanwendung*) (cf. Section 19).

It is not possible *in abstracto* to continue the present analysis of the operations hidden in this basic pattern of transitional phases (initially integration and problem generalization; then preparation for renewed specialization). For that purpose it is necessary to distinguish between various types of transitions and transitional phases. They differ from each other in many respects, in fact. There are, first, *cumulative* transitions; e.g., from one counter variant to the next, from investigation to immediate re-investigation, from exploratory examining and trying out of possibilities to deeper investigation, from investigation to striving for a proof, etc. There are, second, *subsidiary* transitions from one variant to the next (improved one), from move to move, from plan to plan, from striving for much to striving for less (drop in the quantitative expectations), etc. There are transitional phases that take place automatically and leave hardly any or no traces in the protocol, but there are also those that take up many lines (e.g. (G5; B), p. 226); in some transitional phases the characteristic broadening of problem awareness is limited and of short duration, in others the subject returns to the main problem in its most general form; the problem transformation may be quite partial and trivial or radical and important; etc.

In the following discussion cases are ordered according to four degrees of transitional phases, from simple and automatic to complex and laborious transitions.

1. The simplest case is that of a cumulative transition from one counter variant of 1. Z_1 to the next within the framework of a positively directed series of calculations. If, before starting, the subject is aware of the composition of the complex operation 'to calculate Z_1 ' (e.g. in (E5; A); '1. $N \times N$, then two, actually even three ways to retake'), then the transition from one variant to the next can be automatic. This simple case resembles Selz's example of copying a hexagon, discussed in Chapter II, Section 20. There is but one disparity in the analogy: instead of the operations actually being carried out, step by step, here they are worked out entirely in the mind.

Whenever a favorable result is obtained which measures up to the expectations, then in theory, indeed, there first follows an integration (a return to the general goal-setting 'to calculate Z_1 ') and then a specialization ('now to calculate the next variant'). But these processes may be completely automatic. Quite often they are, in fact,

automatic enough to leave no trace of a transitional phase in the protocols.

Mutatis mutandis, the same holds for some subsidiary transitions, e.g., during the phase of trying out: from one apparently unfeasible solving proposition to the next. Here, too, the moves to be tried are often ordered a priori. Sometimes this methodicalness finds its way into the protocols, for example in (G5; A): 'Now, let's look in some more detail at the possibilities for exchange: 1. $N \times B$ or 1. $N \times N$ or maybe 1. $B \times N/5$ or maybe first 1. $B \times N/6$.' Thus the transitions to follow may be automatic.¹⁹ As in the cumulative transition, the ensuing determination to investigate the next move may be fully explained by the preceding determination-to-carry-out-a-composite-operation and by the quantitative results of the just completed part of the investigation. The transitional phase itself, if at all perceptible, does not furnish any new problem information in such a case.

2. In general, however, the transitions from one variant to the next, respectively from one solving proposition to the next, are not so automatic. Composite operations whose complete structure and sequence are known beforehand to the subject (and expressed in the protocol) are relatively rare. They occur mainly in checking calculations and in re-investigations during the striving for proof. Much more often the subject begins with 'Let's first... (e.g., investigate plan, move, or variant *a*)' without bothering about what exactly he will do afterwards. He knows that after the possible elimination of *a*, a new investigation must follow; but he need not yet know which one. In order to know which one, separate means finding operations are necessary. This means that during the transitional phase a *special abstraction process is directed towards finding a new solving proposition to be investigated*.

After finding a satisfactory variation in reply to countermove *a*, the subject says, for example, 'But he can probably reply differently. Yes, ...' (investigation of countermove *b* follows). The branch is sought and found during the transitional phase by means of a revision of the problem undertaken to that end. Or, after the elimination of a tested (own) move, the subject may look for other pertinent moves

¹⁹ This need not be the case, however. One may find several cases in the material where a composite operations plan is *not* completely carried out, contrary to the original determination. Whenever the subject, after completing one part, does not proceed with the next but instead starts on something else, an intermediate transitional phase with an explicit return to a more general problem can be expected.

via a 'general inspection'; e.g.: 'Let us look around for other possibilities' (M2; A). These are separately *determined processes of means abstraction* carried out on the position facing the subject. With most transitions to a new countermove or other pertinent move the searching process is gone through so quickly, we must assume, that no goal-setting is stated in the protocol. It is, in general, only stated when the process is slow, when the search is not going smoothly, and/or when the pertinent moves seem to be exhausted. The two quotations above, from (M2; A), stem from just such situations.

In these less automatic cases it is obvious that the transitional phase contains relatively independent processes (searching and means finding operations). These processes may yield important contributions to the 'preparation of what follows' and to the transformation of the total problem conception. The choice of the next variant or move cannot be predicted from the previous protocol text before the transitional phase.

3. In order to describe the processes in those transitional phases that proceed even less automatically we will begin at the point where quantitative and qualitative results are obtained and generalized. It is true that the obtaining of results belongs to the preceding elaborative phase, but we must go somewhat further back to be able to draw a vivid picture of the path followed during the integration process.

(a) The quantitative *appraisal* of the variant, move, or plan just investigated provides a good starting point.²⁰ Even though each partial investigation also yields qualitative results in answer to special, non-quantitative questions (e.g. the attainability of a certain strategic or tactical goal), the final rating, the evaluative moment in the judg-

²⁰ The boundary is carefully chosen: to the *elaborative phase* belongs the ascertaining of attributes (quantitative and qualitative results) of the most recent subproblem and its solution; to the *transitional phase* belongs each generalization, either pure, amplified, or combined with previous results. The definition is precise as long as we do not change the level of refinement (into subordinate problem cycles and their results) of our protocol analysis. In case of a shift from a more macroscopic to a more microscopic view of the thought process, 'results' from a preceding (large) elaborative phase will often appear to be produced by a process of generalization and combination of results from subordinate (small) elaborative phases – so that the more microscopic view of the thought process would necessitate a shifting of the boundary. If we use large elaborative phases as units, not only do all the transitional phases between subordinate problem cycles within each elaborative phase fade out, but the transitional phases 'of higher order,' which remain visible between the elaborative phases, also become smaller.

This does not detract from the practicability of the criterion for establishing

ment of the envisaged end position of a variation is the main thing. The quantitative is especially decisive near the end of the thought process. In fact, the overwhelming majority of direct formulations of results in the protocols are of a quantitative nature. Let us, therefore, begin the analysis with the process that leads to this evaluation: namely, the appraisal of the envisaged end position(s).

The appraisal is often a fairly difficult and complicated process in its own right. In Section 41, D2, where we discussed the investigation of envisaged positions as a subproblem of the elaboration, it appeared that here, too, static, dynamic, and evaluative moments are distinguishable (cf. Section 40). The first two moments, however, generally serve the last, the striving for an evaluation. From the formulations of the results (more than from the often meager goal statements and problem formulations) we are often able to see how – as in the first Phase sometimes – *the evaluation comes about via the noting and weighing of static and dynamic features of the position*. For example in (G2; A):

(After re-investigation of $1.N \times B, P \times N \dots$)

36 pressure on QB6, the pair of Bishops: also very agreeable, but the position promises more.

The position attributes on which the subject bases his judgment are often mentioned after the judgment itself, as a kind of *post facto* justification – probably reflecting a subsequent checking process during which the relevant factors are made explicit. For example:

(G5; A):

94 $1.B \times N/5, P \times B$ is therefore forced. But that's good for White. The Knight on KB6 is weak, the Bishop at K7 hangs – and the Bishop on QB3 stands badly.

boundaries. In principle, the application is not difficult. If we consider the investigation of a certain sample variation of $1.Z_1$ to be one elaborative phase, while ignoring the substructure, then the establishment of the unfavorable result of $1.Z_1$ belongs to this elaborative phase. If the subject concludes from this unfavorable result (and possibly from the results of other sample variants) that $1.Z_1$ is no good, or a fortiori that the entire plan containing $1.Z_1$ might not be good, or that the position is not so favorable after all, then the processes basic to these conclusions belong to the transitional phase. Likewise the ascertainment of and generalizations from qualitative results may be differentiated.

From the text of the protocols it is often difficult to decide whether the subject refers to a specific or a more general solution attribute. When he says after calculating the sample variation $1.Z_1$: 'That's no good. He can always play P-KB3,' he may mean it's no good in this variation (with its subbranches), or against $1.Z_1$ in general (thus $1.Z_1$ is no good), or even against the whole plan (the plan is no good). These difficulties are encountered especially with qualitative results even if the complete protocol context is taken into account.

A statement of the various factors involved in an overall evaluation occurs frequently in the re-investigation of possibilities, towards the end of the thought process when the subject searches for definite results necessary for his argument. Aside from their argumentative function – to justify and support the evaluation – these factors have meaning as independent qualitative results, as a matter of course; they may be generalized or combined with other results and thus assist in more accurately concretizing the qualitative aspects of the total problem conception.

(b) So much for obtaining elaborative results, roughly characterized as a process of combinative and abstractive operations. Quite naturally, however, this process runs on into the *generalization and combination of results*: the theoretical beginning of the transitional phase.† The (abstracting) attitude of the subject remains about the same, but now the quantitative and possibly the qualitative results are viewed in an enlarged context. A variation is conceived, for example, as a *sample variation* for a whole solution proposal (move, plan, idea). The result then functions as a prototype for a more general result. For example:

(M₄; C): (Calculation of *one* (sample) variation of 1... P-K₄ yields):

70 Good position. Yes, 1...P-K₄ suits me much better. [Better than 1...N-K₅ investigated so far.]

Often, however, results are combined: the conclusion is primarily a *summary* of the results of formerly calculated variations. Basing his conclusion on the investigation of dozens of variants subject M₂ in (M₂; C) arrives at: 'So, 1... N-K₅ is playable' (line 124). Immediately afterwards some new (first) moves are examined and their results broadly summarized and generalized: '...I really have even less faith in all that. Everything is going badly' (line 134).

Akin to the evaluation of a move which is based on investigated variants, we find the evaluation of a plan based on the results of moves, and the evaluation of a position based on the results of the investigation of possibilities in general. The protocols abound with examples of such combinations and generalizations:

(M₄; B₂):

(Investigation of 2... K-B₁)

57* Doesn't work so well. For that matter, I don't at all see how I can make anything out of the attack.

† See note 20 on p. 232.

(M₂; C):

(Investigation of 1... P-Q₄)

38 No, then I get a hole at K₄. Maybe my position is not so good after all.

(M₃; A):

(Calculation of 1.N×N)

31 ... – nothing. There is no decisive combination.

(M₄; ... Q-K₂):

(After investigating several forceful moves)

27 Maybe 1... B-B₁... no; it doesn't go so quickly as I thought. Not so simple. (and when re-investigating further on):

33 1... B-B₁ (*variation follows*)

34 then you're not exactly better off either. It's not so easy to break through his position. White will be able to hold his own on the QB file.

In the last example as a result of the generalization we see an anticipation with respect to a general qualitative solution attribute ('White will be able to hold his own on the QB file'). Similarly, the frequently stated 'So that is forced' is also a qualitative solution attribute (or possibly an anticipation of it), which stems from a combination and/or generalization of partial results, namely, the results indicating that other defenses or replies 'won't do.'

Another form of combination of results is *comparison of alternative continuations* and of their qualitative results, as occurs rather frequently in the final Phase (compare Section 43, p. 177). Actually, comparing results with existing expectations may also be considered a part of the integration process. In (G₂; A) G₂ states: 'also very agreeable, but the position promises more' (maximal expectancy!). (G₂; A) is, however, one of the rather rare protocols in which the comparison with expectations is expressed. It is not even certain that we may always assume a separate process, for, in most cases, presumably, the judgment is a priori relative to the maximal expectancy. A remark like 'doesn't suit me much' means 'not so good as I had hoped (here and now).'

The list of possible forms of generalization of results is still far from complete. But they all boil down to the following: the partial result is allotted its proper place (integrated) in the developing outline of reasoning; its (generalized) value and significance for the outline are assessed; it is then liable to be used in the convergence to a partial argumentation or to the final argument.

This whole process is already known to us for the special case of the final Phase, just before the move decision. There, too, the most recent

results together with all preceding ones are fitted into the argumentation to produce its final form: the final argument. In the final Phase both this process and the re-checking recapitulation of the reasoning in the light of any new insights often show up very clearly.

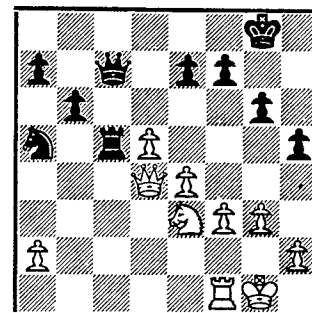
(c) Now for the '*preparation of what follows*.' In the cases treated under point 1 (p. 230) no preparation is needed: the transition is automatic. In the cases discussed under point 2 (p. 231) a process of means abstraction leads to the finding of new moves to be investigated. But this process may run dry so that no new moves can be found by 'inspection.' In that case the subject will finally have to choose from the moves already considered. Generally, he then reverts to a deeper re-investigation in some direction (new phase of deepening) that has been prepared by processes in the transitional phase. A brief discussion of these preparatory operations follows.

Undoubtedly one of the most general methods in thinking, discussed more than once in *Denkpsychologie*, is activated when our first attack on a certain problem fails; we then try to get a clearer picture of what the problem really is. A deeper, more conscious analysis of the problem: an attempt to focus the structure of the total goal (or problem) conception, an attempt to arrive at a sharper formulation of the problem. We met this phenomenon earlier whenever the subject had 'difficulty' in choosing between two plans, in eliminating a continuation, in finding a partial proof, in evaluating a position, etc. Here – and almost only here – the subject's methodology (the operations he uses) becomes quite clear in the protocol text through an abundance of problem formulations. 'Degrees' of sharpness in these problem formulations were discussed in Section 41.

This basic process is often best described as a recapitulation, as a new and *more systematic arrangement of relevancies*. The subject strives towards better defined groupings, sharper boundaries, a structural overview of his argumentation. The result can be a clear alternative: try either *A* or *B*; or can be a precise and detailed definition of the strategic or tactical goal. In other cases, however, we would do better to characterize the phenomena in the run-dry-type of transitional phases as a fresh and now more consciously systematic *position investigation*, in which the estimated *value* is again at issue, the maximal expectancy being in danger of dropping. In still other cases the emphasis is on the *qualitative* aspects of the *board problems*.

The following example shows a little bit of all three aspects: improved systematization, endangered expectations, deeper insight into the board problems; but the emphasis is clearly on the latter aspect.

In (M5; R-Q1) little comes from the first attack on the main problem (investigation of moves: P-K5, K-N2, P-B4, and P-N4). Subject M5 winds up the fairly extensive investigation of 1. P-N4 with a brief remark about the sense of the move:



POSITION ...R-Q1

White on move

(M5; R-Q1):

74 1. P-N4 is actually a bit warped; only makes sense if a sharp attack comes out of it. If not, discard it.

After this M5 ventures another attempt at solution through 1. P-K5 (re-investigation); when 1. P-K5 fails to produce the desired effect, a deeper analysis of the problem follows:

80 1. P-K5 (*variation follows*)

84 doesn't suit me particularly. The thing is to keep the Knight from exchange. (Pause) Something must happen immediately; if Black can exchange pieces, White won't win so easily. The (Black) Knight threatens to come to B5, and R-B8 is also possible. Then it's not so easy anymore.

For further examples see Section 41.

As a result of the renewed problem analysis in situations of the run-dry-type, sharper formulations emerge, which lay the groundwork for the coming specialization. The subject now has a better idea of what he is getting at and of the obstacles he must overcome; he knows with more certainty which (groups of) moves are pertinent and which certainly are not. The more sharply focused problem conception enables him to use more narrowly directed methods of investigation. The following investigation may, for instance, be directed towards systematic completion of the analysis of a group of possibilities; or towards their elimination. There is more room, too, for indirect selection and for comparing possibilities. The thus sharpened problem may already imply an alternative formation with a favorite at one end – and thereby determine the transition to the following special investigation.

As in the cases described under point 2, the moves for the coming investigation are 'discovered' via a special survey in the transitional phase. Only now the moves – or at any rate the ideas – are not new and uninvestigated but old ones in a new guise. The question is not so much to 'look around for other possibilities,' (see p. 232) but rather to recapitulate, delimit, reflect, and become conscious of what is possible (relevant). In this respect, too, there is a similarity with the final Phase: only, here these mental operations do not close the thought process as they do there.

4. Finally, the course of the transitional phase is least automatic when a 'crisis' must be overcome, when the subject must give up a strong favorite (plan, idea, continuation) and/or a cherished expectation. Most of the characteristics of such transitional phases are already known from the discussion of drastic problem transformations in Section 47. It will suffice here to summarize the typical processes.

Frequently, working out the preceding (negative) result is already a laborious process. We find *lengthy investigations of the final envisaged positions*, with an *increasing awareness of the factors* pertinent to their evaluation. We find *conflict analysis* in this stage already (Why is it that the position is so unsatisfactory?). We find *checking operations* and *doubt* of the correctness of the evaluation (Is there really nothing to be done about it?). The *integration* and result *generalization*, too, require more than normal effort and are worked out *very explicitly*; as a result of the conflict analysis many problem and solution attributes are expressed. The explicitness of the subject's total problem conception attains a maximum: if a sharper problem formulation (as in point 3) is at all possible, it will certainly take place here. *Checking operations* and *increased consciousness* of all factors occur here, especially those concerning the *nature and causes of the difficulties on the board* (again conflict analysis). There are many *pauses in the thinking* that undoubtedly denote 'I guess I am taking a little rest again to collect some new ideas' (G5; B2) – and energy.

After such a highly conscious process of preparation, it is generally only just before the beginning of the next special investigation that the problem is definitely transformed. A decision to investigate in a new direction is needed to dispel the subject's doubts.

We close this section with a brief summary of the function of the transitional phases:

It is in the transitional phases that the most recent partial results

are integrated, are fitted into the total problem conception which is thereby updated. This implies a return to more general problems. The return then forms the basis for finding new means: new moves, plans, ideas, subgoals, new methods of investigation, a new outlook on old possibilities, better defined groupings, new evaluations and expectations. The more pronounced transitional phases, especially those which contain actual pauses in thinking, have the additional function of providing some rest for the subject from the preceding, strongly focused concentration and to allow him to collect new energy for what follows.

Section 49: Types and examples of problem development

In no two protocols is the course of the problem development and thus the structure of the thought process the same. Even if we look just at the formal structure of the process, the differences remain large. In the present section we shall examine them in some detail.

As a starting point for a formal description of types of problem development we shall follow the metamorphoses of *the group of pertinent moves*. Although the protocol is not an entirely reliable basis for determining exactly which moves belong to this group, by taking into account the protocol text, the (formulas of) successive solving propositions, and the structure of calculations it is possible to arrive at an acceptable interpretation.

In the long run, of course, the pertinent moves cannot but decrease in number until finally one remains to be played. The subject arrives at this result by applying general operations and methods (groupings, eliminations, etc.) of the sort which have been discussed in the previous pages. The actual sequence of problem changes, however, differs from protocol to protocol in many respects.

In some cases the subject starts immediately after the first Phase *to try out* various moves among the pertinent possibilities, apparently without other specific selective criteria (e.g. subgroupings); in other cases there are *subgroupings* from the very start. The whole protocol may express *one continuous process of convergence* to an alternative problem: Z_1 –(non- Z_1) with Z_1 the favorite; in other cases *intermediate groupings* in plan alternatives and the like occur, in cumulatively linked phases of convergence. In still other cases the sought-for convergence and rounding off of both argument and thought process fail, so that there are *changes* in the *alternative groupings* and *favorite formations* (subsidiarily linked phases of convergence).

In the following empirical classification the different structural types refer solely to the problem development after the first Phase. Each of the four main groups is found in at least ten (out of 49) protocols. The classification is based on an interpretation of protocols as a whole – a somewhat hazardous undertaking, it is true – but an attempt was made to stick to the protocol text as closely as possible and to avoid unwarranted assumptions. In view of the unavoidable incompleteness of a protocol (cf. Section 23) it is likely that the simpler types of problem development structure are somewhat over-represented, viewed from the point of the actual experimental thought processes. The only general pretension of the classification is that the various types do occur with some regularity, in actual chess thinking as well as in the experimental series.

I. *Rapid and definitive formation of a favorite*

During or shortly after the first Phase the subject's preference for one particular move becomes evident. A favorite is formed; from here on the whole process is one continuous process of convergence up to the point where the final argument is ready. How long the convergence process lasts may vary but finally the first and only favorite is played: $[Z_1-Z_i] \rightarrow Z_1$.

Subdistinctions can be made within this group according to the relative emphasis given to the positive or negative part of the argumentation. On one side are the extreme cases where the negative part is completely or practically absent and on the other where the negative part (investigation of 'other moves') is predominant. If move Z_1 is considered satisfactory in its own right, the choice is *direct*: other moves (Z_i) are hardly found in the protocol. If other moves prove unsatisfactory, the choice is *indirect*: there may be practically no analysis of the move Z_1 that is finally played. Formula:

$$[Z_1-(Z_i)] \rightarrow Z_1 \text{ respectively } [(Z_1)-Z_i] \rightarrow Z_1$$

II. *Definitive favorite formation after trying out*

Here the formation of a favorite is preceded by a phase of trying out in which the consequences of a few of the pertinent moves are explored. After the trying out Phase the convergence process collapses into a type I. Here, too, the first and only favorite is finally played. Formula:

$$\text{Trying out} \rightarrow [Z_1-Z_i] \rightarrow Z_1.$$

The duration of the trying out process as well as the number of moves explored, of course, differ from protocol to protocol. The convergence is often of the type: $[Z_1-(Z_i)]$, due to the fact that the pre-favoritism phase of trying out has already scored negative results for other moves (Z_i); the negative part of the argument is then superfluous.

III. *Simple cases of change of favorite*

Here the convergence process after the formation of the first favorite is cut off and the favorite move invalidated. Either immediately afterwards or somewhat later on another move gains favor, solidifies its position as favorite, and is finally played. The general formula, covering cases with varying degrees of trying out, is the following:

$$(\text{Trying out}) \rightarrow [Z_1-Z_i] \rightarrow (\text{trying out}) \rightarrow [Z_2-Z_k] \rightarrow Z_2$$

Within this group there are found various specific forms; see the examples under 4 and 5 below.

IV. *Step-by-step convergence, possibly with changes of favorite*

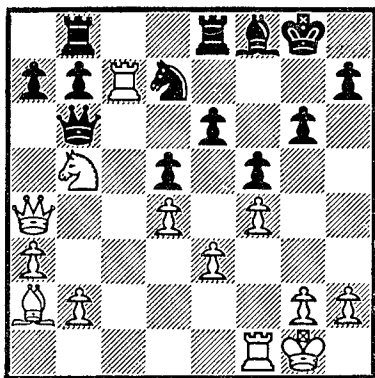
Here the convergence, or at least one of the convergences in the protocol, proceeds in discrete steps, via subgroupings of moves. The distinctive characteristic of group IV is the occurrence of plan alternatives (or other groupings of pertinent moves) with corresponding favorite formation and convergence possibly with other complications as well. Group IV consists of highly heterogeneous but generally quite complex processes so that it is impossible to establish one general formula for the group. The one characteristic feature is the occurrence of plan alternatives: $[P_1-P_i]$, $[P_1-P_2]$, etc.

One obvious subdivision separates those protocols with only one convergence process (IV, A) from those with more than one (IV, B). Within IV, B we find, apart from step-by-step convergence, shifts in the move groupings such as changes of favorite. Most of the protocols with drastic problem transformations, as described in Section 47, belong to subgroup B (if not to group III). Subgroup A corresponds roughly to groups I and II and subgroup B to III, the characteristic difference in both cases being appearance or non-appearance of intermediate plans or other move groupings

A few illustrations of problem development are now worked out and roughly ordered according to increasing complexity.

1. (M₂; ... P-QR₃) Formula: a - a - a T = 3 minutes.

In this objectively solvable position the key move 1... P-QR₃ leads to a gain of a piece. After a very brief first Phase subject M2 hits upon the right idea:

POSITION ... P-QR₃

Black on move

(M₂; ... P-QR₃) S: NICO CORTLEVER E: DE GROOT T = 3 MINUTES APRIL 8, 1943

Count pieces. Equal. Well, the Knight is attacked.

1... P-QR₃; 2 R×N - maybe the Rook can be trapped. (Time = about 1 minute)

(E: How do you arrive at that?)

The hanging position of the Rook gave me the idea; after 1... P-QR₃, the Queen threatens to take a piece. Not until *after* that did I see that the Black Knight was attacked. Furthermore:

1... P-QR₃; 2. R×N, P×N; 3. Q-B₂ - 3... Q-B₃ must be stopped, can that be done in another way? No - 3 Q-B₂, R-B₁; and 4... Q-B₃; so that won't do. (Pause) Let's look at 3. Q-Q₁ immediately. No, then 3... Q-B₃ right away. 3. Q-B₂ doesn't help either. It would help if the KN Pawn were on KN₃. The idea is to play R×QP as soon as he plays Q-B₃. Everything's checked. Yes, I play 1... P-QR₃.

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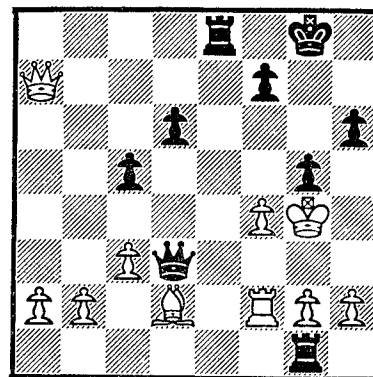
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We can safely suppose that the experimenter's question had no seriously disturbing influence on the structure of the thought process. The idea to catch the Rook was there, the favorite move was already formed; the remainder of the thought process was merely convergence: striving for proof. The subject only enlarged on the positive part, but for good reasons: when the favorite move promises and then appears to yield such a large advantage, a comparison with the results of other moves is superfluous. Thus type I, in particular: [Z₁-(Z_i)] → Z₁.

Objective solvability of the problem, or, rather the subject's anticipation thereof, obviously favors this type. In the special series of combinatorial tasks (cf. Section 28; also 5 below), the anticipation or

hunch that there is a winning or at least one best move was replaced by certainty. Then one-sided convergence processes [Z₁-(Z_i)] did occur more than once - although, of course, the first trial need not be a hit. Protocol (M₅; ... R×Pch) shows an extreme case. We have to do here with a mating combination for which the negative part of the proof is superfluous even from a theoretical point of view, as we know from Section 9:



POSITION ... R×PCH

Black on move

(M₅; ... R×PCH) S: A. D. DE GROOT E: CORTLEVER T = ½ MINUTE APRIL 15, 1943

Curious position. Circles around the King: mate in three or four. 1... R×Pch, 2... P-B₄, 3... Q-R₆, and 4... R-K₃. Is it already mate then? Yes, or 1... P-B₄ch - look for a combination 1... R×Pch.

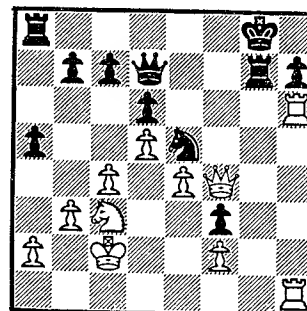
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Interesting in this protocol is the increasing specialization of problem and goal: a step-by-step convergence process within the first Phase.

2. (M₂; ... P-R₅) Formula:

a - b - c - c - c - d - e - f - a - a where
a=1... P-R₅, b=R-N₇, c=N-N₅, d=R-KB₁, e=R-N₅, f=Q-N₅

POSITION ... P-R₅

Black on move

(M2; ... P-R5) S: NICO CORTLEVER E: DE GROOT T = 10 MINUTES APRIL 8, 1943

- Count pieces. Black's positionally better off but on the defensive on the King's wing. Trapping the Queen doesn't work here. He has no threats for the moment.
- 5 Counterattack with 1... P-R5? 2. N×P, N×P? 1... R-N7 to threaten the Pawn on KB7. 1... R-KB1 chases the Queen, 2. Q-R4; then he still doesn't threaten anything. (Pause) No, at first sight: 1... P-R5's still the best. [to be continued]

So much for the first Phase. The three moments, static (lines 1 and 2), evaluative (line 1) and dynamic (lines 3 and 4) are clearly represented. From line 5 on a somewhat elaborated orientation to possibilities follows as a result of which a few pertinent moves are established. After that the main part begins with the investigation of the move 1... P-R5. In this stage the move is already weakly preferred: 'No, at first sight 1... P-R5's still the best.' In any case the alternative-grouping: 'Either 1... P-R5 or something else (better)' is already dimly visible. Still the rest of the protocol can best be described as a process of trying out within the group of pertinent moves. In the course of this trying out process the move 1... P-R5 really becomes the favorite. It is difficult to specify exactly where this happens, but by the time subject M2 returns to the move he appears to be ready for the decision.

- 10 (No, at first sight: 1... P-R5's still the best.) But if he just does nothing, lets me take, there's not much to it. First lure the Knight away (1... R-N7) and then P-R5 or P-QN4?
- 15 1... R-N7 - a bit strange. 1... N-N5 doesn't appeal to me either - let's get a closer look: 1... N-N5; 2. R/6-R4, N×P; 3. Q×BP, R-KB1; not so crazy. Still I've got the idea that there's a hitch somewhere. 1... N-N5; 2. R×RP? No, doesn't work. 2. R/6-R3 - loses the exchange. 1... N-N5. Let's examine precisely what he must do. 2. R-K6 and after 2... N×P; 3. R-KB1, for example, and after
- the Knight goes away, take my Pawn on B6 - or 3... R-N7; 4. Q×BP. First 1... R-KB1 after all and then N-N5? 1... R-KB1; 2. Q-R4, N-N5; 3. R-R5, R-B5 threatens to win the Queen, 4. K-N1. First 1... R-N5 and then R-N7? I don't think so. 1... Q-N5 doesn't make much sense either. Maybe it's better after all to play 1... P-R5 first, then we can always get funny later. Yes, I play 1... P-R5.

The argument implied in the protocol text is of a somewhat indirect character: the choice seems more to be based on the negative outcomes of other moves than on the positive features of 1... P-R5. Curiously enough, the consequences of the move itself are hardly worked out. The problem development is of group II:

Trying out $\rightarrow [Z_1-Z_i] \rightarrow Z_1$

3. In some of the type II protocols the phase of trying out is so very brief that there is more of a resemblance with type I. For instance, protocols (E2; A) and (E5; A) with the formulas:

$$(E2; A): g - e - f \mid - f - f \quad (T = 12 \text{ minutes})$$

$$(E5; A): d - c \mid - c - c \quad (T = 7 \text{ minutes})$$

The vertical line indicates roughly where the favorite formation is assumed to start. So: Brief trying out $\rightarrow [Z_1-(Z_i)] \rightarrow Z_1$.

At other times we find a very long phase of trying out while the favorite formation proper is only a matter of the last few moments. After an apparently unstructured review of several possibilities one move is preferred and played shortly thereafter. The convergence process is so brief that one gets the impression that there is a rather sudden choice after an exposition of all sorts of possibilities. These thought processes seem to roughly correspond to the layman's picture of a process of choosing from a number of possibilities. Curiously enough, however, this picture is found almost only in the protocols of some of the less strong players - in particular subject W2's. The characteristic (for strong players) convergence is lacking; there is no progressive deepening of the investigation; $r = 0$ even though N and n_0 may be rather large (cf. Section 36). An example is protocol (W2; A) with the formula:

$$c - e - l - k - r - i - s - g - b \mid - b - b \quad T = 28 \text{ minutes}$$

The formula yields $N = 10$, $n_0 = 9$, $c = 1$, $r = 0$, quite an exceptional combination of values. Even more curious is protocol (W2; C) because of the subsidiary final linking:

$$b - b - c - a - a - e - j - h - f - f - d - c \quad T = 20 \text{ minutes}$$

Although the chosen move, 1... Q-K5, may have been weakly favored during the process - as was move $a = 1... P-R5$ in (M2; ... P-R5) above - there is no indication of it in the formula nor in the protocol itself.

4. Protocols (M4; C) and (E1; A) have been briefly discussed in Section 47 as examples of drastic problem transformations. In spite of their rather long duration and composite structure they still belong to the simpler cases of favorite change (group III). Below, the formulas of successive solving propositions are given, complemented this time with a schematic representation of the course of the problem development after the first Phase.

Trying out of P_1, P_2, P_3 , etc. $\rightarrow [P_1-P_1] \rightarrow P_1$

$$\left\{ \begin{array}{l} \text{Trying out of moves} \\ \text{as examples of} \\ P_1, P_2, P_3, \text{ etc.} \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \text{Investigation of} \\ \text{moves within } P_1 \\ \text{as examples and} \\ \text{towards elimination} \end{array} \right\} \begin{array}{l} \text{(in } P_1: \\ \rightarrow [Z_1-(Z_i)] \rightarrow Z_1 \end{array}$$

First the battle of the plans is decided in favor of P_1 – curiously enough in a largely indirect way. After the plan decision there follows as a separate question: ‘Which move should I play first?’ Such a clear-cut state of affairs is rather rare, though.

7. Somewhat more complex than (G₅; A) is the problem development of protocol (M₃; A). A number of moves are fairly systematically investigated immediately after the very brief first Phase, namely, 1. B×N/5 (a), 1. N×B (b), and 1. N×N (c). The search is for a ‘decisive (center) combination,’ roughly the same plan, P_1 , that G₅ indicated with the term ‘possibilities for exchange,’ although M₃’s expectancy is higher. When M₃ fails to find something decisive he says, ‘Then maybe an ordinary attacking move,’ (P_2) is worth considering. To this end the moves 1. P-KR₄ (f) and 1. B-R6 (e) are investigated. But subject M₃ cannot really give up the idea of P_1 and cannot help returning to it:

(M₃; A):

38 1. B-R6 maybe. See if anything comes out of that. (Short Pause)

40 Still, I keep looking at taking on Q5; there might be something in it. 1. N×N or 1. B×N/5.

The move 1. B×N/5 is then re-investigated with a favorable result (due to the fact that the maximal expectancy has decreased in the meantime (cf. Section 50, p. 257). So the move 1. B×N/5 = a = Z_1 not only becomes the favorite within P_1 but also in an absolute sense. The rest of the thought process is a matter of convergence of the type Z_1-Z_i , i.e., the subject tries to prove the superiority of his favorite. The general schema:

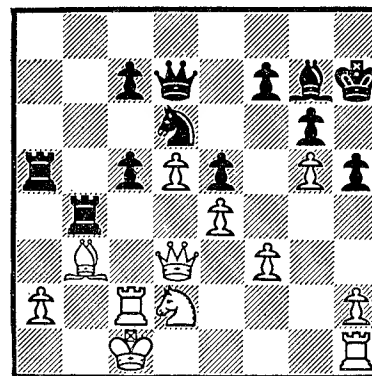
(M₃; A) T = 15 minutes

$$\underbrace{a-b-b-b-c-f-e-a-a}_{\text{Trying out of moves within } P_1} \underbrace{P_2}_{P_1} \underbrace{P_1}_{[P_1-P_2]} \rightarrow [Z_1-Z_i] \rightarrow Z_1$$

The different levels symbolize the synchronized development of overlapping and interdependent problems. Compared to the details of the protocol this super-structure and sub-structure schema is still a simplification.

8. We shall now try to follow the problem development of one thought process in some more detail. The first 26 lines of protocol (M₄; ... Q-K2) are:

- (E: Do you know this position?)
 No, I don't know this position. Let's have a look. Count Pawns, pieces. Certainly there's something to P-B5. But no, that can be sufficiently repelled.
 5 Nothing to be said immediately. Black stands somewhat nicer. King's wing is deadlocked, not much threatened there. How to forge ahead with the attack?
 10 Let's look at something with 1... R-Q5. But then comes 2. Q-K2; not much in sight.
 1... P-B5 or first perhaps prepare it with
- 15 1... R-R6. I'm looking at direct combinations first, otherwise something like 1... N-N4 should be considered. But then comes R×P, yes 2. R×P, that is not much, then 2... B-B1, Rook to... B6, no B3, no B2. ... The intention is to bring the Knight to Q5. I'm looking at
 20 1... N-N4; 2. R×P, B-B1; 3. R away; oh, no. Rook to B4 is best. Then White needn't worry. So that doesn't work very well.
 25



POSITION ... Q-K2

Black on move

Lines 1–8 give the usual picture of the first Phase: static and dynamic orientation, and in line 6 a rather sharp evaluation of the position (a rating of about 6). The evaluation is a kind of summary or conclusion stemming from the preceding observations: its appearance hints at the integration of the position and the completion of an important

part of the problem formation. In line 9 the board goal is formulated: 'How to forge ahead with the attack?' – which implies that an attack already exists. At this point the Phase of problem formation can be considered over; the group of pertinent moves is defined, be it indirectly.

The main part begins (line 15) with a typical partitive transformation which defines a subgrouping within the pertinent moves: 'direct combinations first.' The moves 1... R-Q5 and 1... P-B5 contain direct threats; they are in fact forcing. But the results do not satisfy the still high expectations of this stage (the *maximal* expectancy can be estimated at 8).

Accordingly, other possibilities are taken up that could be called direct improvements of position. The intention of the move 1... N-N4 is 'to bring the Knight to Q5'; the move is direct in that it contains a Pawn sacrifice – that on closer inspection, however, 'doesn't work very well' (line 25). The next move is again a preparation for a direct positional improvement:

- 27 Maybe
 28 1... B-B1 to prepare for N-N4 – no; it doesn't go so quickly as I thought. Not so simple.
 31 1... Q-R6 ... no. Indeed, I should ... (reverses himself again). Let's look.
 33 1... B-B1. What can he do then? 2. N-B4, N×N; 3. B×N or 3. R×N –

The formulation of the negative result in line 28 runs into a clear transitional phase. The subject's 'Not so simple' remark obviously refers to more than the last move considered; the result is generalized and the originally too high expectations are lowered somewhat (the maximal expectancy drops from 8 to around 7). Along with the expectations the direction of the investigation also changes: the next, more systematic examination of 1... B-B1 (e) functions primarily as a sample investigation of a calm (preparatory, waiting) move.

The result is insufficient:

- 34 – then you're not exactly better off either. It's not so easy to break through his position. White will be able to hold his own on the QB file.

Again the result is generalized: 'It's not so easy' to attain anything tangible with a calm approach either. Furthermore the investigation of 1... B-B1 must be assumed to have had an unexpressed but important qualitative result: namely, that is it difficult to activate the King's Bishop since Black's own Pawns block it in. With this assumption the next phase is easily understandable. The problem now becomes one of attaining something positive (by now the maximal expectancy may be somewhat less than 7) by means of a move that

helps to activate the Black Bishop for continuing the attack. The solving proposition is 1... P-KB3:

- 40 1... P-KB3, in order to play 2... B-R3 and in that way threaten something with P-B5.
 43 1... P-KB3; 2. P×P, B-R3. Yes, hold on, that looks good. 2... B-R3. Does that threaten 3... P-B5 then? ... Indeed it does: 3... P-B5; 4. B×P, N×B; 5. R×N, B×Nch, etc. Well, so it threatens something. But, he can parry 1... P-KB3 with 2. P-KR4 – but I can take then, and I have a protected passed Pawn.

But not even this has the hoped for effect. In the first place 'he can parry 1... P-KB3 with 2. P-KR4,' and in the second 'it doesn't yield so much, even if I can carry out that combination.' This conclusion is formulated during a typical transitional phase after the composite investigation of 1... P-KB3:

- 51 I still don't know quite what I should play. (Pause) I keep on thinking of 1... P-KB3. I don't know what else to go for. But it doesn't yield much, even if I carry out that combination.
 56 1... B-B1, to get the Knight to N4 – but then 2. N-B4 is the retort; we've already seen that.

Apparently subject M4 undergoes a slight crisis of quantitative expectations here: the move 1... P-KB3 does not lead to much, but M4 does not know what else to do. It is obvious that the maximal expectancy will have to drop again: let us say from somewhat less than 7 to 6, a drop with concomitant deeper investigation that M4 resists. In this stage the opposition of 1... P-KB3 to other moves (Z_1-Z_i) already makes up the grouping, but the move cannot yet be said to be a real favorite. After a brief re-investigation and definitive elimination of 1... B-B1, the transitional phase continues as subject M4 now starts a deeper and more systematic analysis, including counter possibilities among other things:

- 58 Have a look, what all can White do anyway? Something like P-B4? (Short pause)
 61 1... P-KB3; 2. P-B4 – no, that takes too long; 2. N-B4 is better for white. Something else to do. Let's have another look.
 65 1... Q-R6 again. I'm looking at all kinds of possibilities. 1... Q-R6. ... Let's look again at pushing on B5.

Although the text is not wholly clear, what is clear is that during this transitional phase the problem has become more structured due to the subject's striving to survey and order the existing possibilities.

The alternative becomes more of an acute choice problem: 'pushing on B5' (= 1... P-KB3) against 'all kinds of (other) possibilities.' Indirectly, the move 1... P-KB3 becomes increasingly preferred, and in

fact the rest of the thought process can be described as a continuous convergence process, Z_1-Z_i . The following deeper investigation of 1... P-KB3 – for the time being only the main variation, 1... P-KB3; 2. P×P, B-R3 – finally leads to a relatively favorable result (line 78):

- 68 1... P-KB3 again: 1... P-KB3; 2. P×P, B-R3. Now if he gets out of the bind with 3. K-Q1, for example, then – no that doesn't work: 3... P-B5 and then comes 4. N×P and if I take on Q2, 4. K×B; R-Q5, etc. 3. K-N1 instead; then take on Q7 and 4... P-B5 forks. Oh, no, the Queen takes back on Q7 and the Rook on N5 hangs. But that can probably be improved upon. Yes, first
78 4... Q- N4. That is probably not good for White.
80 1... P-KB3 rather suits me. Yes, I think that I shall play that.

But now a (checking) investigation, cumulatively linked to the preceding positive result, leads anew to a consideration of the other variation of 1... P-KB3:

- 81 Look at other possibilities. White doesn't have to take, he can play 2. P-KR4; then I don't see how to proceed.

So the state of affairs is still not satisfactory, mainly because subject M4's quantitative expectations are still too high. When a brief investigation of one other move (1... P-B4) leads to radical elimination, another transitional phase appears in which the difficulties are clearly expressed:

- 85 1... P-B4 right off; got something there? – but I don't see what good that is. (Short pause) Really the whole thing is to know what Black has been aiming at. The plan, I should like to know his last move.
91 I should play
1... P-KB3 in spite of everything.

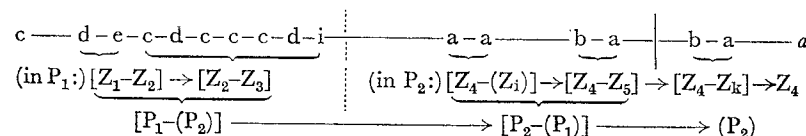
It is obvious that Subject M4 is searching for a decisive argument in the competition of possibilities. To this end he wants to know the kinds of board goals that Black has had in mind earlier in the game. It is no accident that M4 raises the 'It's not *my* position' issue at this moment. True enough, move 1... P-KB3 is clearly the favorite now, but the subject is not yet used to the idea that he will have to settle for a very small positional advantage (somewhat less than 6). 'In spite of everything' thus means: in spite of the fact that it is not so much as I hoped for. Accordingly Subject M4 says: 'I should play 1... P-KB3' – but he does not yet do it. There first follows another checking investigation that broadens the positive part of the proof; then a near-decision: 'Well, probably I'd play 1... P-KB3' until finally the decision is made, albeit still with some hesitation: 'I play 1... P-KB3, yet I don't know how I shall proceed.'

- 92 Look to see if White has nothing. (Pause) 2. Q-B3 maybe; something to do with the Queen.
96 1... P-KB3 and now maybe 2. N-B4 right away, going to attack the Pawn on B4 then. 2. N-B4, N×N; 3. R×N – well, then you take on KN4. That doesn't work, costs a Pawn. Well, probably I'd play
102 1... P-KB3 here – then he probably answers 2. P-KR4. ... (E: What do you play now?)
105 I play
106 1... P-KB3, yet I don't know how I shall proceed. The combination rests on his taking and especially B-R3 of course. And I thought of 1... Q-R6 in order to rule out P-KR4, to separate the Pawns.

The composite problem development in this protocol cannot very well be represented in a formula, what with the increasing specialization and sharpening of the problem, the continuous abatement of the maximal expectancy, and the long sequence of so many subsidiarily linked methods.

9. As a last example let us again consider (G5; B2). Its problem development has already been amply discussed in Section 47 so that only its formula is given here. Again the interesting feature is the overlapping of subproblems and -processes with super-problems and -processes, schematically expressed by the different 'layers' of formulas. The number and complexity of overlapping and simultaneously developing subproblems is quite impressive; even so it must be remembered that the formula is most certainly a drastic simplification compared to the structure of the problem development in the thought process itself.

(G5; B2) T = 15 minutes.



where move $e = Z_1$, $d = Z_2$, $c = Z_3$, $a = Z_4$ and $b = Z_5$;
and P_1 is the blockade (plan B), P_2 the King's side attack (plan A).

CHAPTER VII

VII. THE ORGANIZATION AND METHODOLOGY OF THE THOUGHT PROCESS

A. THE SEQUENCE OF PHASES

Section 50: The principle of interaction

In Chapter IV we came to see the alternation of elaborative and integrative phases as the basic structure of the thought process. In the light of the organization and methodology of the thought process this means that *the subject (player) periodically returns to more general problems, especially to the main problem*. The structure of all thought processes in which a difficult problem must be dealt with is characterized by this alternation.

In Section 44 we saw that the alternation of phases corresponds to an *interaction between elaboration and total goal (problem) conception*. 'During an elaborative phase the problem in its present state of development determines the course of the thought process via the schematic anticipation contained in the goal awareness; conversely, in the transitional phases the results of the elaborations cause a more or less drastic transformation of the problem' (p. 184).

In many respects the processes involved in this interaction have already been analyzed in the previous pages. The influence in one direction – the results of elaborations on the total goal conception – was discussed in Chapter VI, while in a way Chapters IV-VI are one continuous affirmation of the importance of the influence in the other direction. In particular, there has been much emphasis on how the moments or components of the total problem conception, via the schematic anticipation, influence the direction and methodology of the ensuing elaborations. Type, core problem, estimated value, each plan, each goal-setting and problem formulation, each solution proposal that is more than a blind stab, each alternative grouping, each shade of favoritism, each hunch about the value of the position or of a variant, or of a possibility, each hunch about necessity or desirability of certain actions, etc. – all of these components of the total goal conception may, at any moment, exert a decisive influence on the course of the thought process. The total goal conception also contains the subject's appreciation of the board problems at a given moment

(cf. Section 45) which in its turn is often of decisive importance for the ensuing thought operations inasmuch as operations-goals follow from board goals.

From the protocols we can easily infer the following relationship between components of the total goal conception – anticipations in particular – and ensuing thought activities:

1. Via the schematic anticipation(s) in which the components of the total goal conception become operative, the components determine¹ the general direction of the investigation, especially *the selection of moves and plans to be considered*.

2. The components of the total goal conception, in particular the subject's appreciation of the board goal and board problems, determine – often in considerable detail – the specific (board) means that are supposed to be adequate: pertinent strategies and tactics. In terms of the investigation of possibilities, this means that the selective influence of the subject's board goal conception is not restricted to immediate solving propositions in the present position but extends to *the selection of pertinent continuations in envisaged positions*.

3. At a certain point in the thought process the components of the total goal conception determine to a large extent *the manner in which the following investigations are to be set up*: whether the subject will explore fairly neutrally, try to build up a preference, try to maintain a favorite (deeper investigation), or try to prove the superiority of a nearly chosen move, either by recapitulating its merits or by final strengthenings and/or rejections of alternatives (positive and negative proof).

4. Further, the components play an important role in the transitions to new variations, moves, or plans. In particular, the *decisions to reject or accept* (temporarily) certain solution proposals or possible continuations appear to be based on the relation between quantitative outcomes and quantitative anticipations. Both the mathematical sign and the magnitude of the difference between the subject's current expectancy and the quantitative results of his investigation of a proposed solution are of decisive importance for what will happen to the proposal.²

¹ The question whether or not the immediately following operations are *completely* determined by the components of the total goal conception – as would correspond to Selz's conception of thought dynamics – will be deferred until Chapter IX.

² The importance, from the point of view of machine simulation of human thought, of the ever present *pre-set expectancy* – a major component of the subject's total goal conception – will also be discussed in Chapter IX.

5. The same holds for *the detection as well as rectification of errors, weak continuations, and inadequate solution proposals*. Whenever a good or bad partial result appears inherently 'improbable' it is distrusted: an improvement for the underdog is sought and mosttimes found. It is clear that a result can only be judged 'improbable' if a pre-set quantitative anticipation exists – either the subject's general expectancy or a specific expectancy for a specific variation. The mathematical sign of the difference between anticipated and calculated result determines the direction in which an improvement is sought, while the magnitude of the difference is of importance for the subject's determination and/or thoroughness in his search for improvements. If a solution proposal (move) is found at fault in this way, an immediate re-investigation will follow – or finally a rejection (cf. point 4 above). Again, the exhaustiveness of the re-investigation is related to the magnitude of the difference.

In the foregoing chapters we have given so many instances of the influence of various anticipations on the ensuing elaborative operations that a general review of the individual components of the mechanism of this influence appears superfluous. In the rest of this section, therefore, the analysis of the interaction will be restricted to the quantitative moment, i.e., to the remarkable regulatory function that the estimated value, the maximal (and minimal) expectancy, and the more specific quantitative expectations appear to wield in chess thinking.

In the game of chess, in its present form at any rate, the quantitative argument is always the decisive force. Perhaps it would be nobler to defeat one's opponent in a risk-fraught attack on the King rather than to win smoothly in an uninteresting endgame that follows an exchange of Queens, yet the modern master will elect to exchange Queens if this course leads to a clearer and greater *advantage*. The manner of playing, the character of the position, of the plan of action, and of the entire game are but of secondary importance compared to the advantage or disadvantage derived from the quantitative reckoning. One need only look at the way in which some insipid chess columnists and analysts annotate games: they talk only of advantages and disadvantages, errors, less strong moves, the best move, a somewhat better position, etc., without ever mentioning the qualitative elements. Although such analyses are certainly not the most fascinating, the mere fact that a chess game can be treated in such a fashion attests to the importance of the quantitative.

In the protocols, too, the quantitative predominates as, for example, in the formulations of results. Indeed, obtaining an 'advantage' or avoiding a 'disadvantage' is the only objective and overriding motive. Wilhelm Steinitz, who laid the foundation of the modern positional game, speaks of the 'accumulation of small advantages' as the aim in a positional game. (STEINITZ 1889 and 1895). When the player supplants one qualitative goal (plan) by another, the character of the game changes drastically, but the more important quantitative relationships remain the same – if both sides play well. Especially during the transition from the middle game to the endgame – but by no means exclusively at this stage of a game – one frequently finds that 'advantages' of one type are traded in for advantages of another type.

From all this it should be easy to understand the important regulatory function fulfilled by the quantitative anticipations in the thought process. The following instances are taken from protocol (M3; A) – see below.

At the beginning of (M3; A), after some ten seconds, the estimated value is formulated (line 1): 'White's position is superior in any case.' Expressed in a rating: estimated value = expectancy = 8. But the position is complicated and contains combinatory possibilities. For that reason there remains a wide margin of uncertainty; it could even be that the position is won. In other words: Subject M3 can tentatively set his maximal expectancy at 9 or 10. Thus he directs his search towards a decision. This maximal expectancy is now the operative quantitative anticipation.

The quantitative goal determines the direction of the investigation (point 1, p. 255) as well as the means to be considered (point 2). Only forceful moves offer a chance of success, i.e., of attaining 9 or 10. More important, however, is the fact that *the results of the variations are measured against the maximal expectancy*. Every result that does not stand up to the (rigorous, self-imposed, anticipatory) test is rejected. The outcome of this (relative) measurement is decisive for the 'success' or 'failure' of the attempted solution and hence for the next linking (whether it be cumulative or subsidiary). The maximal expectancy thus regulates to a large extent the sequence of operations; it helps to regulate the entire methodology involved in rejecting and accepting solution proposals³ (point 4).

3 This regulatory function is characteristic of actual games, where the maximal expectancy varies – not so in puzzles, chess problems, or studies where it is fixed (e.g., White to play and win).

(M3; A) S: S. LANDAU E: DE GROOT T = 15 MINUTES DECEMBER 8, 1939

- Let's have a look. White's position is superior in any case. To search for a combination in connection with KB6 and Q5.
- 5 To figure out
1. B×N/5, with possibly N-Q7 afterwards, in case Black takes back with the Bishop. 1... B×B then later N-Q7; so 1... P×B is forced; but one doesn't get very far with that either.
- 10 Let's look at
1. N×B. Also important. Calculate 2. N×N, to see if that provides any direct advantage. 1. N×B, R×N;
15 2. N×N, N×N; 3. B×N, R×R probably leads to nothing. 4. R×R, B×B; but wait a moment, also 4. B×R is possible. What then? Let's have another look:
20 1. N×B, R×N; 2. N×N, N×N;
3. B×N, R×R. Now something else:
4. B×B, R×Rch; 5. Q×R, R-K1;
6. B-B5 probably to the advantage of White. But it can be played otherwise:
25 5... P×B; 6 B×R – haven't achieved much then; leads indeed to nothing. Look for something else:
1. N×B, R×N; 2. N×N, N×N. Now something else? No, doesn't yield much.
30 1. N×N, N×N; what then? 2. B×B, N×B – nothing. There is no decisive combination. Then maybe an ordinary attacking move:
1. P-KR4 for instance; but what then
- on 1... Q×NP? Is there any compensation then for the Pawn? Probably not.
1. B-R6 maybe. See if anything comes out of that. (Short Pause)
Still, I keep looking at taking on Q5; there might be something in it. 1. N×N or 1. B×N/5. If 1. B×N/5, then 1... N×B is impossible. 1... B×B would be forced, therefore – but that has its drawbacks too: loss of the exchange. Yes,
1. B×N/5 is the move. With that White gets the advantage. 1... P×B is forced, and then a favorable position is reached. Play 2. P-B4 for instance. Maybe we can get even more out of it.
1. B×N/5, P×B; 2. N×B in order to win a Pawn possibly? No, that doesn't work. Let's look at 2. Q-B3. Then 2... Q-Q1 is forced. What then?
1. B×N/5, P×B. A combination with 2. B×N, B×B; 3. take on Q5 and N-Q7 – insufficient.
1. B×N/5 followed by 2. P-B4 is probably the best. Maybe a better continuation? Yet possible perhaps to get material advantage? Search for something.
Once again:
1. N×N, N×N; 2. B×B, N×B. Does that yield anything? No, better is 1. B×N/5, P×B; and now? What further? In any case
1. B×N/5 is good.

This regulatory role becomes especially clear as the maximal expectancy fluctuates during the course of the problem development. (M3; A) is again a case in point: Gradually, negative deviations undermine the maximal expectancy (interaction) until finally, in line 31, the conclusion is 'There is no decisive combination.' The margin of uncertainty surrounding the estimated value is reduced by the fact that the maximal expectancy drops. As an immediate consequence, other moves now come up for consideration: 'Then maybe an ordinary attacking move' (points 1 and 2). But these ordinary attacking moves (1. P-KR4 and 1. B-R6) do not satisfy the expectancy either, and

M3 says, 'Still, I keep looking at taking on Q5; there might be something in it.' But now he does so with somewhat tempered expectations. Curiously, the ensuing re-investigation of 1. B×N/5 ends up with the same variant of which he said in line 9: 'but *one doesn't get very far* with that either.' Now he avers: '1. B×N/5 *is the move*. With that White gets the advantage' (line 46). This example once again proves that formulations of results must be taken in a relative sense.

Are there also 'absolute' results? Are there results which are unfavorable and unpleasant in an absolute sense, which just 'won't do'? Does it occur that results are unequivocally and absolutely rejected, or that they are absolutely favorable, advantageous, or pleasant so that they are unequivocally accepted? The absolute conception would correspond to a sort of 'trial and error' theory of thinking as proposed by, e.g., THOMSON (1924). Under the relative conception what is decisive for the success of a thought operation is not the (absolute amount of) 'pleasure' created by the result but *the agreement* of the result with the (quantitative) anticipation – in line with Selz's theory. For chess thinking we must concede that *both* cases occur. For clear-cut cases – the outright winning or losing results, such as checkmate, winning a piece, etc. – there is no need to evaluate in relation to a maximal expectancy. Since most quantitative results are not so extreme, however, results are generally 'measured' against quantitative expectations.

A psychologically interesting question is whether we must think of this process as one of 'comparing' the two values, expectancy and outcome. Such a description does not appear adequate though: it is too intellectualistic. We should rather say that the subject is analyzing variations with an anticipated *degree or level of satisfaction* in his mind. In fact, subjects often express themselves in terms of feelings, such as 'not (completely) *satisfactory*,' or 'that does not *appeal* to me either,' especially when the difference between expectancy and outcome is slight. The abundance of terms like '(un)pleasant,' 'nice,' '(un)satisfactory,' 'appealing,' etc., suggests an interpretation of the expectancy as a (temporary) level of *aspiration*, or in LEWIN's (1926, transl. 1951) terminology: as a quantitatively measured 'quasi-need' that must be fulfilled. Even when towards the final Phase of the thought process quantitative results are no longer measured against expectations but are actually compared among themselves, the psychological process is not just a matter of intellectual comparison of quasi-numerical values. It is rather 'satisfaction (or pleasure) values' that are compared: 'I *like* that better,' 'This *appeals* more to me,' etc.

We return now to the regulatory function of the quantitative expectations. There are still a few complications to be mentioned.

First, as appears from points 4 and 5 above, negative deviations from the expectancy lead to *rejection* of a solving proposition only after several attempts at strengthening have been undertaken: minor corrections, variant reinforcements, re-investigations (cf. Section 51). Rejection is a last resort, which is only called upon when the means of strengthening run dry. Exactly where the 'exhaustion point' will be reached depends – apart from the objective degree of exhaustion which could be defined by an objective, expert analysis of the solving proposition in question – on the phase of deepening of the thought process, i.e., on the state of development of the subject's total goal conception. During an early, still exploratory investigation the exhaustion point will be reached much sooner, *ceteris paribus*, than during deeper investigation near the end of the thought process (cf. Section 52).

Second, positive deviations from the expectancy hardly ever lead to immediate *acceptance* of a solving proposition or variant. Again, attempts at improving the opponent's counterplay (broadening), checking operations, re-investigations, and, particularly in the final Phase, a recapitulation generally precede the decision to accept (the validity of) the possibility. Moreover, before the decision is taken, the (maximal) expectancy goes up – possibly beyond the level that was attained: 'Maybe we can get even more out of it.' In any case, at the moment the move decision is made there is no longer any positive deviation but rather *agreement* between anticipated and estimated result. This is in line with Selz's conception: in its quantitative component the solution corresponds to the schematic anticipation in the mind of the subject.

Note again the regulatory function of the quantitative expectations in the process. One result of this regulation is the serious consequences that a wrong appraisal of a position may have. If the expectancy is too low, for example, the upshot may be an inadequate search for improvement possibilities, in this case in the subject's own play. As a result the stronger variants are not likely to be discovered. In practice this is a common source of weak play: the player overlooks a winning, favorable, or sufficient defensive move because he does not realize that the position is as good as it is.

Protocol (M3; A) provides – as does practically every other protocol for that matter – a few examples of expectancy-induced checking investigations and attempts at strengthening. Thus the thorough investigation of 1. N×B (lines 20-26) renders a negative outcome.

relative to the maximal expectancy ('haven't achieved much then; leads indeed to nothing') which first elicits an attempt at strengthening:

- 27 Look for something else: 1. N×B, R×N; 2. N×N, N×N. Now something else? No, doesn't yield much.

It appears that now the 'exhaustion point' is reached for immediately afterwards another move has its turn: 1. N×N, etc.

The fact that the result of the elaboration is at variance with the expectancy thus induces a search for improvement, the discrepancy being a *signal that something is probably amiss*. In the following fragment from another protocol this specific anticipation is explicitly expressed:

(M4; ...Q-K2):

- 75 Oh, no, the Queen takes back on Q7 and the Rook on N5 hangs. *But that can probably be improved upon.* Yes, first 4...Q-N4.

The great practical importance of this inducement to checking is indirectly demonstrated by the observation that omissions and calculational errors especially tend to remain undiscovered in situations where a discrepancy is lacking! This occurs, for example, when in a calculation the subject is unlucky enough to choose a weak move for both sides. The end result may still agree with the expectancy because the errors cancel each other out.

Striking examples of cancellation are found in protocols (M2; C) and (G4; A). Both subjects overlook a straightforward defense in an investigated continuation and thus get involved in the calculation of a number of lengthy variations that are completely irrelevant to an objective analysis.

The conclusion should *not* be, however, that these calculations are worthless for the thought process. They always retain some value as sample variations. They illustrate the possibilities of the position and can also have heuristic value. In (G4; A) subject G4 via the calculations 1. KR-K1, Q×NP?; 2. B×N/5? (2. N-B4 wins a piece, even after 2...B-N4!) finally arrives at the correct solution 1. B×N/5.

Overlooking such errors and omissions need not have an adverse influence on the choice of move; indeed this seldom happened in the experiments – at least not with the master subjects. The primary function of a sample calculation is often to *sharpen and test the current quantitative and qualitative anticipations*. But if the intuitive appraisal is reasonably accurate a priori (as it mostly is with master subjects) spurious verification cannot do much harm.

Apart from the expectancy, i.e., the overall appraisal of one's chances, other more specific anticipations, both quantitative and qualitative, can have a regulatory function. For each it is possible to trace the mechanism of interaction. A few specimens must suffice, however.

The productive significance of the *anticipation of urgency* is quite clear. It is found in particular in the master protocols of position B. The subject would not work so hard on the King's side attack, plan A, without the 'impression that I must proceed actively' (G5; B1), line 62, or without the hunch: 'If I do nothing, I have the idea that I am gradually going to lose' (M2; B2), line 8. The same holds for (M5; R-Q1): 'Something must happen immediately; if Black can exchange pieces, White won't win so easily' (line 85). Here as in (M2; B) it is the unsatisfactory results of the search for calmer moves which intensify the feeling of having to hurry; another example of the converse influence in the interaction process; calculational results modifying the anticipations and therewith the subject's total problem conception.

The *anticipation of (objective) solvability* is another case in point. Like the anticipation of urgency it may have a productive influence on the energy expended in investigating in a certain direction; for example (M5; B-R7ch): 'I've a distinct feeling that it's somehow or other over' (line 153).

An *intuitive faith in a certain possibility* may have the same effect ('intuitive' in the sense of: based on evidence that is experienced as insufficient). In addition, such anticipations – as for that matter any specific expectation – often have an important influence on the setup of the investigation (point 3); for example, the order in which the variants and moves are investigated. In trying out sample variations one first examines the *putative* main variation; conversely, in calculating the branches of the proof one begins 'with the *probably* bad counter moves – in order to be able to eliminate those' (M1; A)*, line 14. Again in both cases it is obviously the hunches and expectations – thus anticipations – which help to regulate the course of things.

Section 51: *The hierarchy of subsidiary⁴ methods*

In the analysis of thought processes certain typical sequences appear to support the hypothesis of a constant *hierarchy* of subsidiarily linked methods (or 'program') in the subject's mind. Selz's investigations led him to formulate this general law (cf. Section 20): If a solution attempt fails, the same method is again applied but with new material; a new method is only applied when the available material is exhausted. When

⁴ It must again be stressed that it is the etymological meaning of 'subsidiary' that is intended here, i.e., the notion is of a reserve supply, one free from an inferior or subordinate status or capacity. (From Latin *subsidiū*, the troops stationed in reserve in the third line of battle.)

all available methods are exhausted a 'delay-until-later' follows – itself a method of thought (cf. Section 33). In terms of chess: If a calculation leads to unsatisfying results, other *variations* are tried out (attempts at strengthening) while the same plan and first move are retained; if the variations fail, other *first moves* are tried out; if these also fail the entire direction of the investigation is changed (another *plan*). There was just such a development in (M3; A); cf. Section 50.

However, except for the possibility of a provisional delay in the investigation of a certain plan or first move (non-immediate re-investigation, cf. Section 42; progressive deepening, cf. Section 52) the method delay-until-later, that is, delay the solution of the main problem, cannot be applied by the chess player since he has to decide on a move. We have seen in the case of (M3; A) what happens instead: there is a change of direction again (another plan or move), but this change is coupled with a *drop in the maximal expectancy*. Such a drop occurs in many other protocols as well, particularly in the later stages of the thought process when all available plans fail to satisfy the quantitative expectations. Indeed, this is again related to the important distinction between an assigned problem in which a 'right' solution must be found and the choice-of-move-problem in chess in which *the player himself defines the requirements of a solution*. In the former, in which a quantitative goal is given, the subject *must* 'delay-until-later' when he cannot find a solution; in the latter, when the problem solver (chess player) is stuck, he has another recourse: simply lower the quantitative goal (expectations).

Aside from the repeated appearance of certain sequences of operations, the existence of hierarchical subsidiary linkings may appear from the formulation of operations-goal statements in the protocols. In particular, the typical expression 'Let's *first* investigate (try out, look at, etc.)...' implies – at least if a future *subsidiary* linking is actually meant – the existence of a priority that may result from a general, methodical hierarchy. In favorable tactical positions such as position A, for instance, the subjects generally investigate forceful moves first; they say 'Let's first have a look at what can be taken,' (G1; A) or 'Now, let's look in some more detail at the possibilities for exchange,' (G5; A). The investigation of other types of moves: non-forceful, non-taking, non-exchanging moves only takes place if the former category becomes barren. The priority of forceful over calm moves is so general, indeed, that we have every reason to suppose that a fixed hierarchical order of subsidiary methods underlies the systems of methods (programs) of all subjects. Analogously, other

types of subsidiary linkings in the protocols suggest the existence of hierarchical orderings that all experienced subjects have in common.

In the following *hierarchy of subsidiary methods* all presumably general 'priority rules' that could be derived from the analysis of the entire experimental material are summarized.

Assume that the subject has just calculated a certain variation of a move possibility that fits into a plan and that this variation yields a (quantitatively) unsatisfactory result. What happens now? Which types of operations can replace the one that failed? Which priority rules obtain?

1. The simplest and the first possibility which the subject may consider is the investigation of another own-variant of the same continuation; that is, an *attempt at strengthening*. This attempt is not (immediate) re-investigation but simply continued investigation.⁵

2. If such clear strengthening possibilities are absent, the subject will turn to the investigation of a different first move; if, in the previously generated *backlog*⁶ of moves implementing the same plan, there is one move which was already subsidiarily linked (considered as a second or substitute) to the rejected one, this move has highest priority.

3. If there is no (longer a) backlog, then certain *qualitative results* of the prior investigation may have led to or now *lead to the discovery* of a different first move (or moves) within the same plan.⁷

4. If no more first moves present themselves, then the subject may consciously *search for another first move* in the transitional phase: 'Let us look around for other possibilities' (M2; A), line 23.

⁵ With regard to attempts at strengthening it is somewhat disputable whether the linking is really subsidiary. If there is any difference with *Selz's* (conditionally) composite solving methods of 'corrective completion,' where a *subordinate* method (correction) is supposed to be cumulatively linked to certain types of 'incomplete' – instead of unsatisfactory – partial results, the difference is slight. The reader is referred to Section 20, p. 72, and to Chapter IX, where this ambiguity, along with others, is more fully discussed.

⁶ Definition: A *backlog* is any sustaining reserve.

⁷ The order of priority of subsidiary methods 2 and 3 may be reversed – depending, among other things, on the state of development of the problem (exploratory versus investigatory and proof calculations) and on the type of position. In a still rather exploratory analysis of a tactically complex position, for instance, the backlog, if any, cannot be expected to cover all relevant forceful moves, so the subject generates the new move(s) as a modification of the previously tried one(s). Method 2 may then be used at a later stage of the thought process (a later phase of progressive deepening).

5. If the moves within the framework of the plan have been (temporarily) used up without having yielded satisfactory results, then there frequently follows a *re-investigation* of the move with the smallest negative deviation from the expectancy.

Here, too, we may speak of attempts at strengthening (as under 1) but now in the form of a re-investigation, either immediate or non-immediate.

6. If such attempts at strengthening are not possible or have already failed, then the subject will relinquish the plan and fall back on his *backlog of plans*. This transition may be attended by a drop in the quantitative expectations, or at least of the maximal expectancy. From here on (7-10) a drop is likely to occur.

7. If there is no longer a backlog, then an alternative plan may be sought that is a *modification of the previous plan*.

8. If this does not work, the subject searches for basically *different ideas* (a new plan).

9. If no new conceptions of the solution emerge and if other plans have already been used up without having yielded satisfactory results, then, after a short problem analysis (transitional phase), the subject resorts to a *re-investigation* of the relatively best plan.

10. If such a re-investigation of one or more plans already took place, possibly via several phases of deepening, then a more penetrating *conflict analysis* certainly ensues in a lengthy transitional phase and bears the characteristics of a *crisis of expectations*. Finally, the subject either decides upon an even deeper and more systematic re-investigation, or at least upon a recapitulation of the results obtained. On this basis the choice then has to be made.

This hierarchy is certainly not complete, but it does include the most important types of subsidiary linking ranked according to priority of applicability.⁸ Of course there is no protocol that contains all ten one after another; many steps are skipped in an actual sequence of operations. Indeed, the hierarchy is only meant to provide the framework from which appropriate subsidiary methods can be *successively* drawn. Exactly which methods and how many of them the subject

⁸ In machine simulation terminology, the way in which the subject tackles the problem is very similar to the way in which a clever programmer might build a pop up (push down) list. The programmer could first put in the most drastic methods (thus, e.g., plan changes go to the bottom of the stack) and last the least drastic ones (e.g., move alternatives near the top). The accessibility of the subsidiary methods would then more or less correspond to the subject's priority of applicability.

will select depends on at least three factors: (1) the position itself (objective board problem); (2) the subject's individual preferences, techniques, insights, etc. (i.e., his personal program, cf. Section 59); and (3) how far the subject has progressed in the problem (the current content of his total goal conception).

There is one other restriction. The hierarchy is only valid in problem situations where the subject (player) is still *searching* for a solution, for a good move. If he is in the stage of striving for proof, then the hierarchy is of little avail. When the whole purpose of working out a move possibility is its elimination (negative part of the proof), then there is an entirely different relationship between success and failure, between cumulative and subsidiary linking. A bad result is a success since the elimination of a possibility has succeeded. Whereas in the Phase of searching and investigating the subject may resort to an attempt at strengthening (method 1) after a 'bad result,' such an own-branching in the Phase of striving for proof is now a cumulatively linked attempt to complete the negative part of the proof. Likewise, the investigation of a move from the backlog (2) or one just discovered (3) now corresponds to the broadening aimed at including several moves in the negative part of the proof. Indeed the parallel might possibly be extended as far as 10. That is not necessary, however, since the only point is to show that, important as this hierarchy is, it cannot in its present form be generalized to all phases of the thought process.

Section 52: The method of progressive deepening and the scrapping of a possibility

In Chapter IV, Section 32, p. 106, the concept of 'progressive deepening' was introduced. The term denotes a remarkable phenomenon peculiar to rather lengthy thought processes that are needed for solving difficult choice problems. The analysis of a certain idea (plan), move, or variant proceeds progressively in successive phases of (re-) investigation, either immediately or non-immediately. The investigation not only broadens itself progressively by growing new branches, countermoves, or pertinent own-moves, but also literally deepens itself: the *same* variant is taken up anew and is calculated further than before. The term 'progressive deepening' is meant to include both aspects, which can only artificially be separated out.

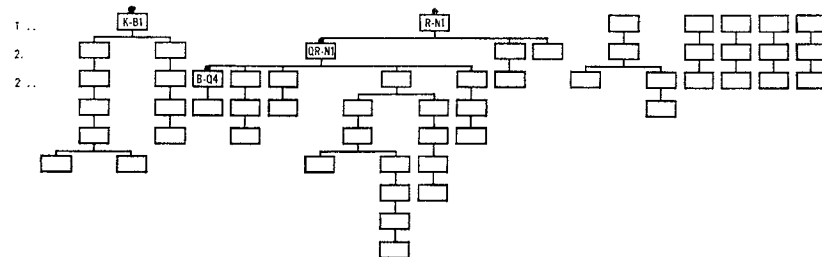
Before taking up this phenomenon we shall give a graphic description based on protocol (M2; B1) (cf. Section 30). Careful study will

reveal five phases of deepening, namely: (1) lines 1-19 (first Phase); (2a) lines 20-35; (2b) lines 36-48; (3) lines 49-61; (4) lines 62-78 (striving for proof, final Phase). The diagrams show clearly how much and what is calculated in each Phase; Phase 2a and 2b have been taken together to collectively form the second phase, in order to increase the generality of the structuring.

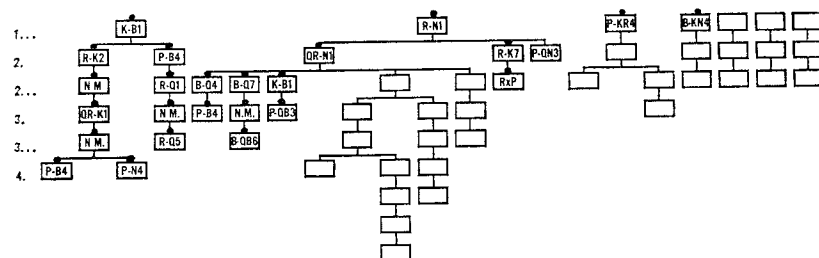
The mature tree contains all of the moves mentioned in protocol (M2; B1). Each quadrant illustrates clearly what occurs during the Phase with which it is concerned, how the calculations successively broaden and deepen. For examples of 'deepening' in the literal sense: 1...P-KR₄ and 1...B-KN₄ are cursorily investigated in the second Phase and several moves deep in the third. The first move 1...R-N1 reappears in each of the four Phases (even twice, non-immediately connected, within the second Phase of exploration). The re-investigations of 1...R-N1 are all non-immediate, except one, namely, the clear-cut transition to the final Phase (striving for a proof), lines 62-67, after a favorable result is obtained for the first time.

In principle, such Phase-trees may be constructed for every protocol (cf. Section 43), but only the lengthier thought processes show a marked structure of progressive deepening. Most of them can be divided into *four Phases of deepening* (that is, one more Phase than was distinguished in Chapter V in the discussion of the basic structure; the 'main part' is now broken down into two Phases). The four Phases are:

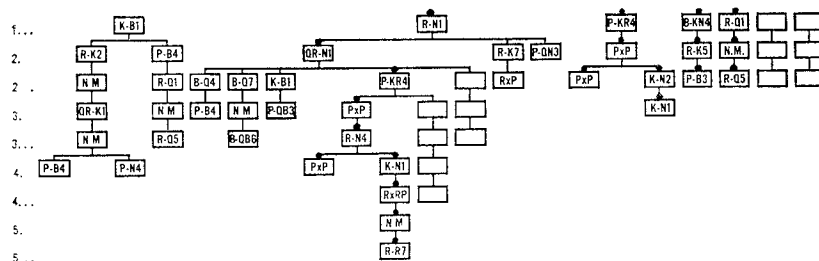
1. The *First Phase of Orientation*, especially orientation to possibilities. What we find here is largely 'looking at' the consequences of moves and general possibilities in a certain direction.
2. The *Phase of Exploration*. The subject tries out rather than 'investigates' possibilities for action. He calculates a few moves deep a few sample variations, or what he considers to be the main variation; if these are unsatisfactory he puts the move(s) in question temporarily aside.
3. The *Phase of Investigation*. There is a deeper, more serious search for possibilities, strengthenings, etc., that are quantitatively and qualitatively quite sharply defined. The investigation is more directed and much more exhaustive: more variants are calculated and they are calculated more deeply.
4. The *Final Phase of Proof*. The subject checks and recapitulates, he strives for proof; the obtained results are made into a subjectively convincing argument. A certain completeness is sought in the calculation of results, be it for the positive or negative part.

FIGURE 7. *Progressive deepening from protocol (M2; B1)*

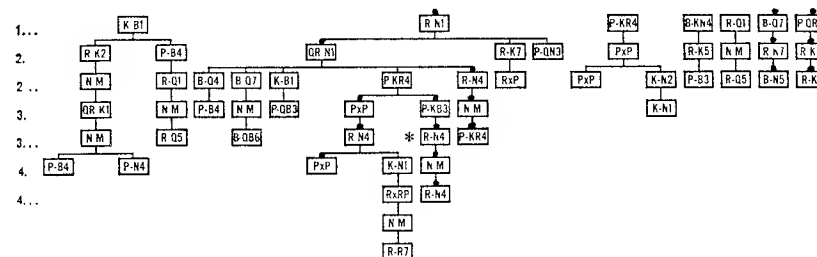
I. FIRST PHASE
(Orientation to possibilities)
lines 1-19



II. PHASE OF EXPLORATION
lines 20-35
lines 36-46



III. PHASE OF INVESTIGATION
lines 49-61



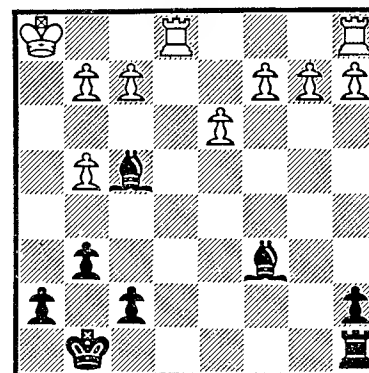
IV. FINAL PHASE
(Striving for proof)
lines 62-78

* Subject M2's statement here was '3.P-KB3 is not good, then the Rook comes to KN4'. Instead of the interpretation given above, M2 may have been envisioning the position after: 1... R-N1; 2. QR-N1; 3. P-KB3, P×P; 4. P×P, R-N4; 5. N.M., R-N4.

EXPLANATION OF SYMBOLS

- Moves yet to be discovered.
- The move (of a variation) mentioned, whether or not for the first time, in the Phase under consideration.
- The move (of a variation) already mentioned in a previous Phase.
- Assume that *some* move is made at this juncture; continue calculating the variation, however, without specifying *what* move. That is, make a 'no move' (N.M.).

The numbers on the left represent the move depth to which the position on the board is calculated, starting with Black's first move (1... Z1).



POSITION B

Black on move

In accordance with Selz's conception of subsidiary methods (cf. Section 51) a subsidiarily linked new method is only applied when the available material (for the first method) is exhausted. The principle of progressive deepening now shows that such '*exhaustiveness*' must be taken in a relative sense. Progressive deepening comes into being because in the first parts of the thought process the subject is content with orientation and exploration of possibilities, that is, with a provisional and incomplete investigation; thus he drops a possibility before the material is objectively exhausted.

Remarkably enough, a truly exhaustive coverage is rare even in the third and fourth Phases. The total argument as implied in a protocol as a whole, although 'subjectively convincing,' is usually incomplete and lacks objective proving power. Certain parts, not self-evident yet essential for an objectively correct solution, are missing. This is often a matter of *force majeure*: the possibilities simply cannot be completely worked out in one's head – nor even in an analysis if a position is objectively unsolvable. In other cases it is clearly impractical to strive for exhaustiveness. When, for instance, a certain move is probably the best one and cannot do much harm in any case, then it is rather pointless to conduct too thorough an analysis to prove it. Even when verification is possible, precious time is rather saved for more difficult moments – a thought habit from practice which carries over to the experimental situation. Finally, unintentional incompleteness also figures in many protocols: overlooking essential possibilities in own- and counterplay.

Thus the measure of 'relative exhaustiveness' that determines a transition remains as crucial a question for the final choice of move as for the preceding transitions in the process of progressive deepening: What requirements must be fulfilled for the subject to consider the results of his investigation convincing enough to reject or accept a move? In particular, to accept *the* move and then to play it? On what factors does this decision depend if the criterion of exhaustion of the material is variable and relative and therefore insufficient as an explanation?

Let us confine ourselves to the quantitatively most important case: *the mechanics of provisional or definite rejection, the scrapping of a possibility.*

We have said that this always happens when an attempted solution 'fails' – but this expression cannot but mean that an 'exhaustive' investigation of the possibility in question yields only unsatisfactory results. How should this conception be specified? Which factors must

in general be taken into consideration to account for the fact that at a certain moment the subject drops a possibility?

1. In general we may retain the rule that says that the subject will drop a possibility (move, plan, idea) when the results of the elaborations are unsatisfactory, that is, when they are *lower than the maximal expectancy*.

However, this rule does not always hold during the orientation to possibilities in the first Phase where a highly favorable (looking) move may temporarily be relegated to the backlog. Even in later Phases a favorable result may occasion a rise in the maximal expectancy on the ground: 'If it's that easy, I can perhaps get even more out of it some other way.' The satisfactory possibility is then set aside (temporarily rejected) as a minimum solution which guarantees a minimal expectancy.

2. The more unsatisfactory the results of the investigation, i.e., *the higher the (negative) proving power*, the easier and more definitive the rejection will generally be. Whether or not unsatisfactory results have sufficient proving power depends on several factors:

a. The *completeness* of the elaborations. One sample variation has little proving power while an exhaustive calculation of all relevant branches has much. We are concerned of course with the subjective proving power which depends on the degree of completeness the subject *thinks* he has reached.

b. The *differences* between the quantitative results of the variants and the maximal expectancy. The greater these negative deviations, the stronger, in general, the tendency to reject the possibility. A continuation that could entail a mating net or loss of a piece is dismissed sooner than one that might involve the loss of a slight positional advantage or an unimportant Pawn.

c. The *degree of uncertainty within the appraisal of the envisaged end positions* of the variants. The narrower the margin of uncertainty,⁹ the sooner the subject arrives at a rejection, especially a definitive rejection. The degree of uncertainty is dependent not only on the subject's skill and self-criticism (a personal factor) but also on the character of the envisaged position. In general he strives for as definite results as possible by calculating the variants until a point of relative calm is reached, that is, an envisaged end position in which the compli-

9 In programming terminology: *The reliability of the evaluation function* for the type of position evaluated is judged and taken into account by the human subject. In the terminology of this book this is another example of an *anticipation* determining the course of the process.

cations are over and the appraisal may be based on a reasonable degree of certainty, that is, until he reaches a *dead position*. He tries to avoid end positions where pieces are still *en prise*, where pieces must be recaptured, where the King is in check, or where serious threats are still present; in such cases he rather calculates a step further, even during an orientative investigation. He strives for an even 'deader' position. Of course, this is not always possible. All subjects investigating the position A variant 1. $N \times N$, $N \times N$; 2. $B \times N$, $B \times B/N$; etc., feel compelled, at least in the provisional calculations, to stop in the middle. G2 says in line 24: '...with many capturing possibilities, but at first sight not convincing.' If need be, experienced players are able to assess such positions, but the margin of uncertainty is of course wider.

d. Finally the proving power depends on the *reliability of the quantitative expectancy* against which the results are measured. If, for instance, the maximal expectancy is rather speculative and if the subject tries for an immediate win without really believing (anticipation!) that one exists, then a move that renders no more than a 'favorable position' is not rejected so quickly or so definitively. Had his expectations of gain been more certain, then he would be more apt to reject it. Naturally, we have to do here with a subjective estimate (anticipation), namely, the reliability the subject himself ascribes to his quantitative expectancy. This measure plays an important part in the (likewise subjective) proving power.

3. The presence of a negative deviation (1) and the actual degree of proving power of the results (2) do not yet fully explain the mechanism of rejection, since *how much proving power is needed to satisfy the subject varies greatly* from case to case. It appears to depend on:

a. *The skill of the subject and the energy he is willing to expend* on the choice of move.

b. *The character of the position*. The degree to which it is possible to attain a reasonable completeness of the analysis and/or a reasonable certainty of quantitative expectations, on the one hand, and of result appraisals, on the other, varies greatly. The subject judges (anticipates) this and adjusts his requirements correspondingly.

c. The specific function and place of the (preceding) *investigation* within the thought process. The subject's requirements for proving power (and, with that, depth and extensiveness) differ, depending on his present purpose: orientation, exploration, investigation, or proof (see Figure 7). The stage of progressive deepening determines, as it were, the approximate amount of energy to be spent on a particular

investigation. In the early Phases the subject's requirements for rejection are easily satisfied; he strives for a quickly attainable, tentative rounding off with a tentative decision; in the later Phases he strives for greater certainty and completeness, at least in the elaborations essential for the final argumentation. It is, in fact, the gradual increase in the subject's proving power requirements that brings about the phenomenon of progressive deepening.

d. *Other special anticipations and components of the total goal conception* may dispose the subject to be satisfied with less proving power or, on the other side, to strive for greater certainty. A few examples:

– The interval of uncertainty around result appraisals may be asymmetric in that the gain, at best, is small while the loss – if something goes wrong – may be fatal. Such a situation, as judged (anticipated) by the subject, naturally expedites the rejection of the possibility concerned. The *risk* involved detracts from the attraction of the possibility even if the (incomplete) results attained by actual calculations are hardly negative.

– Since risky moves require keener and deeper calculations than safe moves, another element is present in such a rejection; namely, *the avoidance of massive calculations*. Indeed, this may be an autonomous motive. Continuations that can only be reasonably evaluated via elaborate calculation are deferred sooner. From (M1; A)*, comments after the experiment:

20 (1. $B \times N/5$), $B \times B$; 2. $N-N_4$ would be very strong. White stands better; temporarily let it be.

Even though the envisaged position after 2. $N-N_4$ is anything but a natural resting point (dead position) and even though the tentative result is only slightly negative as to the maximal expectancy, M1 puts 1. $B \times N/5$ aside. He explains this himself in the following lines:

23 A stupid move perhaps?
1. $B-R6$; if that should prove even stronger, then I don't have to figure out all the variants of 1. $B \times N/5$.

This remark clearly expresses a shirking away from elaborate calculations and again shows the economy of thought inherent in the method of progressive deepening.

In a similar way it is possible to enumerate a number of factors pertinent to the transition to the move decision. Of prime importance is the subjective conviction, in this case the proving power of the total,

usually dual, argumentation. The degree of proving power needed to satisfy the subject depends on roughly the same factors as pointed out in 3 above: the skill and the available energy of the subject (the latter often determined by the clock in an actual game), the character of the position, the size of the risks, etc. Here, too, the argumentation is almost always left incomplete; there remains a certain amount of uncertainty about the estimated value and the quantitative results of the elaboration. This uncertainty is subjectively removed by what could be called an *intuitive and emotional completion of the argumentation*. As a last resort a 'feeling that...' an impression, a hunch, sometimes even an unadulterated emotional preference tips the balance in favor of a particular move.

The principle of progressive deepening and the many other factors on which transitions and decisions are dependent make the entire methodology of the thought process much more complicated than the simple schema *Denk-psychological theory* would have it be. The strict linkings: failure of a solution attempt → resort-to-the-subsidiary-method, turn out to be only definable with many qualifications and only conditionally applicable, in reality. The concept 'failure of a solution attempt,' which was already made relative by the introduction of a variable self-imposed maximal expectancy, now proves to be even less of a mainstay. All kinds of temporary 'failures' are possible, depending on variable proving power requirements. These requirements, including the criterion of satisfactory completeness (exhaustiveness) of the analysis, in their turn depend on the stage of the thought process and on a number of the subject's specific judgments and anticipations. Finally, the objective 'incompleteness' of the final argument together with its 'intuitive completion' by the subject point to the important role played by nonobjectifiable and even non-rational factors that the rigid theory of *Denkpsychologie* does not take into account.

Section 53: The elaborative phases and their sequence

If anywhere, it is within the elaborative phases that we can speak of sequences of automatically performed operations. The more microscopically we examine the phase structure, the more we come across automatic processes and sequences that must be governed by an extensive system of fixed linkings. Calculating a variant and working out a plan are, in fact, operations which in any experienced chess

player are completely controlled by routine. These operations, including that of grasping a position, are executed so often in practice that the process runs almost automatically, be it faster and easier with one player than another. Each elaboration is, of course, controlled by its own specific goal-setting but consists in itself of a complicated, sometimes highly branched system of subproblems and of mutually subordinated and co-ordinated subphases – as we have seen in former sections.

We shall delve no further into the micro-dynamics of these elaborations for the simple reason that the material obtained by thinking aloud does not allow of a systematic analysis. A few special points will be taken up in Section 55 when methods are described.

The question we now pose concerns the sequence of elaborative phases. We have just examined why a subject scraps possibility p; the question now is why he should turn to possibility q rather than to, say, r. *Which factors determine the order in which possibilities or variants are to be investigated?*

In answering this question we shall disregard the exact moment when the subsidiary linking of q to p comes into being in the mind of the subject. We did see in Section 48, however, that it can happen before, during, or after the investigation of p, in the last case in the transitional phase. Here we are only concerned with the relation between p and q – if there is one.

1. The first factor may be called *indirect determination*: the subject turns from possibility p to q because q is the only remaining possibility yet to be considered, either by itself or within the framework of a certain plan. When does it appear?

First, whenever one of two pre-stated alternative possibilities is (provisionally) rejected and the other is taken up. In other words: Whenever there is an already established binary choice between moves, plans, variations, strategies, attacks, defenses, etc. From the importance and high frequency of alternative groupings, the importance of indirect determination is clear. Second, a subproblem may divide into three or more parts. In such a case the last part, when taken up, is indirectly determined. Indirect determination is actually the rule later on in the thought process when the subject's thinking gets organized and structured into circumscribed groupings.

2. The investigation of possibility q may follow p because q promises to be *relatively the best* on the basis of prior scanning (and temporary

dismissal) of q and other moves aside from p . Earlier quantitative results and expectations (anticipations) now determine the order of investigation, the most promising possibility coming first. This factor is very important, though limited by the necessary condition of prior scanning.

3. A third factor is the *relation* that may exist between p and q with respect to their board function. Very often the subject tries q on the strength of q -anticipations resulting from certain qualitative experiences with p . Possibly q may, for example, embody a modification of the idea basic to p ; q may be a move that prepares the way for p ; q may result from a permutation in the order of moves, or be an accelerated execution or otherwise improved version of p , etc. Possibility q may also be the reverse of p ; namely, when p is disqualified by the qualitative results of its calculation so that a search in a completely different direction is begun; thus q is investigated.

Whatever the specific connection between the ideas basic to p and q may be, it is the subject's insight into the intrinsic causes of the relative failure of p which leads to the new solving proposition q . When the strength of a certain defense by the opposition becomes obvious during the investigation of p , that continuation is tried (q) which makes such a defense impossible; if the enemy King can retreat to safety after a normal chase (p), then an attempt is made to cut him off at the pass (q); if the launching of a carefree counterattack (p) is slightly retarded, then defensive measures (q) may first be called for; etc. Many similar p - q relationships appear in the protocols. If this relationship is of a typical nature, we find ourselves in the next group.

4. The choice of q after p may result from a general *priority rule*. In fact, we find *fixed orders*, *natural hierarchies of methods* at all levels. The existence of general subsidiary linkings of this type can be inferred from typical sequences in the protocols and accompanying remarks. At the beginning of the investigation of move p the protocol text reads, for instance: 'First (investigate) direct moves' – by way of a principle or rule – while in fact q is second to p in 'directness.' Statements of this tenor are found in sundry protocols. In general, direct moves appear to have priority over calmer moves. Protocol (M_4 ; ... $Q-K_2$) – presented in full on pp. 249-253 – provides a more differentiated hierarchy: subject M_4 looks 'at direct combinations first,' then for aggressive positional moves ($1...N-N_4$), next towards its preparation ($1...B-B_1$), and finally for a waiting move ($1...Q-R_6$). Only few priority rules

appear to be completely independent of types of position, however. Thus, in positions where there is 'really very little cooking' tactically (M_2 ; $K-N_2$), line 22, the priority of forceful moves over calmer ones is hardly applicable and may be replaced by another: for instance, moves according to some active plan first, then waiting moves.

In general, priority rules are sensible and understandable. Thus, in most situations the investigation of calmer moves which maintain the tensions and the *status quo* is not economical until it is known that there is no immediate advantage to be gained or threat to be parried. For the calculation of *sample variations* of a move, in the Phase of exploration, the priority rule 'first direct continuations' may take on the meaning that the first variant to be examined must be the one that best expresses the underlying board intention or idea. Suppose that the idea is through one or more sacrifices to ensnare the enemy King in a mating net, or to gain space and open lines, then the acceptance of the sacrifices has priority over their refusal. Another example, taken from protocol A calculations: The *point* (underlying idea) of $1.B \times N/5$ (and not $1.N \times N$) is that in reply to Black's $1...N \times B$, $2.N \times N$ wins a tempo (by attacking the Queen) and thereby wins a piece. That is why the variant $1...N \times B$ is primarily the 'main variation' and takes precedence in every protocol where the subject calculates the three branches ensuing from $1.B \times N/5$.

A different priority principle obtains with *proof calculations*. Here it is a striving for liquidation of subproblems that often appears to determine the order of priority. The subject may, for instance, first examine the least promising moves or continuations in order to delimit what is worth considering and so be able to focus his attention and to economize in his analysis. Subject M_1 's classic remark will be remembered:

(M_1 ; A)*:

14 Now begin with the probably bad countermoves – in order to be able to eliminate those.

5. Finally we must allow for chance, for the factor of *serendipity* – a factor that entered the theory of thinking with Selz.

'Bad luck,' of course, is not an acceptable excuse for 'not seeing' something that cost a player the game. He should have seen to it that he did see it. Conversely it is, generally speaking, not 'good luck' when a player detects a hidden winning possibility. Still, the element of chance may be present within the thought process and, indeed, very often is. A player can be blamed for not seeing a possibility, but he

cannot be blamed for not seeing it immediately. He may not see a certain possibility until the very last check-over, while his hand hovers over the wrong piece. In other words: Serendipity certainly plays a role in determining the order in which the subject discovers and investigates certain possibilities.

The fact that in position A subject G4 only 'finds' the move $1. B \times N/5$ after six minutes of thinking and calculating doesn't say anything against him; nor is G5 any less a grandmaster because he discovers by chance – while calculating – the advantages of $1. B \times N/5$, that is, without prior insight. It is quite obvious that the point in time at which the idea registers depends to some extent on chance. In both thought processes we are concerned with 'calculational serendipity,' that is, during routine calculations there may be a sudden, coincidentally evoked means abstraction with respect to the main goal. In (G4; A) for example, the tactical possibility $1. B \times N/5$ emerges in connection with a calculation of $1. KR-K1, Q \times NP; 2. B \times N/5$, etc.

In addition to calculational serendipity 'perceptual serendipity' may lead to coincidental or immediate means abstractions that influence the order of elaborative phases. While scanning the board during the innumerable pauses and transitional phases, a certain constellation, certain relationships between the pieces, certain possibilities, 'may strike the subject's eye.' Thus one subject may see¹⁰ move *a* first, while another subject's eye may fall primarily upon possibility *b* – resulting in a different order of solution proposals and elaborative phases.

Considering the attention that the more romantic conceptions of thinking and creativity have generally devoted to the phenomenon of sudden, apparently coincidental and unforeseen finds, flashes of intuition, inspirations, brain waves, *Aha-Erlebnisse*, etc., it is of some interest to evaluate their frequency and their importance in chess thinking generally. If such a find really is mainly coincidental, it can be expected to have an element of surprise for the subject that may be expressed in the protocol in the form of some exclamation like: 'Wait a minute...' or 'Aha,' or something similar. What does the analysis of protocols yield in this respect?

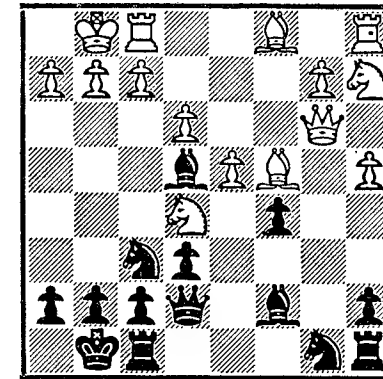
First, exclamations of the 'Aha'-type appear to be rare: in the entire material – including the short thought processes – we do not find even ten clear cases.

Second, among those found, a few represent the finding of a

¹⁰ As always when we talk about 'seeing' in chess, it is the perception of the *relations* between the visible objects and not the perception of the objects (board and pieces) themselves.

solution attribute rather than a new direct means for solving the main problem. When, for instance, it is stated in (C5; A), line 118: 'Wait a minute, I can make him choose (by playing $1. N \times B$)...', then no new move is discovered but solely a relationship: the necessity of the opponent's choosing between two possible lines of defense might be exploited in some way or other.

Third, notwithstanding the subject's surprise in the other cases, the discovery takes place in a phase of searching in a fairly well defined direction as seen from the text of the protocol. One example: the end of the first Phase of



POSITION ...N-B3

Black on move

(M4;... N-B3):

- 11 Let's see ... if there's something to be done. It would seem as though something must happen with those two Bishops.
- 15 I'm looking at something like
 $1... B \times N$ and $2... N-N5$. But that's not a very serious combination: he plays
 $3. P-B3$ on $2... N-N5$.
 Aha, if I just play
- 20 $1... N-B3$ and $2... B \times Pch$ if he takes! $3... N-N5ch$; the King must go to $N3$ - ...

After the failure of the 'not very serious' first solving proposition, the goal-setting 'Let's see... if there's something to be done,' reclaims its rights; indeed, in the chance discovery something does 'happen with those two Bishops.' Thus we may certainly speak here of a search in a certain direction; a clear schematic anticipation precedes the discovery.

The same holds for most of the other cases.

We must conclude from this that the actual *brain wave* coming suddenly and surprisingly, apparently independent of a goal-directed

search, plays but a minor role in chess thinking. In interpreting this finding we must, of course, keep in mind that a brain wave, an *Aha-Erlebnis*, is an experience: an insight accompanied by the *emotion* of surprise. Therefore an expert in any field is much less likely to be 'surprised' by his own discoveries than a beginner is. Also, the surprise criterion, although part of the definition, by no means guarantees the find to be of any value, nor does it seem to contribute very much conceptually to an explanation of the dynamics of productive thinking. These considerations are in accordance with general results of *Denkpsychological* investigations. The conspicuousness of quasi-unprepared, that is, 'surprising' findings, has led the more romantic schools – Gestalt psychology included – to overestimate the functional importance of such finds. Compare the reduced significance of the phenomenon of inspiration in musical creativity (BAHLE 1936 and 1939).

On the other hand, the scarcity of brainwave-like experiences might to some extent be peculiar to chess thinking that is geared to actual game playing. In situations where a chess problem can be 'carried around' like creative problems for any length of time – e.g., an adjourned game position, some analytic position, or a problem of chess composition – actual brain waves might occur more often.

These last explanations detract nothing, of course, from the importance that serendipity can have for the moment of 'finding' some solving means and, with that, for the sequence of investigative phases.

B. THE METHODS OF CHESS THINKING

Section 54: The most important methods as typical problem transformations

Following Selz we may define a 'solving method' as an *Aufgabe*-transformation which goes with a specific class of *Aufgaben* (cf. SELZ 1924, p. 11; DUNCKER 1945, p. 8). Whenever a particular transformation is found to occur predictably in a number of solving processes (protocols) that result from *Aufgaben* of a distinct, definable type, the transformation can be called a solving method. This definition is largely maintained in the ensuing analysis, except for the following modification: '*Aufgabe*' is replaced by the more inclusive 'problem.'¹¹ A *solving method* may thus be briefly characterized as a *typical problem transformation*.

This definition needs some clarification.

First, it is not limited to solving methods in the narrower sense, i.e.,

methods accompanied by 'insight' (*einsichtige Lösungsmethoden*). The definition does not require that the subject be aware that the operation applied serves as a means of solving the problem (cf., e.g., SELZ 1924, p. 11). The general concept of 'solving method' – or thinking method, method of thought – that is adopted by Selz and the present writer is much broader. It is so inclusive, in fact, that 'every directed thought process up to the attainment of the goal or to the subject's abandoning his solving attempts can be described as a chain of sequentially corresponding solving methods that are either cumulatively or subsidiarily linked' (*Ibid.*, p. 11). From the point of view of protocol analysis the concept of 'solving method' is defined by the operational criterion of the resulting transformation, with the one added demand that this transformation be 'typical.' Moreover, not only is the occurrence of a 'typical problem transformation' in a sequence of mental processes *the* discernible criterion indicating that a method has been applied: the typical problem transformation *is* the method itself.

In the following pages the reader should continually bear in mind that the concept of 'method' has acquired a much greater scope here than it has in daily usage. Even in pre-Selzian *Denkpsychologie* it was generally restricted to (solving) methods in the narrower sense, where the subject *consciously* 'applies a method,' i.e., knowing that he does so. (Cf., e.g., LINDWORKY 1916, Ch. II, where he states that 'methodical solution attempts' are relatively rare.)¹² In the present con-

¹¹ Selz himself hardly ever gives clear-cut definitions. In his extensive texts the terms 'operation,' 'solving method,' '(problem) transformation' and even the more inclusive 'manner of behavior' (*Verhaltensweise*) often seem to be interchangeable, except that they emphasize different aspects of the same phenomena. 'Solving method' and 'problem transformation' are, of course, only applicable to problem solving processes, while the other two are more general. Further, a problem transformation must be *goal-determined* (not coincidental) and typical (corresponding to a *specifiable* class of problems) in order to be interpretable as a solving method.

¹² The prime reason for not maintaining the requirement that the subject be aware of his own methodicalness is that the requirement cannot be of much importance for a dynamic explanation of thought processes. Like the surprise element in a sudden find (brain wave, etc., cf. Section 53, p. 278), being or not being aware of methodicalness is rather an epiphenomenon. Experienced problem solvers may apply exactly the same method automatically (without awareness) on one occasion and deliberately (with awareness) on another. Teachers, particularly of mathematics, are well acquainted with the effectiveness of automatic problem transformations as well as with the possibility of becoming aware of them – either anew or for the first time. When actually teaching it is especially important to make one's own solving methods explicit to be able to work with them and teach them as *heuristics*. (Cf. SCHUH 1944; POLYA 1957).

text any approach, procedure, or operation that (1) recurs in the protocols, (2) can be related to specific types of problem situations, and (3) actually transforms the problem, is considered a method.

Second, the reader should bear in mind the broad scope that the concept of 'problem' (and of problem transformation) has acquired in the preceding chapters. The term does not refer to an objective problem but to any psychological problem – main or subproblem – that may be present in the subject's mind at a certain moment (cf. Chapter V).

The substitution of 'problem' for '*Aufgabe*' thus extends the concept of 'solving (or thinking) method' far enough to make it applicable beyond the sphere of experimental tasks and for subproblems as well as for the main problem. From the translated Selz quotation above (pp. 280-281) it would seem that this extension is in line with his general conception of a thought process as a 'chain of solving methods.'

In summing up it appears that two large groups of thought methods gain admission as a result of the expanded concept. In addition to the old group of consciously applied methods there are (1) the completely automatic, relatively simple, and 'self-evident' problem transformations which as methods went unnoticed in the old days, and (2) the full-fledged methods (procedures, approaches, heuristics) which were not acknowledged as methods solely because they were not consciously applied.

Let us now turn our attention to the most important, most frequent, and most general methods of chess thinking. Can they be conceived and described as typical problem transformations? If so, how?

Plan formation. 'We should really make a plan' (M₂; ... N-K₁) means that Subject M₂ wants to set himself a concrete strategic goal within relatively easy reach, in contrast to his long-term mating or drawing design. Making plans is a well-known and often consciously applied method in chess thinking since Steinitz' theoretical work in the last century. Its purpose and function are no different from those of planning in any other area. In chess manuals the directive and selective value is clearly explained. Indeed, the complicated and elusive choice-of-move-problem is remolded: a typical problem transformation. The plan allows for long-term developments; it gives the player a grip on the position; it organizes and delimits the multitude of possibilities; and it narrows down the moves to be considered. Only what fits into the plan need be investigated. Even if the plan must be abandoned, it may have been meaningful as a working hypothesis.

Alternative groupings. Many cases of plan formation largely amount to reducing a multiple choice problem to a binary choice problem: again a typical problem transformation. The pertinent moves are divided into two groups: those fitting into the plan and the remaining ones (perhaps belonging to another well defined plan). The same holds for clustering along other lines (see Section 43, p. 175): order is created and possibilities are reduced, often to two (cf. Sections 43 and 49).

Progressive deepening of the investigation. This, too, is undoubtedly a method. It corresponds, in fact, to certain problem situations, namely, those that are difficult for the subject. When subjects say 'Let's first look' (first Phase) or 'Let's first try something out' (exploratory beginning of the main part) they are purposively carrying out the investigation in such a way as to be able to set about it more thoroughly and calculate it more deeply later on: indeed a transformation of the investigative problem and thus of the main problem. The exact form that the investigation will take is, of course, not determined a priori; progressive deepening is carried out 'if necessary': it is a conditionally composite operation (cf. p. 263). Even so, the intention 'let's first...' already expresses a problem transformation in essence. Later on when the subject decides on another round (deepening phase), i.e., to re-investigate some move or plan, this decision clearly transforms the problem. Sometimes the transformation is a very drastic one as we have seen in Section 47. At these very moments the *method* of progressive deepening can be said to be applied.

Stipulating the order of investigation. Many of the cases of 'Let's first...' do not refer to phases of deepening but, e.g., to different parts of the investigation – variations within a calculation, parts of the reasoning process – that are ordered in advance. Here, too, we often find a conditional order, and therefore conditionally composite operations. The next phase is only entered upon if the preceding one fails to yield the desired results, e.g., when investigating a pre-ordered series of own-moves or -branches. There are unconditional cases as well, e.g., when the subject systematically disposes of a pre-ordered set of counter-branches during the positive part or of own-branches during the negative part of a proof-directed investigation. Naturally stipulating an order is a method; that it is a typical problem transformation is also clear.

Trying out. This is an extremely important general method to which the whole of Section 56 will be devoted. It is not difficult to see that in deciding to 'try out' something – in the mind, of course – the subject either consciously or unconsciously transforms the problem. It

is nearly always a matter of trying *if* ..., seeing *whether* ..., e.g., seeing whether a certain sample variation leads to anything (cf. Section 40, p. 151). The subgoal-setting changes radically at the moment the preceding perceptual search is replaced by a quasi-empirical exploration; and so the main problem changes.

Typical ways of reasoning. As early as Chapter I it was pointed out that only rarely is a pure and objectively convincing argumentation, complete with positive and negative parts, contained in the thought process. The subject (the player) is generally satisfied with less than 100% certainty, not only because he often has no choice – exhaustive analysis being impossible – but also because he has developed the general thinking habit (method) of adjusting, as economically as he can, his certainty requirements to the situation. The degree of completeness with which he is satisfied depends primarily on the situation on the board: complexity of the position, anticipation of objective solvability, quantitative expectations, etc. In competitive games other factors may play a role, such as the importance of the game, the available time, etc. In any case characteristic adaptations, *typical transformations of the problem of argumentation*, are found in chess thinking – and are thus methods.

Of course the (required) degree of completeness or of certainty is not the only variable factor in the final form of the argument. The following examples of a few typical forms of reasoning are illustrative of structural differences:

1. Assume that the opponent has a threat which can be parried in three ways: *a*, *b*, or *c*. Investigation shows *a* and *b* to be unsatisfactory defenses but *c* averts all danger; ergo, *c* is played.

If *a*, *b*, and *c* are analyzed up to envisaged positions which can be evaluated with certainty, then we have a fairly complete argumentation with a positive part (*c*) and a negative part (*a* and *b*) – provided that the unplayability of moves other than *a*, *b*, and *c* has been demonstrated. If, however, the results of *a* and *b* are so disastrous that *a* and *b* are just as inappropriate as the other groups of moves already eliminated, the subject may *indirectly* decide on *c*. The investigation of *c* becomes the less meaningful the more thoroughly *a* and *b* are investigated and the more disheartening the results become. In such an *indirect argumentation* the positive part of the proof is either omitted or else carried out cursorily and incompletely: *c* is a 'forced move'; what happens after *c* is not now examined.

2. Assume that the position offers various possibilities for forcing, combinatory continuations: *a*, *b*, and *c*, while calm moves allow the

opponent to launch a counterattack or enable him to exchange off the pressure. Upon investigation, *a* appears extremely advantageous whereas *b* and *c* are less convincing; therefore *a* is played.

Here, too, the argumentation may be fairly complete; namely, when the results of *a*, *b*, and *c* are quite certain and the other moves have been justifiably eliminated as being 'nothing special.' If, however, the result of *a* is so favorable that *b* and *c* can hardly be any better, the subject may choose *a directly*, without further analysis of *b* and *c*. The investigation of *b* and *c*, and a fortiori of the other moves, becomes the less meaningful the more thoroughly *a* is investigated and the more heartening its results become.

In such a *direct argumentation* the negative part of the proof may even be completely omitted; e.g. (M2; ... P-QR3), see Section 49.

3. Assume that in a relatively quiet position several moves are pertinent, all within the framework of a fixed plan. Several moves are successively investigated; some are eliminated along the way until finally move *a* is played because it turns out to be relatively the best.

Both a positive and a negative part are clearly present, although the two may be more or less incomplete. The incompleteness of the negative part is sometimes expressed in the text of the protocol as: 'I guess I'll play *a*; I don't see anything better.' That is to say: 'I don't feel I've proved that no other move is better than *a*, but if there is one I don't see it.' The opinion on *a* itself may also be incompletely supported, e.g., by just a few sample variations: '*a* offers good chances.' In some cases an emotional or intuitive preference clinches the argument for *a*; in others a deductive argument may settle the issue: *a* is, for example, the 'most logical' move within the plan. In their details the forms of reasoning may differ from each other in all such cases.

The final form of reasoning adopted by the subject is always a subtle and precise adaptation to the situation on the board (that is, to his perception of it) and to the way his total problem conception is developing. The problem of how to structure the 'subjectively convincing argument' transforms itself to conform to this development. To the extent that completeness is considered difficult to attain or less essential the subject then completes it with non-probative arguments: emotional, intuitive, and/or deductive arguments of a general nature. This *intuitive completion of the argumentation* (see p. 274) is in itself an important method of chess thinking.

4. Assume that two plans compete for supremacy – compare (M2;

B) and (G5; B). In such a case the subject's outline of reasoning develops along with that of the total problem conception – described most recently in Sections 47 and 49. If the subject in the struggle of the plans decides in favor of one of them before his move choice, such a decision can hardly be based on a complete analysis because plans are by definition incompletely defined long-term schemes for action. At most it may occur that the subject can completely prove the uselessness of plan *A* so that the acceptance of *B* (or non-*A*) follows indirectly. In general, however, the choice of a plan is based on: (a) 'general considerations' of a largely deductive nature – means (actions, moves, maneuvers) being derived from a strategic goal –; (b) results of sample variations of *A* and *B*; supplemented with (c) intuitive and emotional arguments.

After the choice of plan (*B*) the choice of move, within the framework of *B*, may then proceed along various courses, again; compare the possibilities discussed under 1, 2, and 3. The final argument has, in any case, a composite structure since two interdependent choices have, in fact, been made by the subject.

Many more typical forms of reasoning could be distinguished. There is, in fact, a way of reasoning peculiar to each of the types of problem development discussed in Section 49. The most important variables with regard to which various forms of reasoning differ have, however, been covered by the examples given. In summing up, these variables are:

- the relative size of the non-probative 'intuitive contribution' to the argument (the subject's tolerance for incompleteness and/or uncertainty as dependent on type of position and problem conception);
- the content of the 'intuitive contribution,' or the way in which the incompleteness gap is filled (e.g., by deductive arguments, intuitive hunches, or irrational, emotional preferences);
- the relative predominance of positive or negative reasoning; direct, indirect, and intermediate (incomplete) argumentation;
- the occurrence of plans and other subgroupings of the pertinent moves and the occurrence of intermediate decisions with regard to these groupings.

Although it is not always easy to describe precisely the position type and problem conception conditions under which a particular form of reasoning will appear, it is clear enough from the protocols that there

are specific correspondences. The chess player has an entire system of typical forms of reasoning at his disposal to be used according to the situation for transforming the problem of argumentation (building up a subjectively convincing argument in favor of a certain move) into a specific and adequate form. Choosing an adequate form of argumentation is clearly a solving method. Experienced players – like experienced problem solvers in any particular field – generally perform it quite automatically, without much awareness of its being a method.

Section 55: Methods peculiar to specific parts of the thought process

This section contains a synopsis of all the more or less special thought methods occurring in the material. Many have already been mentioned in other contexts. The following review makes no pretense of completeness.

1. First Phase methods

That a *position investigation* always comes before the investigation of possibilities may in itself be described as the application of a method. The thought process begins with a typical partitive transformation of the main problem: 'Let's *first* have a look at the position.'

Analogously, the subdivision into *static*, *dynamic*, and *evaluative investigation* (cf. Section 40) corresponds to a typical operation, namely, considering the position from three points of view and in a fixed order. The fact that not all three moments always show up in the same protocol may either be explained by the incompleteness of a protocol with regard to rapid, routine determined processes (cf. Section 23) or – if one must assume an absence of one or more moments – by a dependence of the problem transformation on the type of position; the complete form (i.e., all three moments) being evoked by certain types of positions only. The domain of such types of positions is difficult to define in words; it may even vary from subject to subject.

The same holds for the partitive transformation during the static moment: '*First* the material situation.' Also for *splitting the orientation to possibilities into three parts*: threats, own possibilities for direct action, long-term board developments (see Section 40).

The orientation to possibilities often leads, as a matter of course, to certain *groupings of possibilities*: forceful moves versus other moves; plan *A* versus plan *B*, and so forth. *Plan formation* and *favorite forming* – in every possible gradation of confidence, from 'working hypothesis'

to 'theory' to conviction to certainty – often occur as early as the first Phase.

Also important are the submethods by means of which an evaluative judgment of the position is formed. In the first Phase this is not the main object of the subject's activities, it is true, but even so we saw in Section 40 that in difficult cases static and dynamic position investigations served largely to contribute to an improved evaluation. To this end pros and cons of the position must be weighed; possibilities for action have to be estimated; the subject must 'integrate' the position into one functional whole. Here, too, the subject goes about it methodically. The typical moments of this process will be discussed under 4 below (transitional phases).

2. *Methods for investigating possibilities*

We are already familiar with the overall methodological picture from previous sections (41 among others). We have seen how the investigative problem specializes and splits into subproblems which are tackled in a certain order ('forceful moves first' and other conditionally or plain composite operations). We have become acquainted with all sorts of typical specializations and subgroupings, especially alternative groupings – each of these resulting from typical problem transformations, i.e., from methods. We have seen how these transformations of the substructure of the main problem may create an extremely complex whole composed of co-ordinate and subordinate problems, especially when many branches must be calculated.

The chessmaster is skilled in handling such a system; indeed he can only do this because the 'ordering' operations concerned amount to applying highly typical, routinely determined methods. The operations proceed practically automatically.¹³ In the material there is no clear-cut case in which the subject 'loses the thread' – not even among the weaker players. In practice the thought organization is sometimes disturbed, according to occasional reports of chess players. 'Losing the thread' may occur particularly when time pressure¹⁴ and other

circumstances create nervous tension; but these cases are exceptions. If one can ever speak of a system of fixed thinking methods, it is with regard to the organization of the investigation of possibilities.

After the first Phase an important special method is the *trying out* of moves within the framework of a plan or, generally speaking, within the framework of the pertinent moves. Such trying out is often the beginning of a progressively deepening special investigation of a plan, board method, or move possibility. *Progressive deepening* is another method often applied in special investigations as well as in the general setup of the thought process. *Sample variation(s)* generally form the basis for a preliminary judgment and in this the subject strives to examine *the main variation first*, that is, the presumably strongest or most logical moves of both sides – while more thorough elaboration follows later (continued investigation) which is geared to the results obtained.

At issue now is the system of methods used in rejecting and accepting possibilities (cf. Section 52). When a result is improbably favorable or unfavorable, *checking* (striving for *correction*) of counter-play or own-play, respectively, follows. When the result is unclear, the continued investigation aims at *clarification*. If the results of several variants are still unfavorable, the continued investigation is directed towards *strengthening*; if favorable, towards *confirmation*, etc. Each of these terms represents a typical transformation of the subproblem that goes with the investigation of a possibility.

A few more methods of minor importance should be mentioned.

First, there is the conscious *recording* (express formulation) of attained provisional *results* of a quantitative or qualitative nature; for instance, when a minimum result is achieved.

A much applied auxiliary method is the *no-move principle*. In investigating the strength of own or enemy threats the subject does not bother with a countermove or own-move for the time being, but poses the question, either in the initial or in an envisaged position: 'If now I don't (or he does not) do anything, what can he do to me (or I do to him)?' – cf. the 'N.M.' symbols in Fig. 7 representing the calculations in (M2; B). Sometimes, especially in the endgame, the player may even simply *count the number of own-moves necessary* to carry out a certain maneuver, ignoring what his opponent can do. Only afterwards are the countermoves examined and counted out; then the two results are compared. Although the material contains a few examples of this, they will not be given; the matter speaks for itself.

There is an interesting auxiliary method for finding ways of strength-

¹³ If this were not so, it would be completely impossible to explain why some chess players can still play brilliantly while under the influence of alcohol. This provides an indirect proof for the fact that the thought process utilizes highly automatic methods, almost engrained in the player.

¹⁴ On the other hand, nervous tension in general and time pressure in particular sometimes have quite the opposite effect. Many strong players claim that their powers for concentration and clarity reach their maximal efficiency under time pressure. There are even some indications that *the opponent* of a player under some pressure is the one more likely to 'lose the thread.'

ening a possibility. When a possibility can be refuted in an obvious manner, that is when at first sight the move 'doesn't work,' the subject often *deliberately disregards the provisional evidence* in order to 'try it out anyway,' to see what would happen 'if one were to play ... after all.' At the moment such an investigation is begun (after all), it is generally not (completely) known if there is a weak point in the refutation; but if there is, it will undoubtedly be uncovered this way. In a certain sense this is an example of 'methodical doubt,' a more general method of (chess) thought. Indeed, the entire methodology of rejecting and accepting possibilities, of checking operations, and of attempts at strengthening may be looked at from this point of view: time and again *methodical doubt of the correctness of the obtained results* ushers in, as it were, the following operation. More on methodical doubt under point 4.

3. Calculations

Most of the methods for the investigation of possibilities in general, may be found in more restricted applications within specific calculative sequences. Nowhere is the system of overlapping subproblems and their automatically methodical disposal so clearly to be seen as within specific branched calculations. Besides these a few more specific methods are worth mentioning.

The *striving for partial liquidations or eliminations*, for simplification of the problem by cutting off branches, is always to be found in composite calculations. *First* investigated are those parts, continuations, or variants which can presumably be dealt with quickly and possibly dispatched. These can be:

(a) drastic continuations netting 'all or nothing' results that are presumably definitive – the principle of 'direct moves first' is active here;

(b) moves leading to stability and consequently to an easily evaluated position (to a relatively dead position);

(c) moves or variants that are probably bad and can thus be eliminated; etc.

The last principle is especially meaningful for positively (or negatively) *directed* calculations. The more the striving for proof comes into view, the greater the importance of purposeful eliminations.

Aiding in complicated calculations is the auxiliary method of *repeating the 'trunk' of the calculational tree*. From the multibranch analyses in the protocols we are often able to substantiate that the subject *returns* time and again to the initial position. This method obviously

serves two purposes: re-establishing the organizational structure (preventing 'loss of the thread', cf. p. 288) and ensuring greater accuracy in the representation of envisaged positions.

4. Transitional phase methods

The system of methods found in transitional phases was extensively treated in Section 48. A short summary of the most important methods, especially those applied to overcome difficulties in the problem and solution development, suffices here.

A methodical *return to more general problems* is characteristic of the transitional phases. The partial results of the elaborations in the last phases are co-ordinated, summed up, and generalized. If the outcome is unfavorable, if a crisis of expectations is in sight, then the quantitative results especially are *recapitulated, generalized, and made explicit*. The evaluation itself is checked as well, often by employing an auxiliary method that may be called pro-con analysis or – somewhat more pretentiously – *dialectical deepening*. This process starts with the subject methodically doubting his latest judgment in order to unearth its weak points; then these weak points are in their turn doubted. So one comes to an evermore precise weighing and comparing of advantages and disadvantages as well as to a more penetrating and better supported judgment (and evaluation) of the situation. In such cases the protocols often show remarkable 'seesawing.'

(G5; B1):

48 1... P-KR4; 2... K-N2... but he is not obliged to take on N3. Yes, I have serious doubts whether that will work out. On the other hand my pieces come into play very quickly. (Pause)

62 I have the feeling if you don't proceed actively, you will slowly lose. Materially it is not so good. I have seen games, though, where you could not make any progress. But there the passed Pawns were restrained, here they aren't.

(M5; R-Q1):

96 I have the feeling that White must follow up his advantage immediately. Then again maybe not.

The statement-doubt pattern is a rather general phenomenon in the transitional phases; dialectical deepening is an important method. It can also serve to explain some of the remarkable, seemingly illogical sequences in the protocols. Sometimes the statement but not the doubt may appear in the protocol so that the transition from the statement to the following phases looks incomprehensible. The phase in question is actually related to the unexpressed doubt; for example:

(E₃; A):

52 It doesn't seem auspicious for Black to take on N₂.

54 1. N-B₄ deserves consideration.

The first sentence means: I don't have to defend my Pawn on QN₂. The second: 1. N-B₄, that is, defending the Pawn on QN₂, does deserve consideration. If we insert an expression of doubt such as 'maybe after all,' then the difficulty is resolved.¹⁵

As can be inferred from Section 48 methods of critical analysis are applied to other aspects too. The subject investigates, for instance, the factors on which his negative judgment is based, the causes of his failure (*conflict analysis*). On the other hand, in Duncker's terminology (DUNCKER 1945), next to analysis of conflict and *analysis of situation* or of *material*, there is also an *analysis of goal*, of what is demanded; namely, what the board goal striven for actually is. The subject tries to specify for himself as explicitly as possible what he really wants, what (board) ideal he is pursuing. So he comes to evermore precise anticipations of the kind found in (M₅; B-R7ch) – see the discussion on p. 211ff.

We may also consider as a thought method the transition to less strictly organized or canalized ways of thinking: the transition to 'cogitation' and *pauses in thinking* (cf. Sections 34 and 48). During the transitional phases the subject often *withdraws* more or less *on purpose* from the somewhat too concrete detail problems. For the discovery of new possibilities and the acquisition of new points of view, this is of the greatest importance. If concentration is too tense due to nervous tension during competitive play, the attempts to withdraw from calculational details in order again to 'look at the position' is sometimes unsuccessful. The consequences are often detrimental to the quality of the game: blunders generally result from 'overlooking' things one should 'see.'

5. *Striving for Proof*

As important methods in the convergence towards a final argumentation, we have got to know (Section 43) the *clustering* of move possibilities, the *grouping* of possibilities resulting in a final *alternative*, and deliberate *elimination* of moves and entire plans. We have seen how *favorite forming* colored the alternative and how subsequently the

striving was always aimed at *securing confirmation* – through calculations, reasoning, or possibly by supplementary half-intuitive insight. We have seen how, thus, the analyses were changed from 'neutral' to more and more positively (or negatively) directed. We have further seen how the subject attempts to increase the proving power of the argument. On the one hand he tries to provide the favorite with sufficient domination over other moves (*a search for a decisive argument*, direct or indirect, using *comparison of possibilities* as an auxiliary method); on the other hand, he tries to *broaden the basis of the argument* as much as possible. Both frequently go hand in hand with *recapitulatory* and *checking operations* that are found in particular right before the move decision. These are all methods in the sense of Section 54.

Section 56: *Trying out as a general method*

Trying out, by actually doing and trying out as a mental operation, forms a very important (auxiliary) method in thinking and decision making in general. But in chess thinking this method is of prime importance. The entire thought process may be conceived of as *an empirical thought investigation*: by trying out moves and plans in his head, the player determines their worth. No other argument can stand up against the empirical argument.

What light do the chess protocols throw on this general method of thinking? Most psychologists, both before and after Selz, allege a close similarity between the mental operation (method) of trying out and the process of trial-and-error (cf., e.g., THOMSON 1924). This similarity warrants closer examination.

As is known, Selz opposed this similarity view, even for the case of trying out by doing. According to him the subject always shows a clear sense of direction: he tries out within a pre-set, goal-determined, limited domain of solution possibilities. Trying out is based on a *partial insight* into the problem situation. Trying out behavior (*probierendes Verhalten*) occurs in cases where, to be sure, the subject has certain anticipations of the result, but these anticipations are in some respects vague or uncertain. This is exactly the case in chess thinking. Selz describes the *method* of trying out as follows (somewhat freely translated):

'The trying out of several solution possibilities is a general operation for the finding of means. It must be applied by (animal and) man wherever it is impossible via determined immediate means-abstraction, to discover structurally based solving methods and where firm empiri-

¹⁵ Dialectical deepening through methodical doubt is related to Selz's Law of Checking (*Gesetz der Berichtigung*) but is of a more general scope.

cally based correspondences are not yet at hand'¹⁶ (SELZ 1922, p. 645).

Mutatis mutandis this holds for trying out as a mental operation. Human problem solvers usually proceed to try out by doing only after the primary method of trying out as a mental operation fails.

Trying out in the process of chess thinking always has a *multiple function*.¹⁷ The most important aspects are:

(a) *Hitting on a solution*. Trying out as a direct solution attempt, either of the main problem or of a subproblem. *If* the attempt is successful, the trying out process (method) leads directly to the solution (possibly a minimum solution) or at least to a favorite solution proposal.

(b) *Increasing information on the problem*. Trying out as an exploratory device by way of orientation to the possibilities. This function has been referred to more than once in discussing the method of sample variations. Through trying out the problem becomes concrete and specific; to the total goal conception are added new experiences (qualitative information) and new expectations; the group of pertinent moves may be delimited.

(c) *Furthering discovery of new means*. Trying out as an ancillary operation for the finding of new means, especially by coincidental means-abstraction. Insight may come through calculations; consideration of possible mutations favors serendipity. The reader is reminded of (G₅; A): G₅, for the sake of completeness, tries 1. B×N/5 after elaborate calculations of 1. N×N. Only through the 1. B×N/5 calculations does he notice that and why this move is essentially different from 1. N×N. In (G₄; A) G₄ arrives at the idea of playing 1. B×N/5 immediately via a sample variation of 1. KR-K1 (namely, 1. KR-K1, Q×NP; 2. B×N/5).

Of these three functions the second is completely absent in a process of 'blind' trial-and-error (which only exists in theory) while the first and third are present only in much more primitive form, without insight and without awareness of a sought for solution.

16 Das Durchprobieren verschiedener Lösungsmöglichkeiten, . . . , ist eine allgemeine Operation der Mittelfindung, die auch vom Menschen überall angewendet werden muss, wo sich eine strukturgesetzlich begründete Erkenntnis der Lösungsmethoden durch unmittelbare determinierte Mittelabstraktion nicht erreichen lässt und feste empirisch begründete Zuordnungen noch nicht vorhanden sind.'

17 Since multiple function processes are generally extremely difficult to program for a computer the method of trying out can be expected not to be one of the easiest human processes to simulate adequately – as might seem at first sight – but, on the contrary, one of the most resistant.

Let us now examine point by point the differences between trying out in a process of directed thought, in chess in particular, and 'blind' trial-and-error in general.

(1) The chess player does not touch any of the pieces; he tries out *only in his mind*. In the present enumeration this is the only difference that is not relative.

(2) All trying out by the chess player and by the human problem solver in general is always *trying out if...* (see p. 284). The word 'if' indicates that the trying out occurs within the framework of a fairly well defined goal-setting. The subject is looking for something definite, he nurtures fairly specific quantitative and qualitative expectations, and a schematic anticipation is undoubtedly present. This 'sense of direction' is clearly expressed in the goal-settings and problem formulations that often precede a trying out (see Section 41) and is indirectly shown by the small proportion of the existing possibilities that are actually tested (selectivity, see Table 4, Section 36). Due to the player's complex of knowledge and experiential linkings the choice-of-move-problem presents itself only in a contracted and highly specified form. Furthermore, various hunches, expectations, and suppositions (favorite forming is a form of hypothesis forming!) – in short: anticipations – co-determine the where and what of search and trying, respectively. As we know, all sorts of anticipations figure in the sequence, content, and character of the elaborative phases generally. Apart from (1), the strong *directive, selective, and evaluative role played by anticipations* might well be the most important difference from trial-and-error.

(3) Another characteristic is the more complex and especially more *intensive problem development* during trying out. In trial-and-error there is room only for good and bad experiences, for 'pleasure' and 'pain'; in processes of higher-order trying out a much richer stream of information – on the problem – filters back to the subject: qualitative results and generalizations, evaluative refinements, structural and causal 'insights.' Bear in mind point (b) above: the process of trying out proceeds in a steady interaction with the problem development.

(4) Trying out in chess thought is always *part of a rather complex organized process*. Trying out is generally preceded by a relevant specific goal-setting and itself appears as part of a structured whole. It may, for instance, be embedded in a structure of progressive deepening or be used only for the examination of a specific envisaged position as part of the investigation of a move possibility.

(5) Even one phase of trying out generally exhibits a complex structure. There are almost always branchings and subproblems.

Even the calculation of one sample variation a few moves deep consists of a composite series of thought operations. One should realize, for example, that every next move must be chosen from the group of pertinent moves – in accord with the *idea* underlying the ‘trying if ...’ – and that the envisaged situation on the board changes with every move. To be sure, these subprocesses overlap, but that does not detract from the *compositeness* of practically every phase of trying out.

(6) We have already seen that ‘pleasure’ and ‘pain’ are much less directly influential in determining direction than one would think if starting from the trial-and-error schema. According to Selz the crucial question is rather whether the results of trying out *agree* satisfactorily with the *schematic anticipation*. In the chess thought process, the corresponding criterion is whether the quantitative expectations are or are not satisfied. Apart from the presence of certain qualitative expectations (anticipations), the existence of a specified *quantitative expectancy* is characteristic and distinctive. This expectancy can be measured on a relative scale and, therefore, necessitates a relative conception of the achieved quantitative results.

Even though we may not accept Selz’s belief that it is solely the agreement with the schematic anticipation that is decisive (see p. 259) still the importance of absolute ‘pleasure’ and ‘pain’ – good and bad results, regardless of expectancies – is greatly reduced compared to ideal, blind trial-and-error.

It is true that most of the above differences are relative, especially when the comparison is made not with theoretical schemata of trial-and-error but with actual animal trial-and-error behavior. It remains of some importance, however, not to overlook them when attempts are made to explain – or to simulate – human thought processes.

Section 57: Playing methods: the arsenal of the chessmaster

Quite frequently in the preceding chapters, particularly in discussing the problem formation in the first Phase (Section 45), we have hit upon the *system of playing methods* the chessmaster has at his disposal. Next to general thought methods playing methods are of great importance in determining the course of the thought process. The two can be formally distinguished by the way they are conceived and formulated; playing methods in terms of board goals and -means, general thought methods in terms of operation-goals and -means, i.e., in terms of the organization of the thought process.

The two sets of methods are interconnected, of course. In the first

Phase, for instance, the subject’s knowledge of the specific playing methods that go with the type of position determines the direction the investigation (by thought methods) will take. During the whole thought process, in fact, strategic or tactical board plans, -ideas, and -goals – playing methods – are always embedded in the contextual structure of general thought methods. Board goals and playing methods actually form the content of general methods like alternative groupings, favorite formation, progressive deepening, etc.; whether or not a well organized thought process will lead to a good choice of move depends largely on what goes into the process, i.e., on the subject’s choice of playing methods.

Up until now we have concentrated on the more general thought methods, because of their importance for the psychology of the thought process. It goes almost without saying, however, that chess mastership is primarily dependent on the knowledge and skill of specific playing methods and their specific applicability. To these the present section is devoted.

We have often made reference to the *system of experiential linkings* by means of which the actual operations in the thought process must be explained. Typical situations on the board evoke and activate corresponding playing methods; a typical complex of position characteristics actualizes corresponding combinatorial ideas or strategic goals and plans. The extensive and subtly differentiated system of such correspondences (linkings) is, so to speak, the *arsenal of the chessmaster*. The rapid and adequate specialization of the main problem that was so striking during the first minutes, and even seconds, of the thought process is only possible through his mastery of such a system. That the chessmaster sees in a few seconds ‘what’s cooking in a certain position,’ i.e., which typical playing methods the situation on the board demands, enables him to begin his investigation in a highly specific direction. From the very start the group of pertinent moves is quite sharply delimited and, moreover, divided into subgroups according to function. In the same way this high *selectivity* holds in later stages of the thought process – in principle, in every position that the player envisages in the course of his analysis.

The experimental material abounds with examples.

In position A the characteristic isolated Queen Pawn has many consequences. Every reasonably experienced chess player knows from books on theory and/or from experience that such a Pawn is ‘not so good for the endgame’ ((M2; A), line 2), and that ‘White has to play

for the attack; otherwise he has nothing' ((G₄; A) line 5). This most general and still rather crude kind of reaction is found in a number of protocols, including those of the less strong players.

Another important characteristic of position A is the hanging position of the Black pieces in the center: Black's Bishop on K2, Knight on KB3 (weakened by his P-KN3), Knight on Q₄, and the eventuality of White's moving his Knight on K5 to Q7. Subject M₃ starts immediately 'to search for a combination in connection with KB6 and Q5,' (line 2); G₅ says that 'the pieces on KB6 and Q5 are both somewhat tied down,' (line 17) and searches for 'a possibility to take advantage of that' (line 25); finally he goes on to a detailed investigation of the 'possibilities for exchange' (line 37). In the same way M₁ remarks on the 'possibilities for exchange in connection with the loose Bishop on K7,' (line 7) and says 'Now work out the complications' (of 1. N-K₄ and 1. B×N/5) (line 10). To the less strong players this second characteristic is already less striking. For them 'the complications in the center' are less telling, and the hunch that a combination may be in the air is much less pronounced.

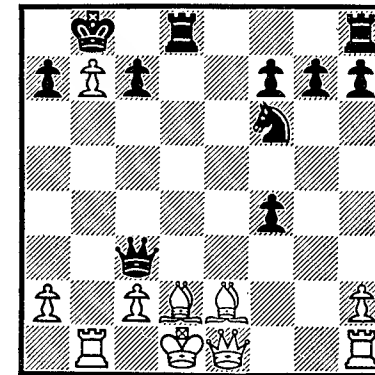
We have to assume an even more highly specialized typical linking for Subjects G₂, G₃, and M₃ (and M₁) since they took up 1. B×N/5 as their first (second) move. They must have had a specific reason for starting with this move considering that exchanging a Bishop for a Knight, particularly the strong attacking Bishop aimed at the King, is not the usual thing to do at this stage of the game. This would seem to imply that the whole idea of the combination struck them as *typical*. The underlying idea could be formulated as follows: In positions 'of this sort' – implying the positions of Black's Queen, two Knights and Bishop on K2, as well as White's two Bishops and Knight on B₃ – one can try a move like 1. B×N/5 in order to win a tempo on the second move: 2. Knight takes on Q5.

As a matter of course such immediate correspondences (position → playing method) are more frequent, more precise, and generally more adequate with strong players. Players' *playing strengths vary mainly in this respect*. In position B, e.g., masters immediately see that something must be done against the threatening advance of the White Pawns and that this is a matter of high priority. On the other hand, subject W₂'s reaction is one of resignation ('a rather hopeless mess' (W₂; B), line 24). In (M₂; ... P-QR3) M₂ almost instantly hits on the idea of trying 1... P-QR3. On being asked how he arrived at it, he answered: 'The hanging position of the Rook gave me the idea' (line 8) (i.e., the configuration: White Rook on QB7 and Knight on QN5; Black

Queen on her own QN₃; see diagram on p. 242). Obviously subject M₂ applied the *method* well known from hundreds of analogous cases: undermine the protection of an attacked piece by attacking its defender.

Quite frequently the simplest of such linkings express themselves in an immediate reaction by the subject, at his first move impulse, the moment he sees the position. True enough the main series with the thinking aloud procedure does not furnish many examples of immediately arresting moves (see, however, p. 148) but many are found in the special series with short exposure times (see Section 28; sub 4: Short processes):

[Instruction: quick decision required with thinking aloud; afterwards retrospection].



POSITION J

White on move

(M₂; j)

Thinking aloud:

'A piece ahead; King's not so well situated. Try to get an attack. 1. B-Q3 threatens to take the Queen. Then 1... Q-B6ch? No, that's impossible. 1... R-K1 does not work either.

1. B-Q3 and B×KBP and B×QBPch later, for example. He can still play: (1. B-Q3), R×B; 2. P×R, Q×P; threatens Q-B6ch but then I play 3. R-N3. Yes, 1. B-Q3.' (1 min., 10 sec).

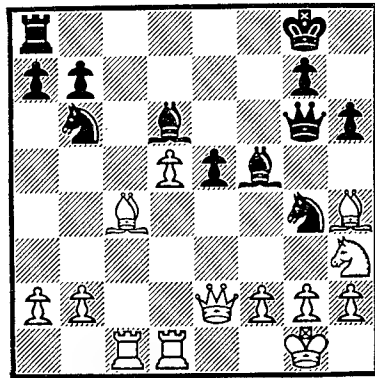
Retrospection: 'At first sight the positions of the Queen and Rook lead you automatically to drive the Queen away, to unpin the Bishop on Q2 – before he pins the Bishop on K2 with KR-K1.'

(In answer to question: Evaluation?) 'I have a good position; wouldn't take a draw.'

(Does the position remind you of anything in particular?) 'Looks like it's been a King's Gambit. I've never seen games that look like this.

My move is actually a sort of automatic reaction; I haven't looked at other moves, and would quickly have played 1. B-Q3 in a fast game.'

[Instruction: 20 seconds exposure time; then retrospection].



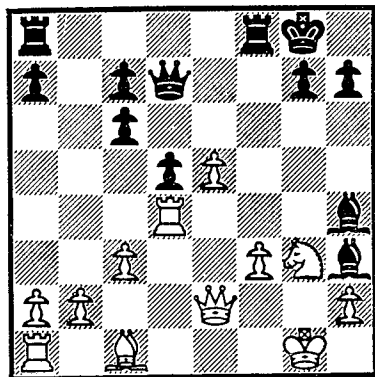
POSITION K

Black on move

(M₂; k)

'You see that you have an attack, but are one or more Pawns behind. First idea is to bring the Rook into it as the last piece that doesn't yet do anything. Then the attacking possibility along the KN file comes strongly to the fore: move the Knight away, B×N threatened; but I should love to bring the Knight to K₄, then his Bishop on B₅ is also attacked, gain of tempo. From there I looked at 1... P-K₅ as a first move. On 2. P-KN₃, N-K₄ anyway, very strong then. On 2. B-KN₃ can also play 2... N-K₄ with possibilities like N-B₆ and B×N, or even 2... B×B. Then I got stuck.'

Position known precisely; material balance not precisely in mind; not seen that Black is an exchange down; had the idea that it was actually more than one Pawn. Judgment: 'a rather pleasant position.'



POSITION L

Black on move

(M₂; l)

'Wunderbar!'

'At first I thought I'm enormously well off with those two Bishops there. Then it became less. First looked at possibilities to checkmate him; that fell through and I had to stop and see if I was better or worse off, materially. *When you see that the Bishop's attacked, then you think of 1... Q-K₂ in order to defend*,¹⁸ then later dislodge his Rook on Q₅ with P-B₄. Also thought of 1... P-N₄ in order to defend. Didn't think of 1... B×N, and didn't take retreating into consideration either.'

Position called off correctly. Judgment: 'Black is better off.'

The italicized lines in positions J, K, and L, are examples of 'automatically' evoked moves of various types. These arresting moves can be considered as primitive, impulsive solution attempts with regard to the main problem (to play a move), solution attempts of a crude but still typical nature. In fact it is often via such primitive impulses that deep combinations are found: the subject may first see the move in its most direct function, e.g., as a sacrifice, only later to discover that it actually has a deeper significance. That is, the move can be upheld after all.

There is more to classifying playing methods than the typical *moves* discussed above. We can also distinguish typical combinations as *tactical methods* and typical maneuvers, plans, regroupings, etc., as *strategic methods*. In each of these groups each method can be considered as being linked to specific configurations, formations, and/or situations on the board. *Opening theory*, finally, can be considered as a system of methods in its own right.

We start with those *simple combinatorial move sequences* that are routine matters to every chess player. Generally known are the fork, the discovered check, the double check, the pin, the tempo move, *Zwischenzug*, *Zugzwang*, the sacrifice, and the quasi-sacrifice. Within each of these classes, especially the last two, a number of well-known subgroups can easily be distinguished. The Bishop sacrifice on KB₇ or KR₇ is commonplace while Knight and Rook sacrifices on these squares are far from extraordinary. There are many kinds of Pawn sacrifices, sacrifices that mostly serve to open lines. Quasi-sacrifices, e.g., of a piece followed by a Queen or Pawn fork, or by a pin which regains the material, exist in scores of subtypes. Then there are many

¹⁸ In fact, this move has four 'obvious' functions: defending the Bishop, approaching the King's wing with the Queen, unpinning Black's Pawn on Q₄, and stopping White's King Pawn. It is multi-function moves in particular that immediately 'strike the eye' of an experienced player.

types of mating combinations: on the last rank, doubled Rooks on the seventh, smothered mate, etc. In some of his books Max Euwe has classified a number of the most important combinatorial themes, such as the desperado, over-burdening, masking, obstruction, several types of geometrical motifs, etc. At the time his book was first published (1935) Euwe's terminology was largely new, but the corresponding types of combinations were practically without exception already well-known to the old masters.¹⁹

Types of combinations can be distinguished on the basis of the themes on which they are built but also according to other criteria, such as the goal. Euwe provides an interesting empirical classification but one that remains far behind the highly differentiated system of types in the chessmaster's experience. The main difficulty in classifying combinations is that most of them are chain combinations consisting of a number of sequential or multi-functional motifs. What is instrumental in pulling off the combination is the existence of *several* tactical weaknesses in the opponent's camp. Often the chains and multi-functions involved – combinations in the literal sense – are again typical. For the expert, in fact, even the most original combinations are put together from parts and themes that he finds commonplace.

Of high practical importance is the general linking between noticing certain tactical weaknesses in a certain context and actuating the *general method* of *combining*. The master has cultivated a feel for the presence of combinatorial possibilities. This means that there is a superordinate class of tactical features of a position that makes it combination prone; if present these features evoke the general thinking method: search for a corresponding combination.

As to *strategic methods*, the development of chess theory itself has played a vital role: in systematically naming, describing, and classifying typical situations and corresponding playing methods. An important precursor was the Parisian opera composer and strongest chess player of his time, André Danican Philidor (1726–1795; see PHILIDOR 1749). Wilhelm Steinitz (1836–1900), the first official world champion, was the founder of modern positional play (see STEINITZ 1889). Among the more modern theoreticians and systematizers Dr. Emanuel Lasker (1868–1940; see LASKER 1932) and Dr. Siegbert Tarrasch (1862–1934; see TARRASCH 1912) who used to call himself *praeceptor Germaniae* might be mentioned, along with Richard Réti

(1889–1929; see RETI 1923), Aaron Nimzovich (1887–1935; NIMZOVICH 1929) and Dr. Max Euwe (born in 1901; EUWE 1937). We shall not go into their work, however, since it has been sufficiently described and commented on in the chess literature.

The theory of chess strategy is generally formulated in 'rules,' rules that connect typical positions with corresponding strategic schemes. Although these rules are of great didactic importance in substantially abbreviating the learning processes of younger generation players, they have the drawback – not very surprisingly – of never being rules without exceptions. Pawn grabbing in the opening is not always fatal; the Rook need not always stand behind the passed Pawn; the pair of Bishops or a Queen side Pawn majority or a protected passed Pawn or a massive Pawn center is not always an advantage. Chess theory, therefore, is continually being developed not only by formulating more and more rules which remain of practical importance, but also by refining the rules themselves, thus promoting the exceptions to new rules. In this respect the so-called neo-romantic school has played a leading role. Réti, its foremost spokesman, devoted himself to showing the insufficiency of the old rules and to describing new playing methods, as illustrated in the games of Capablanca, Alekhine, Nimzovich, and himself, as well as the younger masters of those years, one of whom was Max Euwe. Under fire, in particular, was the too dogmatic system of Dr. Tarrasch. The validity of a number of rules was successfully undermined; rules such as: first develop the Knights, then the Bishops; don't move the same piece twice in the opening; don't bring the Queen into play too early; occupy the center with Pawns; etc. Both Réti and Nimzovich collected illustrative games in which such rules had to be qualified. Although the 'modern ideas in chess' appeared after all to have less world shaking significance than Réti (1923) and Nimzovich (1929) had originally expected, the neo-romantic rules of exceptions, as they might be called, have indeed been incorporated into the modern master's system of playing methods.

In spite of the extensive body of strategic theory, there exists a large number of typical strategic methods – rules and exceptions to them – that have remained nameless. They are not registered in 'official' theory but still form a weapon in the chessmaster's arsenal. Among these are all sorts of maneuvers, offensive and defensive build ups, ways of regrouping the pieces and getting them to co-operate, etc. The general methods of piece co-operation appear to be particularly difficult to describe in detail, to treat systematically, and therefore to pass on to other students of the game. Thus the state of affairs with

19 In problem composition the terminology is much more differentiated. A large number of different themes have been precisely described and given names – the knowledge of which is a small science in itself.

regard to strategic methods differs only in degree from the tactical methods discussed above: 'book learning' still provides but the groundwork, the substructure on which the master builds his system of methods. The equally important superstructure consists of more or less 'unconsciously' applied or at least not readily specifiable intuitive methods; but more about that in the next section.

Finally a few words about *opening theory*. Here, too, we have to do with playing methods, namely, ways of getting to positions of a certain type. As is well known there exist a large number of openings and defenses: French, English, Spanish (Ruy Lopez), Italian (Giuco Piano), Sicilian, Russian (Petroff), Dutch, Scotch, Indians, Scandinavian (Center Counter Game), Danish, Slav – to restrict ourselves to a few named for nationalities. Then there are gambits, opening systems, and assorted variations that bear the name of their founder: Evans, Colle, Réti, Nimzovich, Alekhine, and many others; or of the place where they were first played: Manhattan, Cambridge-Springs, Vienna, Pittsburgh, etc. Name giving is a product of historical growth, however, and for that reason highly unsystematic. In general, every opening or defense splits up into a large number of subgroups and subvariations, each of which has a name of its own (e.g. Meran Variation is an offshoot of the Slav Defense of the Queen's Gambit Declined).

Opening theory occupies a special position inasmuch as the emphasis is more on factual knowledge (namely of variations) than is the case with most other equipment in the master's arsenal. A chessmaster should really know the important opening variations move by move. With the large treasury of experiences and knowledge that has been collected and preserved through the ages, a special study of opening theory (gained by playing over master games) is indispensable for the developing player. The ratio of knowledge (knowing that...) to experience of a less explicit, less directly specifiable nature – that is the ratio of 'conscious' to 'unconscious' methods – is not the same as in the field of general tactical and strategic methods. Even so openings are ordered and stored primarily in terms of the ideas that must be grasped. Rather than committing opening variations to memory, it is generally preferable to rely on the underlying principles, that is, to have from experience the corresponding general playing methods at one's disposal. So even here the core of the matter is again familiarity with typical playing methods corresponding to typical situations.

In summarizing the above remarks we must conclude that *mastership*

means, to a large extent, mastery of reproductively actualizable specific playing (and thinking) methods – a conception that will gain support in Chapter VIII through the results of some additional experimental data. The fact of the matter is that chess is full of well-known components to the master. Completely new and unknown ideas in chess are hardly imaginable – for the expert. The surprises and delights of the early years, during the developing player's 'first love' for the game can no longer be fully enjoyed by a seasoned master. The joys of replaying master games, endgame studies, sacrificial combinations, and other profundities are attenuated although keen appreciation can still be momentarily aroused by beautiful combinatorial twists or profound strategic conceptions. Quite tritely: There is nothing new under the sun; maturity has already seen everything, more or less. It is frequently impossible to specify why one knows the idea in question, that is, which similar situation one has gone through. But in some manner it is incorporated into the general 'experience' and is essentially old hat.

It is hardly surprising, therefore, that after a quarter of a century of world championship Emanuel Lasker branded the game of chess as 'stereotyped.' But his own further life shows more clearly than words the relativity of this statement. After turning his back on chess for a number of years he started to play anew! Of course the stereotype of chess is relative: it all depends on how broadly concepts like 'typical' methods and situations are interpreted. As a whole every chess game is something new, a completely *new* combination of old motives. This is, in fact, the charm of the game!

Section 58: Reproductive factors in productive thinking: Knowledge and Experience

From the preceding sections it is clear that purely reproductive operations, particularly the routine actualization of means (*routine-mässige Mittelaktualisierung*, cf. Section 19), must be of great importance. The master knows from experience an enormously large number of playing methods – strategic goals and means, standard procedures in certain types of position, combinatorial gimmicks (think of 1. B × N5 in position A; see p. 298) – which the weaker, less effectually experienced player does not have available. What the master, on looking over the situation, 'actualizes' quite routinely, a weaker, less experienced player has to build up from the ground – if such a thing is possible at all.

Apart from this collection of reproductively actualizable typical thought operations (routines), the master has another great advantage over the less experienced player. He is much more likely to find adequate new means by way of means abstraction. It goes without saying that in a visually spatial, geometrical game like chess the operations of means abstraction, *immediate means abstraction* in particular, are of the highest importance. Often one finds, abstracts, 'sees,' immediately from the structure of the situation on the board what is essentially going on and, therefore, what must be done. In such a case the subject derives his means immediately from what he perceives; but then, what a master perceives differs substantially from what a weaker player perceives! The master's 'experience' – thus, a reproductive factor – is operative in perception as well as in operational thinking proper.

The swift insight of the chessmaster into the possibilities of a newly shown position, his immediate 'seeing' of structural and dynamic essentials, of possible combinatorial gimmicks, and so forth, are only understandable, indeed, if we realize that as a result of his experience he quite literally 'sees' the position in a totally different (and much more adequate) way than a weaker player. The vast difference between the two in efficiency, particularly in the time required to find out what the core problem is ('what's cooking really') and to discover highly specific, adequate means of thought and board action, need not and must not be primarily ascribed to large differences in 'natural' power for (means) abstraction. The difference is mainly due to differences in perception.

It is above all his *vast store of ready 'experience'* which puts the master that much ahead of the others. His extremely extensive, widely branched and highly organized system of knowledge and experience enables him, first, to recognize immediately a chess position as one belonging to an unwritten category (type) with corresponding board means to be applied, and second, to 'see' immediately and in a highly adequate way its specific, individual features against the background of the type (category).

It is no accident that the word 'seeing,' as used here, stands both for perception and (means) abstraction. The two processes tend to fuse together; they are difficult to distinguish. But if a master and a weaker player are compared, often the former literally 'sees' possibilities that are deeply hidden to the latter, possibilities that the latter must first try to discover, calculate, think out, or deduce in order in his turn to be able to 'see' them (understand them). In other words:

The difference in achievement between master and non-master rests primarily on the fact that the master, basing himself on an enormous experience, *can start his operational thinking at a much more advanced stage* and can consequently function much more specifically and efficiently in his problem solving field.

It is not easy to appreciate fully the enormous effect of the expert's *reproductive completion of the perceived situation*, as his perceptual advantage might be called. In fact, the more 'experience' a person has collected in any field, the more difficult it becomes for him to understand the behavior of the have-nots. Every experienced teacher knows the pitfall of overestimating his students by fondly assuming that from the given problem situation his students can 'immediately' derive (see) some property or means that he himself finds quite obvious – whereas in reality, in order to 'see' it, much perceptive and abstractive experience is required. The teacher has had this experience for so long that he is no longer aware of it. An experienced problem solver in any field is particularly apt to overlook the primary and fundamental problem transformations occurring even *before* he starts his own consciously operational thinking. This is especially true when these problem transformations have shifted, over the long run, from the field of thought to the perceptual field – as they usually have in chess.

A simple example to illustrate the general idea can be borrowed from Köhler's experimental adventures with anthropoids (KÖHLER 1917, transl. 1925; see also SELZ 1922). We humans are struck by the inability of these otherwise quite intelligent animals to take a ring off a nail – a possibility that we 'immediately see.' Due to our experience with nails and rings and their use, we *see* the situation in a totally different way than the ape does. Similar examples can be given touching upon the relation between adults and children.

The relation between chessmaster and weaker player is – within a somewhat more limited universe – wholly comparable. The master's inability to identify with the weaker player and his difficulties is often striking. Generally it shows up in a lack of respect; anyone who does not belong to the elite is a 'patzer.' The virtuoso cannot possibly respect as a chessplayer a person who cannot even think out in half an hour what is completely obvious to him, since he himself immediately *reads* it from the perceptual situation.

Pure means abstraction is, in itself, never more than one step with a limited scope. Generally, it derives its power from the problem conception or situation perception from which it starts. As a result of the fact that it often appears as a 'sudden flash' to the subject and/or

as a striking find or discovery to the observer, the importance of the operation has often been overdrawn – just as with the composer's inspiration (BAHLE 1936 and 1939). We should keep in mind, however, that *all* 'masterly' achievements – fascinating discoveries included – in general as in chess, are based on a body of 'mastery' that is acquired by experience (and hard work), namely, mastery over an extensive, differentiated system of immediately actualizable dispositions for typical problem transformations; or in other words: mastery over a highly composite set (program) of general and specific perceptual and thought habits (routines).

In effect, the foregoing discussion may be summarized by saying that a master is a master primarily by virtue of what he has been able to build up by experience; and this is: (a) a schooled and highly specific mode of perception, and (b) a system of reproductively available methods, *in memory*.

In using the latter term we must take care, however, to distinguish between *knowledge* and *intuitive experience*. They can both be regarded as mutually interwoven subsystems of experiential linkings (in the domain of chess) that result from learning processes and are 'located' in memory, but they differ, by definition, in that knowledge (knowing that...) can be verbalized while intuitive experience cannot. Knowledge can be explicitly formulated by the subject and thus communicated, in words, to others; it is retrievable from memory by verbal cues. Intuitive experience, on the other hand, is an intuitive know-how – as distinct from knowing that... – that is only actualized by situations (on the chessboard or in the thought process) where it can actually be used. Here, too, adequate methods are immediately available from memory but, if used, the subject could not describe them, let alone write his own heuristic program. In principle, intuitive experiential linkings may at any time become knowledge (knowing that, in addition to knowing how), namely, at that moment when the subject becomes fully aware of them.

It will be clear now that the differentiated system of thought habits (routines) which forms the essence of chess mastership, consists partly of knowledge but largely of intuitive experience.²⁰ The latter is possible

²⁰ Since one element of a large proportion of the experiential linkings in chess is *type of position* (type → operations-goals and thinking methods; or: type → board procedures) and since no chess player is able to describe his own typology (or classification) of positions in any detail, experience must of necessity remain largely 'intuitive.'

In fact, the proportions of knowledge and of intuitive experience in the

and may be highly efficient in chess because the game has in fact nothing to do with verbalization; types of positions and corresponding playing methods – moves, maneuvers – have a language of their own. In this respect the chess player's system of thought habits is basically not unlike a system of motor habits such as that of a billiard or tennis player. In fact, most *skills* depend largely on 'intuitive experience,' i.e., on a system of methods that one cannot explicitly describe.

Terms like 'intuitive experience' – and 'intuitive completion' (cf. Section 52) – imply a fairly specific conception of *intuition*. This conception needs some comment.

The term is used in widely divergent meanings. It may, for instance, stand for a deep and fundamental philosophical insight (BERGSON 1934), or it may denote a supposedly fundamental function of the human mind, along with thinking, feeling, and perceiving, namely, the function 'which tells us of future possibilities and gives us information of the atmosphere which surrounds all experience' (FORDHAM 1956, p. 35; JUNG 1933). Apart from those cases where the term stands for some philosophical idea or belief, it is used rather loosely in everyday language, e.g., in expressions like: 'I chose it by intuition,' 'I knew intuitively that it was wrong,' 'Her intuition told her not to trust that man.' 'Intuition' may also denote a quality, a talent, namely, the ability to judge or predict correctly 'on an intuitive basis.' With regard to the evaluation of social situations women are often supposed to have this quality – as opposed to manly 'reason.'

The meanings of the terms 'intuition' and 'intuitive,' as they were introduced in the preceding pages for chess thinking, are nearer to the usage of everyday life than to the more philosophical and theoretical concepts. In chess, judgments, evaluations, preferences for certain plans or moves may be 'intuitive.' The chess player's so-called feel for a position and other 'feelings that...' his hunches and his anticipations may all be 'based on intuition' or 'contain an intuitive element.' Characteristically, the implied judgments are by definition supposed to be *generally correct* or at least to be better than chance, while at the same time *the subject cannot (completely) explain his motives*; he is not able to explain how his conclusion could be justified on the available

master's system of thought habits differ per individual. Some are more able, others less able to specify verbally their methods of thought and play – quite apart from differences in scope of and differentiation within the system of methods itself (cf. Sections 23 and 59).

evidence. Also, intuitive judgment is based on more than the sum of an incomplete, explicitly rational argumentation plus a purely emotional preference: otherwise the intuitive completion of the argumentation could not be better than chance – which it presumably is.

If intuition is defined in this way, we must conclude that it is highly frequent in chess thinking – see, e.g., the discussion on the intuitive element in the argumentation (Section 52). Most conspicuous are those riveting cases where intuition points in a direction different from the other arguments: the ‘feeling that...’ may at times be stronger than substantial evidence to the contrary.

Now it appears beyond doubt that intuition of this type is primarily, if not wholly, based on what we have already called ‘*intuitive*’ experience. That is, we are dealing here with an effect of a learning process – be it learning without awareness – and not with the working of some mysteriously inexplicable, magical power of the mind which some people enjoy and others do not.²¹ There are, of course, differences: some chess players make more (efficient) use of intuitive ‘methods’ than others; but be that as it may, whatever intuitive powers a player has are based on previous learning.

Let us consider the general case in a given *situation* of an *intuitive preference* for some *method* by which the problem is to be transformed. Such a problem transformation can be said to be intuitively based, if and only if, on interrogation, it appears that:

(1) the subject is not aware that the transition is based on any specifiable experiential linking (reproductive means actualization);

(2) the subject cannot call back to mind any specific, previously encountered analogous case, from which his preference for the method may have been derived (either by cognitive means actualization or by reproductive means abstraction);

(3) the subject cannot completely justify (his preference for) the method used by referring to what the particular problem situation ‘obviously requires’; that is, spontaneous rational insight (immediate means abstraction or possibly coincidental means abstraction) is insufficient as an explanation;

²¹ The term ‘intuition,’ along with ‘inspiration,’ ‘illumination,’ and ‘insight,’ belongs to a class of ideas which tend to have a blinding halo: a halo which keeps both protagonists and antagonists from thinking clearly over the idea in question, out of devotion and scorn, respectively. The fact that many people fail to appreciate the important connection between intuition and reproductive functions results, at least in part it would seem, from the tendency to defend a lofty concept against a down to earth reduction to ‘reproductive factors.’

(4) the subject’s (preference for his) method can be regarded as objectively adequate to a degree that precludes its being explicable as due to a purely emotional preference or personal taste that just happens to fit the case.

Criteria (1) through (3) can be operationalized: (1) and (2) by specifying a method of interrogation and (3) by establishing criteria for ‘complete justification.’ Exclusion of chance and an underlying emotional, non-intuitive preference (4) is more difficult to define for *one* particular case – the main argument against chance being the frequency of correct intuitive decisions in a large number of cases. An analysis of the subject’s answers to interrogation is not, in general, sufficient here since it is difficult, if not impossible through retrospection to make reliable distinctions between intuitive and emotional preference.

Nonetheless, without the further operationalization of this definition, it should not be difficult to prove the existence of intuitively based problem transformations. Considering, on the one hand, the enormous frequency of provisionally phrased – one is tempted to write ‘intuitively phrased’ – preference statements, anticipations, and arguments in the protocols, and on the other, the prevailing tone of expert opinion among chessmasters, we must conclude that the intuitive moment is extremely important in chess.

Its *basis in previous learning* can be argued, in the first place, by exclusion: no other reasonable explanation of the phenomenon seems possible – provided that we want to avoid recourse to parapsychological or magical factors. But it can also be shown that there is no need for such a conception.²² Each of the four criteria for an intuitive problem transformation – or for an intuitive decision, for that matter – can easily be interpreted as a borderline case in a sequence of well-known cases that are of a clearly reproductive nature.

Take the following sequence, for instance: the awareness of an underlying experiential linking that may be extant in varying degrees of explicitness. *First*, the finding of a means may rest upon exact knowing: ‘In position P move Z must be played’ (due to opening theory, e.g.). *Second*, the subject may know that certain specific methods of play are effective in certain roughly specifiable types of

²² Seldom do chess players themselves cherish a magical or mystical conception of their intuition. They are the first to admit that it is based on experience. A pitched battle like the one between the rationalists and romanticists in the world of music theory (BAHLE 1936 and 1939) has never been fought in the world of chess.

positions. *Third*, the method may be known, indeed, but the subject cannot very well specify which features of the position (problem situation) have led him to apply the method; there is only a vague notion of analogy and thus an intuitive element, obviously resulting from 'intuitive experience.' *Fourth*, the knowledge about the meaning of the method itself may be vague (e.g. Subject E2: 'Every once in a while I feel like playing such a move in such a position,' (E2; A), line 10 – note, that the statement itself clearly points to the existence of an experiential linking). *Fifth*, and there is nothing revolutionary anymore about taking this last step which leads to complete fulfillment of criterion (1): the awareness that the linking is reproductively determined may be lacking as well. In that case we still assume that reproductive factors are operative. They may *no longer* be specifiable because the linking has become highly automatic; but there is no necessity whatsoever to suppose that a linking, while operative in thought organization, has ever been specifiable (conscious). On the contrary, we know that humans can very well be conditioned without awareness – and the formation of intuitive thought habits can be viewed as a special type of conditioning. In addition, as a pendant to the 'no longer' case, we must consider the 'not yet' case, namely, that of relatively new, abstract experiential linkings that are not yet specifiable since they are thought habits *in statu nascendi*.

Analogously, for a strict fulfillment of criteria (2) and (3), they can also be shown to be but borderline cases in a sequence of diminishing degrees of awareness of the underlying processes. The borderline case in (2), namely, the absence of any recollection of definite analogous previous situations – not knowing where, when, or how one learned a know-how (by experience) – is such a general phenomenon, as a matter of fact, that here the assumption of an experiential basis for intuitive operations is hardly objectionable. The assumption in (3) is, in fact, that choosing a method from intuitive preference may be a manifestation of partially explicit insight into the adequateness of the method or of insight *in statu nascendi*. On principle, means abstraction, whether with insight or intuitive, leads to the application of a *new* method; but here the reproductive factors are present in the precedent perception or problem conception, as argued above.

The conception of intuition as a product of previous learning (intuitive experience) and thus as a largely reproductive phenomenon appears to be quite tenable. We can now distinguish *intuitive means-actualization* as a special kind of means actualization (without awareness

of its origin); and *intuitive means-abstraction* as a special kind of means-abstraction (with partial insight). As we gradually increase our knowledge about the origin and growth of the system of reproductively acquired operation dispositions, called 'experience,' the problem of intuition will become less and less problematic.

Section 59: Individual differences in the system of methods

The system of thought habits, methods, reaction dispositions, operation dispositions, or whatever one prefers to call it, is in the last analysis an individual one. Accidental elements and variations in the personal experiences from which the system of Experience is built up are, understandably enough, apt to cause individual peculiarities or idiosyncracies.

Among skilled and experienced players there is, of course, much identity and analogy in the system. First, they have a large part of their theoretical knowledge in common: opening theory, endgame theory, general strategy, knowledge of position characteristics and of typical combinations, etc. Second, there is much communality of general thinking methods, in particular the more automatic problem transformations. Even when masters acquire their experience in different ways, the operative deposit is largely the same. We saw supporting evidence for this similarity in the earlier discussions of the protocols; see, for instance, the three grandmaster fragments from the first Phase of position A on page 153. Furthermore, a certain method which is stored in one chess player's memory in the form of knowledge may be present in another's in the form of intuitive experience. For playing purposes, however, such a difference is of little importance: if the situation on the board requires it, both will apply the requisite method.

In spite of this general similarity a few systematic differences can be shown to exist even among players of the same class. They appear most clearly when non-trivial, difficult problems must be solved – problems of finding means, evaluating envisaged positions, or calculating variations. To such difficult problems no immediately applicable specific transformations correspond, so the player may, to some degree, have his own approach. Since difficult problems are both more interesting and more time consuming than the simpler ones and therefore apt to be relatively prominent in the subject's reporting, the protocols provide some basis for commenting on a player's individual thinking methods. Of course, the evidence becomes

more dependable when more than one protocol of the same subject is available. One particular interest lies in the fact that a master's individual way of thinking presumably determines his playing *style*.

In the present section only stylistic differences between masters will be discussed (for class differences see Chapter VIII).

The degree to which the thought process is consciously and systematically organized is one of the most important aspects in which master subjects clearly differ. Subject G5 (Max Euwe), for instance, shows a strong tendency to order his thought process systematically, to make well defined groupings, and to construct well considered plans. This high degree of *conscious organization* is apparent throughout his entire thought process; correspondingly, the extra task of thinking aloud bothered him least (cf. Section 23). *Deductive* arguments, that is, deducing the plan from features of the position or the move from the plan, etc., occur above average while intuitive elements appear to play a minor role. From the moment an idea or plan is accepted it is worked out thoroughly and maintained as long and as consistently as possible. As a result the plan-phases in Euwe's protocols are quite long, much longer, for instance, than with Subjects M1 (Lodewijk Prins) and M5 (A. D. de Groot) whose protocols show frequent transitions from plan to plan and from move to move. This frequent shuttling may reflect a certain 'laziness' in thinking since it serves to avoid possibly superfluous calculations. We have no basis, however, on which to evaluate the relative merits of the two ways of organization: what is 'laziness' in one problem situation may be efficiency or economy in another.

Although G5 frequently employs trying out, the method is more prominent in the protocols of Subjects M2 (Nico Cortlever) and G4 (Reuben Fine). Like Dr. Euwe, these two subjects seem to make relatively little use of intuitive experience while their thought processes are also consciously organized to a high degree, but their attitude is more *inductive* than deductive; they tend to try out and calculate sample variations and to see *empirically* whether an idea has any value beyond that of grouping, systematizing and ordering. Subjects like M2 and G4 could be called 'empiricists' in thinking. Like G5 they do not rely much on intuition, but unlike G5 they do not believe in 'logical' systematization either.

Both types – if we are justified in speaking of types – belong to the group that tend to organize their thought process *actively*. Another group of subjects (players) show a more *receptive* and *intuitive* attitude

during the thought process.²² In extreme cases their protocols reflect speculation rather than directed thought. Intuitive experience is apparently the dominant force: calculations follow the lead of intuitive anticipations. Absorption instead of active concentration is the keynote, and the subject himself seems to have relatively little awareness of his own system of methods. Not surprisingly, therefore, G3 (Salo Flohr) had difficulty meeting the additional instruction to think aloud. G2 (Alexander Alekhine) expressed himself with relative ease but produced little more than a succinct abstract of his thoughts: one must assume a substantial (intuitive) completion. As to G1 (Paul Keres), most conspicuous for the brevity of his protocol, there were a few quite strong anticipatory judgments (cf. the final sentence: 'Well, White will win after 1. B × N/5') that must be intuitively based.

For the rest the material is not large enough to say much more about typology and individual methodologies. It is fairly clear, however, that the study of thinking-aloud protocols is, in general, a good way of gathering some insight into a player's methods and thereby his style. In this context three or four protocols of one subject are probably worth twenty games.

The fact that every system of experiential linkings in a specialized field – be it chess or whatever – has, ultimately, its own individual character makes it difficult to make predictions on a general theoretical basis in the psychology of thinking. In fact, the 'strict determination' of the next step in a thought process can only be maintained if we include among the predictors a number of essentially individual and subjective data. In addition to the subject's main goal with its corresponding schematic anticipation, the current stage of development of the subjective problem must be among the givens. Moreover, the subject's personal system of thinking habits – his program – must be known. Finally, whether or not the system runs smoothly, that is, whether or not the methods are easily actualizable when needed, depends on the subject's 'condition.' All this, however, does not affect the general validity of a core of mastership. It consists largely of a system of chess-specific experiential linkings that is *common to all masters*. In other words: Differences in class are much more important than differences in style.

²² This distinction corresponds roughly to the one made by Julius Bahle with regard to composers. His work type (*Arbeitstypus*) is similar to the actively organizing, his inspiration type (*Inspirationstypus*) to the receptive-intuitive thinker (BAHLE 1939, p. 340 ff.).

CHAPTER VIII

CHESS TALENT

Section 60: Mastership

The term 'master' has more or less gone out of fashion as an epithet for someone who has attained a high, generally recognized, degree of competence in a special trade or field. Though there have been a few recent attempts to restore the title of 'master' to its honored position, the concept has practically disappeared from everyday life. No longer does everyday language recognize the guild master, in the true meaning of the word; the master of healing is now called a physician, the master builder an architect. The academic degrees of Master of Arts and Master of Science retain some of the flavor of the original sense, it is true, but nowadays they are mere titles and no longer indicate real 'mastery' in a field. Again, a few derivatives preserving the old sense are still common usage: 'masterly' achievements, 'mastery' of the piano or of some specific skill, etc. The concept has to some degree been preserved in the arts where we are apt to speak of the immortal works of great masters – of 'the old master builders and painters,' in particular, since it appears that one has to be dead in order to qualify for the title. Music, more than any other field, acknowledges the living 'master': *maestro*, *maître*, *Meister*.

In chess, the term has retained its full impact: the concept still has real meaning. One knows the master by his skill; a chessmaster can hold his own against the strongest players in the world; he can partake in high level analyses with the aristocracy of chess; playing 25 simultaneous games or four blindfold simultaneous games doesn't faze him. These are the external marks of the chessmaster.

We are primarily concerned, however, with the psychological basis of the general concept, with what makes up 'mastership' in general. What are the ingredients?

Skill always appears to be characteristic: *knowledge and command of the métier, i.e., of the typical manual and intellectual techniques and methods in the field in question* – be it the field of shoemaking, painting, building, confectionery, or chess. Mastership is, above all, knowledge and

understanding. 'Understanding' means being at home in the entire field of possibilities; the master can immediately 'home' on to each specific case, i.e. recognize it as belonging to a certain type, one calling for a prescribed treatment. The expressions 'at home' and 'home on to' say clearly what it is all about: where one feels at home, one may go his own way and bank on *the system of typical operations and thought habits* that were formed through experience.

The chess master is no exception. It is true that the element of manual skill is completely lacking, but the same holds for the composer and architect. Language never bothered about this point; in fact *Denkpsychologie* repeatedly demonstrated the basic congruity between the mechanics of motor and intellectual processes. In chess one speaks of opening or endgame *technique*; in a 'won position,' for instance, the rest is 'just a matter of technique.' What is meant is playing technique, the command of playing methods (see Section 57); but in essence this is thought technique, too. What is essential is the know-how, that is, having immediate access to highly adequate, typical methods (problem transformations), whose application immediately leads to a strong specialization of the problem. Just as the master shoemaker at the sight of a wornout shoe, immediately knows which methods and materials go into repairing, just as the experienced composer has his typical methods of harmonizing and instrumentation at hand for creating certain effects, so the chessmaster is immediately able to specialize every board and analysis problem by means of specific transformations. Having fecund *experience*, as distinct from encyclopedic knowledge, this forms the nucleus of mastership.

This idea has been repeatedly expressed but it needs further support.

First, how do the protocols of the masters differ from those of the top class players? When we compare the A-protocols of the five grandmasters with those of the five E-players and calculate the means of some of the most important structural statistics (following Sections 35 and 36) we get Table 11:

Most striking in Table 11 is the *agreement* of G- and E-means. Certainly the table provides no basis for any conclusions to be drawn on systematic *differences* in thinking between grandmasters and experts. The only differences worth noting are those in duration T and 'volubility' V. In light of the striking agreement of the other means the differences between T and V seem only to indicate that the grandmaster's thought process proceeds somewhat more easily. But these differences, too, appear to be insignificant if we take into account the

TABLE 11. Means of structural statistics for five G and five E subjects in position A*

	\bar{T} time	\bar{V} volu- bility	\bar{N}	\bar{n}	\bar{n}_0	$\bar{N-n}_0$	\bar{c}	\bar{r}
5 G's	9.6	2.9	6.6	4.8	4.2	2.4	1.8	0.6
5 E's	12.8	2.3	6.4	4.4	3.4	3.0	2.0	1.0

* See Table 2, p. 117, for an overall comparison and interpretation.

spread within the G- and E-groups. In particular, the differences between the means can be traced to the influence of one extreme value for each of the two variables, namely, for (E1; A), $T = 22$ minutes and for (G5; A), $V = 4.3$.

It seems worth while to expand the comparison somewhat.

Table 12 contains for each of the 2×5 protocols the values of the following variables:

T = the amount of time (in minutes) used for the decision process;

N = the number of fresh starts within the protocol;

n_0 = the number of different first moves considered (after the first Phase);

D_{\max} = the maximal depth to which the subject has calculated any variation in the protocol, expressed in 'half' moves (that is, $1. B \times N/5$, $N \times B$; $2. N \times N$ gives $D = 3$);

M = the total number of both Black and White move possibilities mentioned, be they first moves or later ones;

$m = M/T$ = the number of move possibilities mentioned per minute, on the average;

$n^* = n_0/T$ = the number of different first moves considered per minute, on the average;

R = the value of the move made as rated according to the method discussed in Section 8.

The variables D_{\max} , M , m , and n^* have been included because of the popular conception that a master is primarily distinguishable from others because he calculates deeper and/or more moves.

The rows of new variables, (4) through (7), paint the same kind of picture as Table 11: the differences between the means are not great and for each of the variables there is a substantial overlap. The values

in rows (6) and (7) appear to confirm somewhat the impression that the thought process is generally a bit faster among grandmasters; they mention more moves and consider more different solving propositions per minute. Noteworthy is the close agreement among the values for D_{\max} . It is thus beyond doubt that the depth of calculation cannot be the prime distinguishing characteristic between the grandmaster and the expert player.

TABLE 12. Variables of A-protocols for five G and five E subjects¹

Subject	G1	G2	G3	G4	G5	E1	E2	E3	E4	E5	(\bar{G})	(\bar{E})
(1) T	6	9	10	8	15	22	12	16	7	7	9.6	12.8
(2) N	3	6	4	7	13	12	4	7	6	3	6.6	6.4
(3) n_0	3	4	3	5	6	6	3	3	3	2	4.2	3.4
(4) D_{\max}	5	5	7	10	7	10	4	3	7	9	6.8	6.6
(5) M	22	20	21	36	76	61	16	29	31	17	35.0	30.8
(6) m	3.7	2.3	2.1	4.5	5.1	2.8	1.3	1.8	4.4	2.4	3.5	2.5
(7) n^*	0.50	0.44	0.30	0.62	0.40	0.27	0.25	0.19	0.43	0.29	0.49	0.29
(8) R	9	9	7	9	9	5	6	5	5	5	8.2	5.2

We can express the results so far obtained as follows: *It is not generally possible to distinguish the protocol of a grandmaster from the protocol of an expert player solely on structural and/or formal grounds.* In view of the large difference in playing strength we must assume that differences in thinking exist between the two groups. But these are evidently of a qualitative nature; at least they do not or hardly show up in the structural protocol statistics considered.

The bottom row (8) of Table 12 reminds us that substantial differences in thinking do exist. The gap between the levels of performance of the G- and E-group is enormous: four of the five G subjects would almost certainly have won the game; even the fifth would have had a better chance than the E-players who without exception let

¹ Some of the entries in the table, in particular those of variables like N , D_{\max} , and M , are disputable since they depend on the decision of exactly where the main part of the process is supposed to begin and on other somewhat subjective protocol interpretations. This hardly detracts from the dependability of the overall picture however.

their opportunity go down the drain. Position A would not be a bad test for chess mastership!

If this striking difference in class is not rooted in tangible, quantitatively computable properties of the actual thought process, on what is it based? We have already answered this question: on the fast and efficient problem formation and -specialization which derives from the (grand)master's 'experience.' He immediately knows what it is all about, in which direction he must search; he immediately 'sees' the core of the problem in the position, whereas the expert player finds it with difficulty – or misses it completely as in the five (E; A) protocols. The master does not necessarily calculate deeper, but the variations that he does calculate are much more to the point; he sizes up positions more easily and, especially, more accurately.

Of course, other differences must not be minimized. If need be, it is certainly easier for the master to calculate to a depth of five, six, or seven moves, to analyze a certain situation, systematically to work out an intricate plan, or even to digest multibranched networks of variations. All of the methods and operations found in the ordinary thought processes of seasoned players must be assumed to run more smoothly and easily in the master, to be on a much higher level, and to have a larger scope – but these are all differences of degree which cannot possibly explain the large difference in performance. The visible formal characteristics (variables, statistics) of the working thought process cannot reflect what is basic to mastership, namely, the system of experiential linkings that has been built up over the years. In the master, this system is much wider in scope, worth more, and more highly differentiated than in the expert. On the experiential system hangs the quality of the thought process and of its product: the chosen move.

These data provide admittedly indirect but yet unambiguous support for our concept of mastership. In the next section one last experimentally based and more direct argument will be proffered, one which seems to be decisive.

In closing, a few words about the special feats characteristic of the master. On page 316 we mentioned competitive strength, skill in analysis, simultaneous and blindfold simultaneous exhibitions. We might add the special feats of chess memory, for instance, writing down or indicating all of the twenty games played in a simultaneous exhibition the night before. One master is certainly better in these things than another, he may either be more gifted in such things or

more attracted to them or apply himself more. In principle, however, these feats are nothing out of the ordinary for any master.

Aside from a few really extraordinary achievements (say 30 or more blind games simultaneously) most of these feats can generally be explained by our concept of mastership – or connoisseurship. For the most part this has been expounded earlier (DE GROOT 1938). The gist of the argument is that a chess position, and, a fortiori, an entire game are *typical* to the master. A chess position is easily recognized as one belonging to a certain class, that can be handled in a certain specific way.

It is the presence of a largely implicit, differentiated system of classification principles – with corresponding experiential linkings – which enables the chessmaster to retain easily what he has seen and played and thus to play several games simultaneously.

The Russian investigators (DJAKOW, RUDIK and PETROWSKY 1927) were correct, therefore, when they concluded (even though from insufficient experimental evidence) that chess memory is equivalent to a specifically developed professional memory in other fields and that it is based on 'nothing more than' routine and experience. But – and that is exactly the dangerous error in thinking which one may make along with Binet and the Russians – this 'experience' is not the obvious, not the ordinary thing that can be taken for granted, but precisely the most fundamental and distinguishing hallmark of the master. The very fact that he has managed to build up such an extensive and finely differentiated system of fecund experience, that he has become so extraordinarily skilled, is the pristine proof for his 'masterly' disposition.

Thus we see that of Binet's three fundamental conditions for blind play (*érudition, mémoire, imagination*; see Section 2) the first one, experience as we have called it, heads the list in more ways than one.

Section 61: The first seconds: the perception of a new position

If it is correct that the most important difference between master and non-master lies in the differentiation and scope of the system of experiential linkings and if consequently the master can start thinking from a higher level, then this class difference should come out clearly in the first minutes, nay seconds, of the perceptual and thought process. Indeed, we have been able to confirm this more than once, for example, when discussing the interruption protocols, as in (G5; B), Section 45, p. 200. If this conception of mastership is correct, the

master should distinguish himself during *the perceptual process*. To supplement the thinking aloud experiments and as a test of this conception of mastership, a series of experiments in perception were carried out in 1944.

The Russians (DJAKOW, RUDIK and PETROWSKI 1927) had already done something similar. They presented a chess problem to the masters (mate in three); after one minute the pieces were jumbled and the subject had to try to reconstruct the position. From our point of view, however, the experiment contains a number of deficiencies in method that must be avoided here. These deficiencies are:

1. Only one position was shown. Therefore the subject did not have the chance of getting used to the conditions of the experiment.

2. Because the chosen chess problem position was extremely improbable and atypical from a player's point of view, the 'experience' of the masters was not given a fair chance. This weak point was admitted by the writers themselves (*Ibid.*, p. 40).

3. The one minute exposure time was much too long. Even the short exposure times used in the 1944 experiments (ranging from 2 to 15 seconds) permitted much more than perception and retention alone. It is impossible to keep tabs on other ongoing activities, which undoubtedly differ from subject to subject.

4. No introspection or commentary was requested. This could have compensated for the objection made under 3.

5. The control group consisted of non-chess players (*Massen-experimente von Nichtschachspielern*, *Ibid.*, p. 41). It is therefore not surprising that the masters scored three times as high: top class players would have too. A reasonable familiarity with chess was all that was needed. To this extent the subjects came up with nothing typically 'masterly.'

6. The interpretation and computations of the experiment are open to criticism. Several criticisms will be mentioned later.

We must, of course, remember that the Russian investigators had a different goal in mind. They wanted to find an experimental answer to the question of the striking feats of *memory* of the chessmaster: are or are not these feats based on a general retentive power, a general strength of their visual memory? Our goal, however, was to test a specific hypothesis on mastership.

To this end, Nico Cortlever, upon the author's instructions, kindly made up a series of 16 diverse positions, picked more or less randomly from relatively obscure actual master games. Each position had a

prescribed exposure time, varying from two to ten seconds and in one case as high as fifteen. For the weaker subjects the shortest exposure times were prolonged somewhat, up to three to four seconds, in order to avoid zero-achievements.

As to subjects, a small number seemed sufficient: one representative of each 'class.' Dr. Max Euwe represented the grandmasters; the author considered himself a weak representative of the master class; Mr. A. Fast, 1942 champion of Utrecht, spoke for the expert class; and Mr. G. P. Hauer for the weaker class players. Needless to say, it would have been better to work with more subjects from each class; but the differences found, especially those between master and non-master, are so large and unambiguous that they hardly need further support. The four subjects in this experiment will be designated by plain G, M, E, and C, respectively.

In the instruction for the experiment systematic introspection was requested, and it was suggested that for about half a minute before speaking the subjects organize whatever they could retain. This was to prevent them from losing the thread and forgetting the position while talking.² At the close of the introspective report G and M dictated the position from memory. E and C found this procedure too difficult. After marshaling their thoughts they reproduced the position by setting up the pieces on the board at the same time that they were giving their introspective commentary.

A special method of *scoring* was introduced in order to express the results numerically. In general the following points were assigned: *one point* for each correctly placed and identified piece (therefore the maximum number of points attainable = the number of pieces); *one point subtracted* for misplacing, adding, or omitting a piece (with the following modifications); *one point subtracted* for interchanging the placement of two pieces; *one point subtracted* for shifting one or two pieces over one file (from the Queen file to the King file, e.g.); *two points subtracted* for shifting three or more pieces, e.g., a whole wing or file; *one point subtracted* for uncertainty about correctly placed pieces, whenever there are at least three in question; *one bonus point* for every correctly remembered spatial relation (e.g.,

² The details and the wording of the instruction were developed in a few preliminary experiments with the author as a subject. Mrs. de Groot volunteered as experimenter throughout the M-series; contamination was, of course, avoided.

a Knight's jump distance) between two or more pieces incorrectly reported or unremembered;
two bonus points for correctly reporting the material balance of a poorly remembered position;
one bonus point for almost correctly reporting the material balance of a poorly remembered position.

This scheme is adapted to the most frequent reproductive errors. It does not cover all possible complications nor is it completely objective, but it appeared to be adequate for the purposes of the investigation. The following protocols and diagrams of position VI of the series illustrate the method of scoring. They are fairly representative of the entire material and clearly reveal the sharp classwise decline in achievement.

PROTOCOL (G; VI): EXPOSURE TIME: 5 SECONDS

First impression: awfully rotten position, strong compressed attack by White. Then: How much more material have I got to compensate for this? In counting I find two pieces. This, so to say, answers the question, 'What can I sacrifice?' 1...R×N deserves consideration, the Knight's the most dangerous. Now, afterwards, I believe that it's won for Black, but just a minute ago I wouldn't have dared to say so.

The order in which I saw the pieces was about King on K1, Knight on Q2, White Queen on QB3, Queen on K2, Pawns on K3 and his on K4, White Rook on Q8, White Knight on QN4, Rook on QN5 – that funny Rook that doesn't do anything – Knight on KB2, Bishop on KB1, Rook on KR1, Pawns on KR2 and KN3. I didn't look at the other side very much, but I presume there is another Pawn on QR2. The rest for White: King on KN8, Rook on KB8, Pawns on KB7, KN7, KR7, and QR7, QN7.

(While calling off the pieces he was continually reasoning; e.g.: 'Another piece is on KB2 – the King was completely closed in – that must be a Knight then.')

E: Does the position directly remind you of a position you know?

S: There's a vague recollection of a Fine-Flohr game in the back of my mind.

E: Is the position of a certain type?

S: No, I would not say so. Well, naturally I have a certain feeling of being familiar with this sort of situations.

E: Is it something of a stereotype then?

S: Only partly; on closer inspection there's always an individual character.

E: You called the pieces off one by one. Did you perceive them individually or in complexes?

S: Always in complexes. Here, for instance, that entire cramped King's position as one whole: King on K1, Knight on Q2, White Queen on QB3, Queen on K2, Pawns on K3 and his on K4, Bishop on KB1 at least.

PROTOCOL (M; VI):

The King's position struck me immediately, Knight on Q2 and the White Queen on QB3. Compressed position. Queen on K2 and Pawn on K3. Clearly a defensive position. A tangle. White is highly developed. First saw the Black King's wing and from there the White King's wing. Critical situation. Holy smoke: what an attack that guy has! White Rook on Q8, Rook on QN5. Then I felt the need to check material. I was doing that when I had to stop. Black has two more minor pieces. Does White have a Rook for them? Only later did I realize that the Rooks on QN5 and KR1 make two. I didn't come to an evaluation. The need to check material arose from the need to evaluate the chances. My original notion: two pieces against a Rook, I thought Black's position precarious; after stopping I realized that Black still had both Rooks. Conclusion now: Black will probably have a won position.

First I thought: critical; White has a deadly attack, but it's difficult to be certain about it. The Black King can't move. I'd like to have him to the left – this was almost the first thing I thought when the position was shown. That tangle is rather normal, only is there really a Black Knight on KB2? That would be rather curious.³ I saw the White position in one glance. King, Rooks, and Pawns. Normally and quickly developed, there have been sacrifices in the attack. White Queen can check on my first rank, QB1. That rather frightened me.

The tangle evokes memories of a game, namely, Morphy's in the opera box against Count what's his name. In fact, that memory popped up quite in the beginning, when I had seen the knot and White Rook on Q8 and Queen on K2, White Queen on QB3, and White Knight on QN4 – that is, the combined action on diagonal and Queen's file. I thought, I'm checkmated with sacrifices.

Later on: Is it that bad? ... Counting ... Black's ahead – (The position was called off correctly with the exception of an extra White Pawn on QB2.)

I have looked at one move, namely, N/B×P, with the idea of making room for the King.

PROTOCOL (E; VI):

(Subject sets up the pieces in the following order):

Black: King on K1, Queen on K2 flanked by two Knights, Pawn on K3.

White: Queen on QB3, Knight on QN4, Rook on Q8, Rook on KB8, King on KN8, Pawns on KR7, KN7, and KB6; further, Pawns on QR7 and QN7. Further Black: Rook on QN1, Pawns on QR2 and QN3, Rook on KR1, and Pawns on KR2 and KN2. Then there was a Black Bishop I believe, but where? On KB3? No, I can't place it. Probably wasn't there.

Further White: Bishop on K5? in addition. I had the idea that it was materially equal. But I don't remember the Bishop.

It was an attacking game for White, I had the idea. White's better. Of the dynamics I saw especially that the White Queen on my QB3 is pinning my Knight on Q2, and the White Knight on QN4 can give check; threats. (In answer to a question): I didn't see so much of the activity of the White Rook along the Queen file.

³ Subject M does not doubt his memory here but expresses some distrust in the experimenter who set up the pieces!

PROTOCOL (G; VI):

Strikingly close are the Black pieces stupidly packed around the Black King. White has one or two minor pieces less. The Black King has no elbow room.

5 Both sides have castled short. White Queen is deep in the Black position, and his Rooks stand nicely on open files. White has only one minor piece. Black three. But maybe there is a possibility for an obstruction combination; at any rate there's some compensation.

10 (Meanwhile the subject has been

setting up the pieces):

White: King on KN8, Queen on QB3 (or QB2?), Pawns on KR7, KN7, KB7, Rooks on K8 and Q8. Knight?

Black: King on KN1, Rook on QR1, Bishop on KN2, Knights on K1 and KB2. Something like that. Pawns in front.

E: What about the dynamics of the position?

S: I did see something of a Queen threat on the first rank.

First we turn to the numerical results of the subjects. The maximum number of points possible is 22 because there are 22 pieces (the same as in the Russian experiment; there, however, the average result of the masters was much worse even though the exposure time was longer). G reproduces the position correctly and receives 22 points, that is, 100%; the 'defect percentage' is thus 0%. M adds one (rather unimportant) Pawn to the position: 21 points = 95%, defect = 5%. The picture for E is quite different: 17 pieces are correctly placed, five incorrectly. He thinks the material equal whereas precisely the inequality is one of the most important characteristics of the position. For this, one point was subtracted: E misrepresents the entire character of the position. The result is 16 points = 73%; defect = 27%. Even more marked is the difference between E and C. Subject C did see more pieces than he set up (see diagram); e.g., he more or less correctly reported the material relationships but he was at a complete loss to place the pieces ('Knight?' 'Queen on QB3 or QB2!'). Seven points for correctly set up pieces plus two points for material relationship = nine points; 41%. The diagrams below speak for themselves.

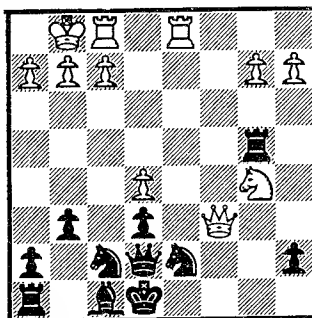


Diagram VI
Black on move

Result for Subject G: 22 points

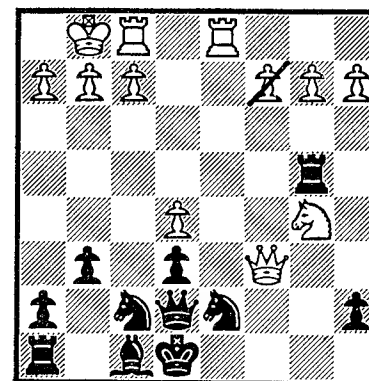


Diagram VI

Result for Subject M: 21 points

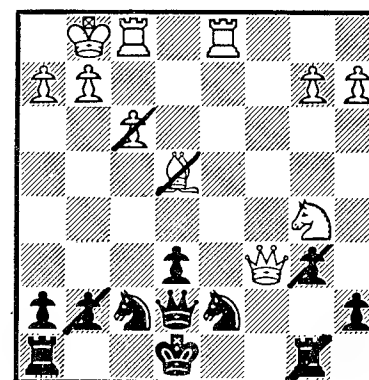


Diagram VI

Result for Subject E: 16 points

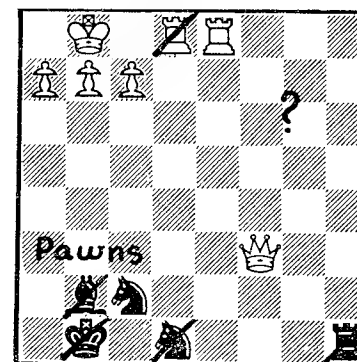


Diagram VI

Result for Subject C: 9 points

Tabulated Results

Position VI: taken from a game Rohacek-Rabar (after the 20th move by White).

	Points	% correct	defect %
Maximum	22	100	0
Subject G	22	100	0
Subject M	21	95	5
Subject E	16	73	27
Subject C	9	41	59

Let us now look at the results of all fourteen experiments (position I was eliminated because it was the first in the series, and position VII because the exposure time was once faulty). These results are summarized in Table 13. In the first column the total number of points achieved in all 14 positions is given; in the second the percentage earned of the possible points; in the third the mean correct percentage score of the 14 positions taken together; in the fourth the mean defect percentage score; the last column contains the number of positions perfectly reported.

TABLE 13. *Numerical results of the perceptual experiments*

	Total points	Total % correct	Mean % correct	Mean defect %	Perfect reproductions
Maximum	234	100	100	0	14
Subject G	217	93	93.4	6.6	5
Subject M	217	93	91.4	8.6	4
Subject E	158	68	69.6	30.4	0
Subject C	119	51	52.5	47.5	0

The scores of G and M do not differ much from each other, but there is a wide gulf between M and E: the gulf that separates the master from the non-master. It makes an enormous difference whether the defect percentage is 9% or 30% and these are averages at that!

If we select a bit more critically and eliminate, first, the two experiments with relatively long exposure times (15 and 7 seconds) and, second, the two cases where Subjects M and E, respectively, failed in their reproductive processes due to some disturbance in concentration – something that can always occur with such short exposure times – a more homogeneous series results, consisting of ten

sets of reports. Within these ten the maximum number of points varies between 9 and 26 (in the series of 14 between 7 and 26 points). Ideally, the exposure times were two seconds (four times), three seconds (thrice), and five seconds (thrice), but for E the time was somewhat prolonged in five positions and for C in six. Strictly speaking these subjects had a slight advantage, but even so the difference between classes tells:

TABLE 14. *Results of the homogeneous series of ten selected positions*

	Mean time (seconds)	Total points	Total % correct	Mean % correct	Mean defect %	Perfect reproductions
Maximum	–	170	100	100	0	10
Subject G	3.2	156	92	93.0	7.0	4
Subject M	3.2	157	92	92.6	7.4	2
Subject E	3.65	121	71	72.3	27.7	0
Subject C	3.85	80	47	51.1	48.9	0

As a result of the elimination of the two positions in which M and E were disturbed, M and E make a somewhat better showing. Nonetheless the differences in class are just as clear as in Table 13.

The entire material consisting of 64 protocols – introspective reports – contains a fund of interesting and important data, which can only be briefly treated here.

From this material it is evident that 'experience' is the foundation of the superior achievements of the masters. It is to be noted that the comparison is with a top class player (ex-champion of Utrecht) who certainly has a reasonable familiarity with the chessboard and pieces. The master's experience enables him to quickly 'integrate' the picture of the position and through this to imprint and retain it within a very short period. The protocols of G and M, in particular, we shall now examine for cues and try to find out how the achievement is brought off.

How does the *perceptual process* run?

The position is perceived in *large complexes*, each of which hangs together as a genetic, functional, and/or dynamic unit. For the master these complexes are of a typical nature. Such a complex – a castled position, a Pawn structure, a number of co-operating pieces ('functional complex'), an interrelated tangle of pieces as in Position VI (see

protocols G and M above; this might be called a 'dynamic complex'), or Rooks on the seventh rank boxing in the King, etc. – is to be considered as a unit of perception *and* significance.

Very often the protocols bear witness to a kind of *scanning of the board*. In this connection eye movements undoubtedly come into play. One can often plot the course followed on the board: first one's own King position, then the build up for an attack, and finally the rest. The position is often perceived in three or four parts.

A *separate process* is sometimes needed to integrate them. Interesting in this respect is (M; XV) where this integration process did not come about. The parts remained separate, the spatial and functional relations between the left flank, on the one hand, and the center plus the seventh rank position on the other, were not perceived during the exposure time. On reproducing the position the subject realizes: 'Hey! Are those Pawns so close to the Black Queen?'

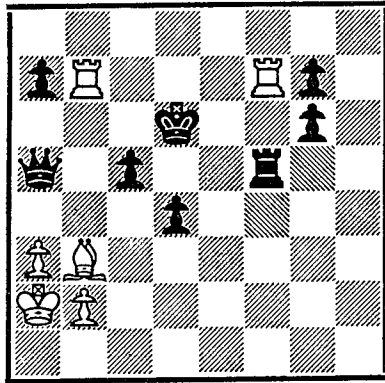


Diagram XV

White on move

PROTOCOL (M; XV): EXPOSURE TIME: 3 SECONDS

Only took in the position. Hardly had enough time for that.

(Pause to imprint the position and organize his thoughts).

- 5 In the center, where I again began, was nothing. Neither on the right, so I went to the left. There I saw the complex around the White King with the Black Queen, threatening him from a distance.
- 10 The King is quite safe for that matter. I didn't find a White Queen but I did find

two White Rooks that had penetrated the position. White's position is not so crazy I then thought. While perceiving the White Rooks on the seventh rank, took in the Black Pawns on QR7 and KN7 at the same time. I was let down when I saw that the Black Rook in KB5 threatened to exchange one White Rook. In looking at the possibilities of the Rooks on the seventh rank – giving check on Q7 and such – I actually got

15

20

- 25 stuck. Not until afterwards did I clearly realize the material balance, so during the three seconds I did not form a definite evaluation of the position. Not until after the Rook on KB5 did the Black King have its turn, which must be on Q6. Meanwhile I had gotten more or less hold of the center position (Black Pawns).
- 30

[(Position called off correctly although there was at first some uncertainty about the presence of the Bishop on QN3. Subject knew that two connected Black Pawns stood in the center and called them off correctly; only then did he realize:)]

35

Hey! Are those Pawns so close to the Black Queen?

40

The different parts of a position claim the attention in varying degrees. Some parts are only seen in passing – one knows almost automatically what that part is – while others for some reason are conspicuous; they attract one's attention and keep it for a while. *Unusual characteristics of a position* (an exposed piece, a far advanced Pawn, a battery of heavy pieces, queer doubled Pawns) stand out against a typical background, claim the attention – and consequently are the easiest to remember.

Further, the *essential relations between the pieces*, their mobility and capturing possibilities, their co-operation or opposition, are often perceived and retained better than the position of the pieces themselves. Thus G in (G; VI) knew (1) that the Black King was completely closed in, (2) that three minor Black Pieces were there, (3) that the Queen was on K2, a Bishop on B1, a Knight on Q2; so there must be something on B2 and: 'that must be a Knight then' (the third minor piece).

(M; VII) is even more interesting. Subject M did not notice that Black was a piece for a Pawn down – due to the experimenter's error in setting up the pieces! Thus subject M tries to place a Black Bishop (two Knights are already in position). 'A piece down is impossible' – indeed this position could not very well occur in a serious game; Black would resign outright. 'Black's white Bishop had been exchanged on KB3 (deduced from the position). So there must still be a black one. But where? Not on his QN5 or QR4 because I would have seen that after 1. N-B3 the Knight can be exchanged. Not on QN3 or QR2, because my Pawn on Q4 was only attacked once. On K2? No, because the Queen could go there. On KB1? Not likely either because Black could castle, I think... It's also possible that it isn't there at all.' The facts that the Knight on QB3 can not be taken, that the Queen Pawn is not attacked, that Black can castle, and that his Queen can go to K2 are evidently better known to M than whether or not there is a black Bishop. The entire perception occurs in fundamental relations and possibilities and is in a sense 'dynamic.'

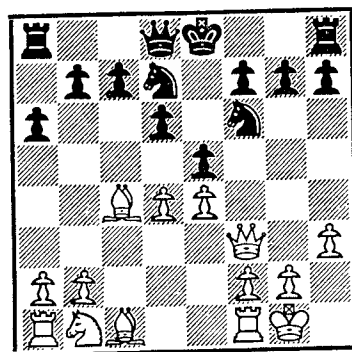


Diagram VII

White on move

PROTOCOL (M; VII): EXPOSURE TIME: 7 SECONDS

The position isn't right.⁴ Should there be a Pawn on QB3?

E: No.

S: White's a Pawn down. First: center position seen, that looks good. Bishop on QB4 trained on KB7, Black Knights are in the way. White has more room, a positional advantage which might make up for being a Pawn down. No, it doesn't ... Probably I'd rather be Black, but still it's nice to play with White.

Order: first the center, then the Bishop aimed at KB7 and White Queen, and the Black Knights (defending K4) and the Rook on the Black Queen's file. I felt inclined to play the move N-B3. The whole position reminds me of the Ruy Lopez, probably because of the position of the Queen on KB3. Haven't been doing anything but scanning the position: I know the position except for one uncertainty: Is the Bishop on KB1 or K2? It's also possible that it isn't there at all. I haven't seen it but both sides are equal of course. A piece down is impossible. I had a feeling that Black was ready for castling. He just can't be a piece down, so Black must have that Bishop somewhere. Where? God only knows.

Perception was in bits and pieces. The

thing immediately reminded me of all sorts of things. Undeveloped Queen's wing together with the Bishop on QB4 have all sorts of pleasant associations; I can develop immediately. Never got to looking at a move. Except for the obvious N-B3 – only looked out for the center.

Position was called off correctly except for the Black King Rook Pawn; put on KR3 instead of KR2. (Eyes opened)

Subject M now reconstructed his reasoning process to make more explicit the difficulties he was having with the Black Bishop:

White has three pieces. From that the conclusion was drawn that Black must also have three pieces (in an opening position of this sort). Black's white Bishop has been exchanged on KB3. So there must be still a black one. But where? Not on his QN5 or QR4, because I would have seen that after N-B3 the Knight can be exchanged. Not on the QR2-KB7 diagonal either because my Pawn on Q4 was only attacked once. On K2? No, the Queen had a free diagonal. On KB1? No, because Black could castle. From this ensued S's puzzlement concerning the position of the Bishop.⁴

⁴ In fact, the Black Pawn on Q3 should have been a Black Bishop!

Even in these short processes the pieces are seen in terms of their functions, that is, as centers of possible actions – in the same vein as Binet's 'blind' subjects reported on how they envisaged the pieces (BINET 1894). When reproducing the positions *relations* are continually mentioned, not only those of a spatial nature ('Queen on K2 flanked by two Knights' (E; VI) but also those of a functional and dynamic nature (pins, capturing possibilities, mobility, control of squares, etc.).

Possible *own-moves* or even *deeper continuations* are often seen during the first five, or even two or three seconds. They pop up of their own accord, as it were; for instance the position calls for 'more pieces in the attack,' or 'a King's march to QB4.' The things seen in this way are almost always essential. In position VI, with an exposure time of five seconds, both Subjects M and G saw one move; the one mentioned by M was played in the actual game, the one mentioned by G was the only winning move!

Apart from the relations and possibilities for action mentioned above, there are other data from which it appears that *perception* often is *primarily 'dynamic.'* M noticed more than once that certain unoccupied but 'critical' squares on the board were perceived in a different way from the others. A square on a half open file where an outpost can be stationed in a single move, a square that is controlled by pieces of both sides, in an endgame position a square in front of an isolated Pawn which cries out for the King: all such squares are specially accentuated in perception. Finally, remarkable support for dynamic perception comes from the errors in reproducing the position: a piece is often put on the square which it wants to be on or on a square that an enemy or another own piece disputes (points of intersection).

Finally, a few words on *the integration of the position*. It consists essentially of taking stock of the spatial, functional, and dynamic relations among the perceived parts – so that they can be combined into one whole. Ordinarily this process occurs 'automatically' and can rarely be charted. We can only consider the integration process complete when the subject has formed a first real judgment of the position – which is more than an unverified impression – when the pros and cons have been weighed. Indeed, only G and M generally got so far, even in the middle game positions with twenty and more pieces and after two or three seconds exposure.⁵ Subject C never

⁵ Already in the first seconds we find all three moments of the first Phase: the static, dynamic, and evaluative moments (cf. Section 40).

achieved a decent integration and subject E only in a few of the experiments – when the positions were simple and the time relatively long.

So much for the particulars of the perceptual process. However, the crucial question remains: Is it really ‘experience’ that makes the achievements of the master so much better?

This question is not difficult to answer. There are a host of obviously *reproductive factors* at work and they are of decisive importance. It is only possible to perceive relatively large complexes as units or wholes because they *are* typical wholes to the perceiver: in origin, function, significance, value and/or prescribed treatment. For instance, one recognizes the opening type from the whole array of pieces, one sees which maneuvers have been carried out from whole wings, one knows immediately what is going on and what should, in principle, happen from typical configurations of pieces – known by experience. It is *because of* the typical nature of the ‘historical,’ functional and/or dynamic relationships that larger units can be perceived as such and thus, that the subject is able in such a short time span to take in the complete position. The protocol text itself often bears witness to the importance of the reproductive aspects: specific knowledge is actualized, typical methods of play are mentioned, typical formations are recognized, memories of more or less analogous situations pop up as evidence of the subject’s implicit classification of positions. Concrete memories are relatively rare, for that matter: the typical prevails over the incidental.

As in the main series, the task assigned to our subjects in the perception experiments was not taken from everyday chess playing. The ‘first Phase’ of perceiving and thinking in a new position is not natural for the subject. The experimental analysis and particularly the finding of such a large difference between master and non-master in perceptual achievement has proved, however, to be extremely enlightening. In evaluating its importance we must realize that it is not only the perception of the position on the board that matters, but every position envisaged by the player in his investigations as well! Here, too, the master has, again and again, the advantage of seeing the essentials faster and evaluating the situations more correctly – on the basis of his experience.

In Section 60 we found, somewhat overstated: M does not *calculate* more than E; in the present section we find: M *sees* more than E, especially the more important things.

Section 62: *Specific traits of chess thinking*

In what aspects does chess thinking differ from non-chess thinking? Does the mentality needed for chess mastership extend to other fields? Can we define specific characteristics, possibly specific methods of thinking, that are peculiar to chess?

In attempting to answer these questions, we can decide right off that *chess thinking is typically non-verbal*. The chess player is concerned with moves on the board, with movements and maneuvers, with spatial relationships, and with the dynamics of captures, threats, and control – all of which can be objects of perception, imagination, and thought, without any dependence on verbal formulations and concepts. It is true that spoken and written language play an important part in learning to play chess and, of course in communication, but for the essence of the game language is of secondary importance. Illiterates and deaf-mutes can learn to play chess; strong ‘natural players’⁶ who never studied any theory still exist. Chess players can communicate very well without benefit of a common tongue: the moves themselves – like the notes as they sound in music – much more adequately than any verbalization express the underlying ideas and mental processes. A game of chess may in itself be viewed as a dialogue, a sharp discussion in moves, not words. In fact, this dialogue can almost be subsumed under current definitions of ‘language’⁷ except that the purpose is not ‘mutual understanding’ but mental competition in a specific field.

Thus chess thinking is non-verbal thinking and especially *thinking in terms of spatial relationships and possibilities for movement*. This is one reason for the particular *importance of intuitive experience* in the field of chess. One need never state why a particular action is undertaken or why a move is played, since the actions on the board are essentially self-explanatory. It is not necessary to formulate playing methods provided that one has them ‘intuitively’ available. The chess player’s experience need not be explicit knowledge – although in the long run ‘experience’ generally tends to become knowledge, particularly

6 In principle, a grandmaster could be an illiterate – as happens to have been the case in the game of *dammen* (Continental Checkers) with the great Senegalese Negro player, Baba Sy.

7 Cf. RÉVÉSZ (1940, p. 1081): ‘Language is the function that enables us by means of a number of articulated and variously combinable sound or movement- or symbol-structures (‘words’ in a generalized sense) to express our perceptions, thoughts, judgments, wishes and to inform others about them for the purpose of mutual understanding.’

when specific circumstances (such as writing a chess column, giving lessons, exchanging ideas with colleagues) induce him to give an account of his own thought habits.

Other specific features of chess thinking follow from the necessity for the player to calculate mentally and in advance what may happen: if... and if then..., then.... It is true that the master more conspicuously distinguishes himself by his extremely adequate selection of what to investigate than by his depth of calculation, but the fact remains that the choice of a good move rests largely on *foreseen possibilities* for action and on the evaluation of their *foreseen results*.

In principle, these calculations can always be considered instances of the general method of trying out. The extraordinary importance of this thought method is certainly a basic characteristic of chess thinking. For this reason we have repeatedly pictured the thought process as an *empirical thought investigation*: 'empirical' since it is by trying out and by (mental) experimentation that one seeks to approach the truth; again 'empirical' – as well as inductive – because there is no a priori or deductive way to establish with certainty how much can be attained and what can be proved. Here the contrast with solving a mathematics problem is striking.

Correspondingly, there is a *striking resemblance to processes of 'real' empirical research*, where processes of thinking and testing reality are interwoven. Especially, if the research goal is a practical one: if the decisions to be taken (moves to be made) depend on expected quantitative results, the parallel can be stretched a long way. First, the progressive deepening of the investigation: ideas recur more than once; solution proposals are tested with increasing thoroughness and are finally compared and weighed against each other. Just such a process is found in the development of scientific research: indeed, the 'subject' may shuttle back and forth between plans *A* and *B*. If, for instance, the goal is to improve the durability of some material, this may be achieved in more ways than one. First, the researcher is likely to do some small scale experimentation with one of the available methods (the provisional favorite). If this proves unsuccessful he will try out a second method – maybe a modification of the first – fitting into the same general framework. If he again fails to attain immediate success, the researcher may return to the first method which will be tried out more thoroughly this time. There is 'progressive deepening,' indeed. Perhaps 'sample variations' and an explicit 'striving for elimination' of certain means may occur; finally, the researcher will probably try to prove scientifically that his 'favorite'

method is the best one. Phases of specialized, detailed investigations can be discriminated from periods in which the investigator returns to a more general goal, or checks and recapitulates his partial results, or analyzes the problem and weighs the pros and cons in a process of 'dialectical deepening.' In short: There occur pronounced 'transitional phases' with all the characteristics we know from chess thinking.

A further important point of resemblance is *the decisiveness of the quantitative moment*. The goal remains throughout to improve the durability of the material, but there is no a priori, objectively fixed limit to the amount or degree of improvement. The 'expectancy interval' the researcher has in his mind will change, and, particularly, get smaller (become more precise and realistic) during the empirical investigation.

Apart from research other *processes of rational choice* parallel the chess player's process provided that the decision is based on rational estimates or experimental determination of various quantitative consequences of a number of given alternatives. This obtains, for example, for certain decision processes in modern management.

Furthermore, *the function of a plan* in chess is no less than the planning procedures in industrial and commercial management or in economics in general – where the final goal is at least as quantitative as in chess.⁸ A plan is a framework for guiding future actions which will come about in the form of separate decisions or steps (moves); it is based on the features of the present situation, one into which a better insight can be obtained through the 'investigation of possibilities' (research, market analysis, etc.). Typically, the planned strategy is never certain to be right: it is formed on the basis of an insufficient set of data, permitting it to be changed in mid-stream.

Here we hit upon another important characteristic of chess thinking: decisions are based on *necessarily incomplete evidence*. Nearly every argumentation is incomplete: it does not generally provide certainties, but at best a high likelihood that the choice is a good one – or the best possible. There is room for 'intuitive completion'; in fact, there is a strong need for this method to enable the subject to build up the subjective certainty he requires for actual decisions. Both the subject's uncertainty (cf. p. 199: the frequency of 'probably,' 'maybe,' etc.)

⁸ The possible parallels are legion. Nowadays, many programmers who play chess or chess players who program report a pronounced analogy and even mutual fertilizations between the two mental activities – in spite of the fact that programming goals are, in general, precisely predetermined. It appears, in fact, that describing thinking in chess is to a large extent describing human thinking in general.

and his intuitive completion (cf. Sections 43 and 58) are clearly reflected in the protocols.

By pulling together some of these characteristics of chess thinking, to wit, the *empirical attitude* of the player, his *relative* examination of possibilities, the uncertainties resulting from the *incompleteness of the evidence*, and finally the decisiveness of *quantitative* results, another one can be inferred: the *relativistic attitude* it requires. The incessant alternation between elaborating ideas and evaluating them leads to 'disappointments' so often that the player should not and, in fact, is conditioned not to 'believe' easily in any particular principle, plan, or method. Everything is tried out, tested, and checked – and often rejected; a priori nothing is accepted as true or taken for granted. In the chessmaster's empirical, specifically inductive way of thinking there are no primary principles from which deductions can be made; nor are there any empirical rules without exceptions. Often a plan or board goal must be given up right after the opponent moves: if shifting to another plan is more 'advantageous.' A dogmatist is just as unfit for playing chess as he is for leading a dynamic enterprise. The chessmaster is of necessity a relativist or even, so to speak, an opportunist in his thinking.⁹

One more characteristic of chess thinking that should be mentioned is the *complexity of the hierarchical system of problems and subproblems* that the player must keep track of during his thought process. He must not, of course, get entangled in his own branches, nor may he disarray the subproblems and partial results, for instance. Indeed, it would seem that this requires a high degree of 'discipline' in thinking as well as a capacity for retaining complex structures of data. In this respect, however, chess is not unique.

Section 63: *The character of game and player*

The serious nature of chess as well as the necessity for a good player to be well grounded in what is called the theory of chess have led

⁹ Curiously enough, this feature is more pronounced nowadays than it was in the early days of chess. The conception of the game as a continuous balance between small 'advantages' and 'disadvantages' is due to the systematically quantitative modern theory, instigated by Wilhelm Steinitz. For a modern master it is impossible to maintain any pre-set idea on how he wants to play. He cannot afford to be only a 'combinational player' or only a 'positional player.' He has to master all styles since he must be ready at any time *to trade in one advantage for another*; that is, as regards 'board ideals' he cannot afford to cherish preferences and beliefs of any kind (cf. Section 63, p. 346).

many people to ask if chess is really a game. Is it not some sort of applied science? Or are the Russians right in honoring their chess-masters as well as their best problem composers with the title of 'artist of merit'? Indeed, is it more sensible to consider chess as a science or as an art?

It would seem that neither position is tenable. Together with other board games like Go, checkers, and *dammen*,¹⁰ the game of chess occupies its own exceptional position. The only heading under which it can be suitably subsumed is that of a serious game of combat possessing a number of specific features that are markedly different from those of both the arts and sciences. We shall delve somewhat deeper into this.

There was a time when what we now call the *scientific conception* of chess did not yet exist. Largely as a result of the work of Wilhelm Steinitz (1836-1900) – and not before – chess began to develop so-called scientific traits. Steinitz analyzed deeper, more systematically, and more objectively than his predecessors did; his theory demanded a search for truth, for the objectively best move and not for the most courageous, most brilliant, or most exotic move. He was the first to connect strategic planning with a systematic position investigation in terms of the features of the position; he was the first to base explicitly the choice of moves and plans on an integrated quantitative assessment of each position. In his time, too, the theory of openings and games developed impressively; more and more books were written on such subjects and were, as a matter of course, studied by the newcomers to the field.

It would be a mistake to suppose that the new school was enthusiastically received by everybody. While Steinitz and his followers believed that the value and respectability of chess could only gain from a more scientific approach – they even dreamed of a chair for chess at a university – a man like G. A. MAC DONNELL wrote in his book *The Knights and Kings of Chess* (1894, p. 172):

'Modern theory of play has done more evil than good to chess. It consists in playing for position, and never, unless when compelled, risking anything; in ignoring the beautiful and having an eye only for the sound; in studying all the best published games and never playing a game without putting forth all one's strength; in regarding victory as the *summum bonum* of the chess player's happiness; in eliminating the poetic element from chess, and degrading it, may I say, into a mere science....'

¹⁰ *Dammen* is the European form of checkers, played on a 10 × 10 board with slightly different rules.

Nowadays, nearly 70 years later, a battle no longer rages around the theory of Steinitz. In a general sense it has been completely accepted: in a modernized and much more elaborated form it is part of the technical knowledge that every chessmaster is assumed to possess.

Nonetheless the game of chess has not become a science. This is quite clear from the development of play since Steinitz' time. It also appears from the analysis of our protocols: the proof supporting the move that the subject chooses generally appears to be incomplete; there is no question of scientific certainty, only of a practical decision based on general considerations and a selection of calculated variations. The argument in favor of the decision to be taken is often completed by an obviously intuitive preference for a certain move. Of course, the quest for the objectively best move is relevant during the thought process – it is known that some grandmasters like Tarrasch, Euwe, and Botvinnik maintain a pronouncedly objective, 'scientific' attitude towards the choice-of-move-problem during the game – but the point is that only rarely can the problem be objectively solved. Even when the choice-of-move-problem turns out to be objectively solvable in analysis, the time limit and the prescribed fixedness of the pieces on the board make it impossible for the player to attain more than an incomplete proof in a normal match or tournament game. In spite of what laymen may think, ALFRED BRINCKMANN (1932, p. 54) is right when he says: 'In the chess battle acting carries more weight than cognition.'¹¹

In fact a requirement for chess players. Experts with a really scientific attitude who no longer train their analytic efforts and motivation on personal success in the chess arena rarely belong to the guild of grandmasters.

Play and *art* are certainly related to some degree. To indulge in either of the two is activity without direct social utility and bears the character of freely chosen conduct (HUIZINGA 1938). Elements of beauty are found in games and play as well as in the arts. To some degree the relatedness of the two is reflected in language: one 'plays' a musical instrument; there are 'plays' of comedy and tragedy and dance. It is obvious, however, that this is at most a matter of analogy, not of identity (cf. WITTGENSTEIN 1953, Sections 65-77).

The above train of thought insinuates a similarity between the chessmaster and the performing artist. It is true, indeed, that the

¹¹ 'Das Handeln hat im Schachkampf das Übergewicht über das Erkennen.'

latter plays a role on the stage or plays the violin for much the same reasons that the former plays chess: for his own enjoyment, to entertain an audience, and/or to earn a living. However, those who want to equate or at least to compare chess with the arts generally have in mind the parallel with the creative artist (cf., e.g., RETI 1923). In that case the game itself is the work of art: it bears the characteristics of the personal style of the player (that is, the winner¹²) and is in fact often beautiful and worthy of being immortalized in the existing literature of chess games. But here, too, scrutiny shows that the similarity between chess and art is rather superficial. The difference between the two is a question of goals. The prime goal of the player is not to play a beautiful game but to beat his opponent; if a beautiful game is the result of the battle it is at most a by-product. It is true that some players strive for a fine, beautiful game, in particular for a nice liquidation or final combination, but this cannot be more than a secondary motive. The game is not created as a work of art but rather reports on a process, reflects a sharp conflict. The jurists who in 1926 had to rule on the question raised by the *Fédération Internationale des Echecs* (F.I.D.E.) stated quite clearly that it would not be possible to establish a copyright on a game of chess. Chess was not considered an *oeuvre artistique* and did not come under the protection afforded by the *Convention de Berne*.¹³

Curiously enough, the (non-legal) status of problem and endgame compositions is different. The creation of a 'work,' although in the form of a problem, rests chiefly on the motive or theme expressed in it as well as on the more or less artistic way it has been composed. As a matter of fact, problemists and study composers often consider their avocation as an art – and their position appears to be tenable.

At this point the layman may ask if this is not an arbitrary distinction. Are not playing the game and composing studies just two facets of the same thing, namely, the game of chess? Yet we feel that the distinction is justified. Psychologically speaking there is a pronounced

¹² The American master Anthony Santasiere has gone as far as to suggest, however, that a brilliancy prize be awarded not only to the winner of a game but also to the loser for his substantial contribution to the creation of an artistic masterpiece!

¹³ This does not, of course, preclude the possibility of protecting chess games by specific legal regulations. Cf. OSKAM and RUEB 1926; and more recently the Statement of Policy by *The American Chess Quarterly*: '1. A game of chess is a work of art representing a joint creative composition. We advocate an international copyright to cover every master game, with a royalty to be divided upon publication of the same (in percentages to be fixed) among the winner, the loser, and the chess federation of the nation which sponsored the event.'

difference between the master of practical play and the problem composer. There is good reason not to consider the former as an artist since he is – apart from a thinker – first of all something else: a *fighter* and a *player*. To a large extent these two words contain the very essence of chess playing as we shall try to explain in the following paragraphs.

In Karl Groos's old classification of games (GROOS 1899) the game of chess belongs to the games of *combat*, particularly to the direct mental games of combat. These games stand somewhere between playing proper and fighting a battle: although played, the battle is quite serious. Although tensions occur in all forms of play (and sport) and can even be viewed as a generally characteristic element (HUIZINGA 1938), they are apt to become vehement in chess.

In fact, scarcely another game can be found in which such high tensions can be aroused, in which so much is demanded from the player's 'nerves.' It is a battle of man against man, with the opponent sitting in the immediate vicinity; during the game hardly any discharging movements can be made; there is no motor outlet for the aggression that piles up during the fight. The temporal tension arc is extremely long (a tournament game frequently lasts some four hours or more) while the continual thinking requires extraordinary effort and concentration. Far from being a cold, phlegmatic game of computation, playing chess is a very suspenseful and sometimes passionate activity. In high-strung competition the foremost aspect is often the battle of nerves. As an expression and concomitant of the necessarily high motivation, emotional strain and tension are indispensable, on the one hand, but must be completely mastered, on the other, i.e., they must not be allowed to interfere with the thought process. It is often difficult for outsiders to understand how much latent tension is concealed by the outwardly calm behavior of chess players.¹⁴

¹⁴ An interesting testimony stems from Mrs. Sonja Graf Stevenson, a strong woman player around the Thirties who still participates in some of the U.S. Women's Championships. Somewhat less self-controlled in her reactions and less reserved in her expressions than most of her male colleagues she has this to say (GRAF 1939): 'What is the game of chess?... It is hard work, struggle, renunciation, insomnia produced by the chaos of variations; it is joy, deep emotion, intimate and full vibration of our whole being....' Some remarks on the players under time pressure: 'it is a tremendous oppression that keeps our whole body in a fantastic tension....'

It is certainly no coincidence that the game of chess produced an Emanuel Lasker: a prophet-philosopher who saw 'struggle' as a motive common to both chess and real life. To him a game of chess was a serious battle that ought to be waged with every lawful means. His motto was 'I love the vigor, the healthy vigor, that risks the utmost to attain the attainable.'¹⁵ Characteristic for his own games was the fact that he did risk very much and often managed against all probability to attain the 'attainable'! World Champion Lasker conceived of Steinitz' theory in a much broader tenor than Steinitz himself ever had in mind. According to Lasker a generalized form of the theory could serve as a basis for a general science and philosophy of combat, applicable to all forms of battle that are known in life.

In spite of the seriousness of the battle playing chess is also truly *playing* in many respects. BUYTENDIJK (1932) has characterized the sphere or climate of play as a climate in which 'images,' 'fantasy,' '(riches of) possibilities' that may or may not substantialize, 'pretense' (as if), and 'symbols' prevail. All can be found in chess. The chess player revolves around and, so to say, lives continuously between 'possibilities'; within certain limits there is room for 'fantasy' – and the ever changing 'images' on the board fascinate both player and onlooker. The game itself is of course a pretended fight with pieces symbolizing human beings: Kings, Queens, Knights, etc.

The element of *chance*, one of the main components in all forms of play according to Buytendijk's exposition, plays an important role, too. If one considers the course of a game psychologically from one of the player's sides, the opponent's choice of move is not to be foreseen and therefore contains elements of chance. It is possible to have 'bad luck' or 'good luck' – as a game should have – in spite of the fact that the forces and the rights of both players are equal. Since, moreover, move decisions are hardly ever based on certainties, there is the possibility of being lucky in one's own choice of move: the right move may be based on sketchy grounds or chosen for the wrong reasons – as happened occasionally in our protocols (e.g., (M1; A)).

In a section of his book called 'Vom Wesen der Schachpartie' ('On

Further on she describes in very strong words – that may sound somewhat exaggerated to the taste of male chess players – the moral difficulties and consequences of losing a tournament game; in fact, for a real chess fighter an experience that is very difficult to reconcile.

¹⁵ 'Ich liebe die Kraft, die gesunde Kraft, die das Äusserste wagt das Erreichbare zu erreichen.'

the Essence of a game of Chess') BRINCKMANN (1932, pp. 20-1) has this to say:

'Uncertainty in chess is the only certain thing! But that is exactly what appeals to us. While on the one hand we feel driven by our mind to obtain (the highest possible) certainty of knowledge, on the other we feel bound by the spell of some dark need for action that draws us into the realm of possibilities, of free creation, into the realm of unforeseen developments and of luck.'

Evidently, this is the realm of *play*.

Psychologically speaking the game of chess is closer to card games – if only poker – and even to gambling games than the layman probably realizes. In the Middle Ages the relationship was still closer. In fact, people used to wager on the outcome of chess positions. On many occasions chess along with gambling and card games was suppressed or interdicted by the clerical authorities. In the year 1061, for example, Cardinal Damiani informed one of his correspondents that the Bishop of Florence played chess publicly one evening while lodging in an inn. The Cardinal told that the next day he reminded the Bishop that according to regulations bishops who have a passion for gambling risk being deposed. When the Bishop replied that chess was, after all, something different from gambling, the Cardinal answered that chess was forbidden just as well, under the same regulations. As a penance the Bishop had to wash the feet of 12 poor people and offer them coins.

Under Bishop Odo Sully (who died in 1208) it was even interdicted for a clergyman to have a board and pieces in his dwelling; and Louis the Holy outlawed the game of chess completely in the year 1254. Another interesting example of the hazardous status that the game of chess enjoyed in bygone ages is afforded by Savonarola: in the city of Florence, in the year 1497, one year before he himself was burned at the stake, Savonarola publicly burned a number of chess sets along with other sin inspiring objects. In a pamphlet that appeared circa 1500 a Russian Orthodox priest went to the extremes of eternally damning chess players along with gamblers, card players, and checker players.

Even though gambling on chess positions went out of fashion, chess remained akin to other games – gambling included – in its power to incite a strong fascination. Striking documentation for chess's compelling passion stems from a brief account by an unknown priest of the 17th century. It can be found in *The Harleyan Miscellany* (from the library of Edward Harley, 1st Earl of Oxford). After stating that the game of chess is the most ingenious and just game that has ever been devised, the priest has this to say on its evils:

'I. It is a great time-waster. How many precious hours (which can never be recalled) have I profusely spent in this game!

II. It hath had with me a fascinating property; I have been bewitched by it: when I have begun, I have not had the power to give over.

III. It hath not done with me, when I have done with it. It hath followed me into my study, into my pulpit; when I have been praying or preaching, I have (in my thoughts) been playing at chess; then I have had, as it were, a chess-board before my eyes...

IV. It hath caused me to break many solemn resolutions; nay, vows and promises. Sometimes I have obliged myself, in the most solemn manner, to play but so many mates at a time, or with any one person, and anon I have broken these obligations and promises...

V. It hath wounded my conscience and broken my peace. I have had sad reflections upon it, when I have been most serious. I find, if I were now to die, the remembrance of this game would greatly trouble me and stare me in the face. I have read in the life of the famous John Huss, how he was greatly troubled, for his using of this game, a little before his death.

VI. My using of it hath occasioned much sin, as passion, strife, idle (if not lying) words, in myself and my antagonist, or both. It hath caused the neglect of many duties both to God and men....'

Comments are superfluous here.

To some extent chess can be an unholy passion in our time as well. It is true that the normal club life which conforms to the roster of one game per week tends to offset over-zealous playing. But where blitz games are played in big-city clubs, in chess cafes, and among masters, chess's resemblance to gambling becomes much more pronounced. Blitz games are played at a fast tempo and rather fervently; something is often at stake, if only a cup of coffee, while differences in strength may be compensated by giving odds of time or material. For those chess devotees who are more 'players' than 'fighters' this way of quenching the chess thirst is much more gratifying than playing long and oppressive tournament games.

Thus not only fighting instincts but also a playing passion (or 'instinct') seems to be an underlying motivating factor. In fact, chess can be considered to satisfy, albeit in a rather sublimated way, quite a number of primitive drives: aggression (fighting, destruction), self-defence, passion-for-playing, ambition, of course; and in addition, a rather sadistically tainted lust for power. The latter lust is particularly obvious in the satisfaction derived from sequences of forced and forcing moves in games where, e.g., the opponent is gradually tied in knots or during mating combinations. This feature has been stressed by the psychoanalyst ERNEST JONES (1931) in his study of Paul Morphy. According to Jones the unconscious motive underlying the chess passion in general, and in Morphy's case in particular, is

the symbolic and sublimated form of patricide that the winner of a chess game commits, especially in the process of mating his opponent. To be sure, the case history of this American chess genius's mental illness points to a critical importance of his relation to his father and to father substitutes. Jones makes quite clear that Morphy's wish (at least) to defeat the Father was an extremely powerful component in his motivation and strong chess ambition. It seems very dangerous, however, to generalize these relationships. This would amount to reducing all of an individual's aggressive tendencies to those directed towards the father – a reduction that is hardly acceptable to any psychologist outside some orthodox psychoanalytic circles. Nonetheless Jones's views have some general significance: they draw our attention to unconscious sources of the chess passion, i.e., to underlying primitive asocial tendencies (cf. FINE 1956).

We now shift our attention from the game to *the player*. Which character traits and temperament can we expect to find in him? Among the well-known grandmasters it is probably possible to point to some who are primarily 'fighters' and to others who are primarily 'players.' Steinitz and Alekhine would then belong to the first category; Schlechter, Capablanca, Flohr and Reshevsky rather to the second. Lasker appears to fit into both. Indeed, the basis for what we could call *the chess temperament* seems to be formed by the traits common to both classes.

First, undoubtedly the player and the fighter are of the *passionate* type, in Heyman's sense (HEYMANS 1932, Vol. II, p. 40 ff.). We shall not now go into the details of Heymans' system, however. Suffice it to say that all people of a strong, long-term ambition belong to the passionate category – and it can be taken for granted that such ambition is needed to attain mastership in chess. In the fighter the accent is on the will-to-win, based on strong aggressive (and sadistic) drive structures. The player must certainly have some of the same drive structure, but he is also motivated by his fascination for the very elements of play: the possibilities, the images, the unexpected developments. It might be said that he plays chess 'primarily because he is unable to leave it alone!'

The above remarks bear on another personality trait that has been mentioned in our discussion of the chess player's manner of thinking: his *relativism*. The relativism required by chess thinking is likely to be found in his other fields of behavior; it appears, in fact, to be a general personality trait of chess players. The typical 'player' is a

relativist, a (playful) opportunist, an unbeliever without stanch standards for thinking or for life. It is no coincidence that the Church of old fought against games and players of all sorts. As to the 'fighter': in chess he fights solely for his own victory and this is apt to become the only standard that counts. We have already stated that from no other type is the chess player so far removed as from the dogmatist: generally he is a skeptic and relativist through and through. Characteristically, in scanning the biographies of living and dead masters and grandmasters practically no religious adherents of the great beliefs are found.¹⁶ Roughly estimated, skepticism and relativism in the form of atheism or agnosticism are even more frequent among chessmasters than, for instance, among scientific workers.

Thus the chessmaster appears to be a thinker with a rather special personality structure. Although the structure can be partially attributed to his frequent practice of the game, it must be grounded on certain inborn temperamental dispositions that should be included among the prerequisites for chess talent (cf. Section 65).

Section 64: The development of chess talent

The issue of the development of talent is complex and many-sided. We shall confine ourselves here to some remarks on how the core of chess mastership – the acquired, highly differentiated system of thought habits – develops in a lifetime, particularly in youth.

The following exposition is based solely on the biographical data of a number of chessmasters, data extracted from the chess literature for the most part but also complemented by the author's personal communications with Dr. Max Euwe and Hans Kmoch as well as his own information on and personal experiences with chessmasters. Most of the better known chessmasters are included although no criterion for complete coverage was applied. The analysis of vitae had to remain rather skimpy – in this respect we cannot compete with JULIUS BAHLE's (1936 and 1939) parallel effort to study the development of mastership in musical composition. Therefore, the following observations and generalizations are offered with some reservations, that is, as *hypotheses that appear to be tenable*.

Bahle – to start with some of his findings – was able to bring forward an impressive sum of biographic evidence to support his

¹⁶ International Grandmaster William Lombardy's monastic leaning towards Roman Catholicism appears to provide a very interesting and exceptional case.

'rational' view, contradictive to such 'irrational' conceptions as an effortless 'growth' toward mastership, inborn genius, unanalyzable, miraculous inspiration, magical intuition, and so on. He showed the prime importance of the factor of self-organization both in the developing composer's creative production proper and in his antecedent learning processes. Here, too, a very strong long-term motivation towards self-development in the field in question leads to a rapid accumulation of fertile, differentiated 'experience.' Instrumental, even indispensable, are the self-organized work and intensive training of the composer's early years. Even to an unparalleled genius like Mozart it was by no means the gods who presented him with the gift of composing: he too had to acquire his system of creative methods bit by bit.

In a very early stage already this self-development becomes self-organizing; more and more the youthful composer comes to order every single detail in his way of life and in the organization of his work in order to serve the one great goal: the maturation of his own creativity. Apart from the prerequisite of being gifted it is this enormous concentration that makes it possible for him to condense in a few years the aggregate of (learning by) experience needed for an ascension to complete mastership.¹⁷ Bahle describes this process in somewhat different terms, but the tenor is the same. The question now though is what do we find in the early years of chessmasters?

First, it is quite obvious from every one of the life records that in their youth chessmasters have known a period of *passionate chess playing*, indeed, of near monomania. It is during this time that the groundwork for later mastership is laid. They devote an exorbitant amount of their time not only to playing but also to analyzing thoroughly every game of their own and often those of others; not to mention studying the theory of chess. During this time the system of knowledge and experience expands vastly and differentiates rapidly.

Quite frequently this fanatic enthusiasm for chess leads to conflicts with plans for study and training. Many a future chessmaster – also, alas, amateurs who aspire to but never reach the ranks! – has used the

17 In fact, the young aspirant is apt to be an egocentric and something of a monomaniac, at least for some time. The creative person's *hierarchy of values and life goals* differs markedly from other people's – every other goal or standard being, at best, second to the goal of creative development – which makes for 'asocial' behavior, 'maladjustment,' neurotic and 'psychotic' traits. Bahle is certainly right in asserting that in most of the studies on 'genius and madness' insufficient attention is paid to the – quite 'normal' – effects of such a devious value hierarchy.

freedom of student life primarily for playing chess to his heart's content. Steinitz, Zukertort, Neumann, Reti, Eliskases – to mention but a few famous names – started but never finished their academic studies. Specimens can be found in every country.

Those who finally manage to finish their professional training generally have to struggle to do it. This struggle is hardest on masters who acquire world fame at an early age, like Reuben Fine and Paul Keres, for instance. In such cases it is not only the seductions of game and fame that have to be overcome but also the responsibilities to the chess world. Dr. Tarrasch, in his autobiography (TARRASCH 1925), described how a tournament disaster after a series of promising successes finally awakened him from his ambitious chess dreams and recalled him to reality: a disappointment enabling him to finish his medical study normally.¹⁸ Numerous are those whose passion for playing chess delays them in the pursuit of their studies, sometimes after having toyed with the idea of becoming chess professionals. Among Western chessmasters a figure like Max Euwe is rather an exception: thanks to his sense of duty and temperance he managed to combine chess successes with a regular course of study and career. Another exception but a less fortunate one is Paul Morphy. In 1857 when Morphy entered the chess world at the age of twenty he had finished his studies, to be sure. His subsequent successes were spectacular; within a year or two he defeated every one of his opponents including the German master Adolf Anderssen. Shortly thereafter, however, he retired from the chess world, cut off practically all social contacts, repudiated the game of chess completely in 1869 and finally died in 1884 after many dark years of paranoia. In the case of the young Morphy it was primarily his father who sharply – probably too sharply – saw the dangers of exceptional chess talent; he therefore guided Paul and when necessary limited his chess practice with a

18 Quite naturally, the acuity of the problem – chess versus professional training – depends largely on the status of chess within the culture in question. In the U.S.A. a talented youngster who devotes practically all of his time to playing chess – that is, to an 'intellectualistic,' 'highbrow,' 'unmanly,' 'mere game' where efforts 'do not pay off' – must of necessity begin after some time to feel guilty of his neglect of more respectable activities. In the U.S.S.R., on the other hand, the development of talent is fostered and furthered by the approval of state and culture. Moreover, the talented chessmaster is even helped to complete his professional training – a psychologically sound policy – more or less regardless of how this future master may earn his living. The practical angle to the question, of course, is whether it is possible at all to earn a living as a chessmaster; in the Western countries, particularly in the U.S.A., it is difficult, to say the least.

severe hand in order not to have it interfere with Paul's studies. According to the psychoanalytic interpretation the fact that it was not Paul himself but rather his father who forcefully solved the problem for him had some influence on his subsequent professional failure and on the development of his mental illness (JONES 1931). Morphy is a highly exceptional case, however. In general one gets the impression that those who have not neglected their studies have been happier and better adjusted in later life than the others who economically and psychologically have been totally dependent on the maintenance of their chess excellence.

Morphy is also one of the few chessmasters about whom the romantic conception of an effortless growth into mastership, of an inborn chess genius, is occasionally championed. At first sight his extraordinary chess successes appear to have required hardly any preparation: as soon as he began to compete formally with others he beat them all. His style is sound, both pure and elegant; to the present day students of his games his moves appear so self-evident that they suggest – as does the music of Mozart – an effortless conception. But in spite of his phenomenal career Morphy does not afford an exception. First, before his official debut into the chess arena he had played a large number of games and had concentrated much time on chess in general. Although his father had allowed him to play only on Sundays, this was after all one day a week replete with chess, one serious game after another. Moreover, the once-a-week rule did not exclude analyzing – if this happens blindly, as it often does to chess enthusiasts, it can hardly be forbidden. In fact, according to the biographies, Paul's father stimulated the development of his chess talent, providing him with chess books among other things. On one of his birthdays he was given a work by the English chess king Howard Staunton on the title page of which, under the name of Staunton, there stood: 'Author of... (books)...' With a pencil Morphy added: '...and some devilish bad games.' Not only does this attest to his critical attitude and early self-confidence, but it also shows that he seriously studied Staunton's games. During this period of his life he did not play against masters but against well trained and strong sparring partners from whom one cannot win without a sober effort.

Further, after he finished his studies and after the death of his father a real hurricane of chess activity broke out. During this time he could make arrears and secure a solid basis of tournament and match experience for his coming career. Important for his steady development to the highest level of chess skill was his good luck of

meeting, even on his European tour, a somewhat stronger antagonist on every new occasion. By the time he finally met Anderssen in 1858 his experience, qua number of match and tournament games played, was quite considerable. True enough the speed of Morphy's rise to grandmaster and world champion is remarkable and probably unparalleled, but his development too was gradual, prepared step-by-step, and no more fell out of the air than any other grandmaster's.¹⁹

Let us now try to get a somewhat more detailed picture of how the player's mastership comes about in these periods of passionate chess playing.

As a matter of course he accumulates more and more 'knowledge and experience.' This means, in our terminology, that a system of highly adequate thought habits is built up in a process of progressive extension, differentiation, and refinement. The player's system of knowledge and experience expands rapidly in terms of thinking and playing methods he has available.

What actually happens is best illustrated by looking at playing methods. First, by means of playing experiences and/or textbooks the player gets to know certain important general strategic and tactical rules; next, he learns to recognize and to handle exceptions to these rules – which in their turn grow into new, more refined rules; with new exceptions, etc. Finally, the player develops a 'feeling' for the cases in which these already highly specialized rules can be applied. Thus, for instance, the player learns how important it is to occupy the center of the board with Pawns; then he finds out that a too broad or too far advanced Pawn center may be weak since it may become an easy object of attack. Next the player discovers that advanced Pawns (such as the White Pawn at K5 in the French Defense) can be both weak and strong – in different respects. Finally he develops an ever finer and more reliable feeling for the types of situations where the strengths and the types where the weaknesses of such a Pawn structure prevail. Another example: A materially equal ending with a blocked Pawn structure usually peters out to a draw as the material is exchanged down; but the 'strong' Bishop generally wins out against

19 STEPHAN ZWEIG'S (1944) well-known 'Schachnovelle' obviously does not take into account the indispensability of competing with others for the development of talent.

In chess, as in any other field of endeavor, it is completely impossible to reach the highest levels of skill without the benefits of what others have done and developed. Only by standing, so to say, on the shoulders of predecessors can the highest laurels be plucked.

the 'weak' Bishop; however, in positions with long Pawn chains where the King cannot penetrate, the strong Bishop may be powerless. A player develops a feeling for those cases where there are winning chances and for the ways in which such chances must be exploited.²⁰ In this manner the player's ability to classify and to apply appropriate treatment (in terms of thinking and playing methods) is steadily refined.

The learning process as described above is, of course, of a very general nature. It can be found in every development of a mental skill by experience in practice. Along with other serious games of combat chess is an exceptional case for only two reasons: (1) its domain is complex enough to prevent any human being from acquiring complete, error-proof mastery of the trade and (2) the irrefutable argument of victory – irrefutable since there is no chance element in the rules as with card games – continuously provides the learner with reliable, objective information on the standing of his achievements and shortcomings. In the latter respect, (2), professional skills like those of the family doctor or practising psychologist are much less transparent. Among family doctors, for instance, it can be taken for granted that there do exist real class differences as well, but they cannot be objectively ascertained. Differences in skill are certainly not adequately expressed in degrees of recognition or success – so that the real master practitioner may remain cloaked. In many other fields as well, it is hard to distinguish clearly between real masters and phony masters, that is, between the 'real thing' and possibly quite extensive but not sufficiently empirically founded systems of 'experience' in which subjectivity, prejudices, and faulty generalizations occur. In the case of chess, the existence of objective skill criteria, (2), in a domain of high complexity, (1), makes it possible to distinguish objectively between a large number of skill levels and to talk of *mastership* in a well defined sense. In spite of the fact that mastership consists of 'nothing but' experience and routine no more than a handful of extraordinarily gifted persons are able to build up a system of experiential relationships (linkings) of a large enough scope and differentiation.

The *order* in which the potential master learns to handle the various pieces of chess technique is not always the same. It is true that many

20 Whenever it is only possible to express what we mean in terms of a 'feeling for,' we obviously enter the domain of intuitive experience. This point is reached at an earlier stage with regard to tactical playing methods than strategic methods (cf. Section 57).

players start as tacticians with a penchant for King side attacks, only later to evolve into strategists; but others begin as endgame experts or positional players. The order depends both on the player's personality and on his environmental influences, early teachers and textbooks, for instance. Thus in the Netherlands the influence in the Thirties of the wealth of literature on the strategic side of the game (Max Euwe's books) produced quite a number of rather good positional players who were not (yet) very skilled in combinational play. In the long run, though, the developing master cannot afford to remain solely a tactician or solely a strategist; he must learn to master both sides of the board.

Since chess has neither anything to do with life experience nor with maturity, chess talent – just like musical or mathematical talent – may show itself and develop into mastership at an early age in the exceptionally gifted (cf. Révész 1921). At the age of sixteen Paul Morphy was already a local celebrity, at the age of twenty he was champion of America, and at twenty-one world champion, although the title did not yet officially exist. José R. Capablanca (1888-1942) was champion of Cuba at the age of twelve; in the year 1909 he won a match from the American champion Frank Marshall with a very convincing superiority (+8, – 1, = 14). Both Alexander Alekhine and Mikhail Botvinnik got their master titles at sixteen. Emanuel Lasker, Aron Nimzovich, Reuben Fine, Paul Keres, and the Dutch masters Max Euwe and David Nobeboom reached the level around their twentieth year. Grandmastership before the age of twenty-five is by no means an exception: apart from Morphy and Capablanca there are Emanuel Lasker, H. N. Pillsbury, Ossip Bernstein, Akiba Rubinstein, Alexander Alekhine, and no less than five well-known candidates for the world championship in the Thirties: Salo Flohr, Mikhail Botvinnik, Samuel Reshevsky, Reuben Fine, and Paul Keres.²¹ In the years after World War I Samuel Reshevsky (born 1911) drew much attention as a child miracle: when eight years old he managed to play simultaneously against twenty adults, with good success. He had the playing strength of a master even before he received much education at school. After this unparalleled entrance into the chess

21 In modern times grandmastership before the age of twenty-five appears to be the rule: Bronstein, Korchnoi, Panno, Petrosian, Spassky, Tal – to mention only a few grandmasters of candidate level – not to forget Robert Fischer who managed to get the title at the age of fifteen and to become one of the world's strongest players at the age of eighteen.

world it looked for some time as though he would not penetrate the ranks of the world's foremost players. Not until 1935 – still prior to his twenty-fifth birthday – could he be deemed one of the world's pre-eminent grandmasters.

Although there are many cases of a rapid development of chess talent, Révész's thesis (1921) that the capacity of chess masters generally reaches its peak at an age of twenty or twenty-one and then soon declines is not right. It is true that strenuous tournaments and matches require a number of physical and mental qualities – like tenacity, perseverance, 'strong nerves,' mental elasticity, fighting spirit – that may tend to fall off somewhat after the early twenties; but on the other hand the amassing experience is likely to make up for any decline and to lead to still loftier achievements. Capablanca, for example, did not lose one tournament game between 1914 and 1924 (from age twenty-six to thirty-six); when he lost his world championship to Alekhine in 1927 he was still at the top of his powers. Lasker (1868-1941) became world champion in 1894 at twenty-six but not until 1899 was he generally considered the world's strongest player. Lasker was rather out of the ordinary, for that matter – as are Botvinnik and Keres – in that he held his own after his fortieth year: he won important tournaments in 1914 and even in 1924, at the age of fifty-six. Bernstein and Rubinstein, both born in 1868, had their most conspicuous successes in the years 1905-1914. Alekhine played his most masterly chess shortly after he gained the world title, that is, around his thirty-seventh year (San Remo, 1930). Keres, Reshevsky, and of course Botvinnik have demonstrated their sustained strength throughout their thirties, forties, and now apparently fifties. In general, though, most of the past's grandmasters have had *their most spectacular successes between their twenty-fifth and fortieth to forty-fifth years*: this is true for Paulsen, Blackburne, Zukertort, Tchigorin, Tarrasch, Teichmann, Janowski, Maroczy, Schlechter, Nimzovich, Réti, Spielman, Bogolyubov, and Tartakower. The last five of these grandmasters have been notably uneven in their achievements, but in no case is there a question of steady decline before their thirty-fifth year. Euwe was a master at twenty, a grandmaster at twenty-eight, but reached his zenith around the years of his world championship, roundabout forty.

Euwe is not the only one to have had a somewhat late 'blossoming,' particularly in the sense of attaining grandmastership after twenty-five. Philidor (1726-1795) won his famous match against Sire de Légal in 1755 and managed to maintain his hegemony in Western

European chess until the French Revolution. Staunton (1810-1874) did not even begin to play regularly until the age of twenty-six; he was probably the strongest player in Europe from his thirty-third to forty-first year. At the London tournament in 1851 Adolf Anderssen's period of greatness was inaugurated. He was then thirty-three but did not play any less strongly ten years later. When in 1866 the thirty year old Steinitz managed to beat the forty-eight year old Anderssen by but a small margin, the result came as a surprise to the chess world. Public opinion did not fully recognize Steinitz as world champion until around 1873; in fact, his style of play became more profound near his fortieth year while he laid the now famous foundations of chess theory in the fifth decade of his life.

It should be clear from the above examples that, contrary to Révész's opinion, it is an exception rather than the rule if a chess-master deteriorates before his thirtieth year.²² Even the well-known and often cited cases of Morphy and Pillsbury appear to lose their validity on closer inspection. Morphy retired from active chess at the age of twenty-one, which does not argue a decline of strength; the relation should rather be to his later mental illness (JONES 1931). Pillsbury did decline appreciably before he was thirty, but this was primarily due to the physical and mental consequences of the then incurable infectious disease that was also responsible for his early death in 1906. The popular myth that his excessive chess activity, his blindfold simultaneous play in particular, was in itself harmful and a cause of his deterioration is unfounded. By the same token nearly all of the other cases that have been cited in support of the thesis of an early and brief fame in chess appear to be spurious: Neumann and Rubinstein were mentally ill; Von Kolisch retired early from chess to be trained as a banker by Rothschild. (He earned a fortune and became a chess Maecenas instead!) Of course, there are chessmasters who died young: von Bilguer, de Vere, Charousek, Breyer, Noteboom, van den Hoek, and others. In fine, it is hard to find any real example of a decline before the age of thirty.²³

22 Cf. DRAPER (1963) who concludes from a statistical analysis of world championship and tournament results that 'the basic chess-ability of a master is not adversely affected by age...' (p. 126).

23 In an unpublished thesis by Paul Buttenwieser of Stanford University, entitled 'The Relation of Age and Skill of Expert Chess Players,' 1935, the author studies statistically the lifetime records of 100 chessmasters and comes to the following conclusion: 'The composite age-curve, based on the median trend, indicates some improvement in skill from the twenties to the thirties, when a plateau is reached extending through the forties. From 50 to 70 occurs a steady drop which, while a

From studying the record of chessmasters' accomplishments it would appear that – apart from the obvious individual differences in the development of talent – two further generalizations can be made.

First, the sturdy theoretical development of the last half century added to the increasing frequency of tournaments and international contacts promotes the rapid rise of the coming player. The process of acquiring knowledge and experience is accelerated by the availability of more and better chess books, by better communication in general, and by more training opportunities via tournament contacts.

Second, there seems to be some relation between the way in which a master's chess skill develops and his personality structure. Where chess talent is only one index of a general giftedness, there are some indications that reaching the highest levels of skill goes hand in hand with the maturation of the whole personality: though it takes longer to acquire mastership, once secured, it may last longer. Conversely, the more one-sided and specific the chess talent of a player is, or the poorer or less developed the remainder of his personality structure is, the earlier the (grand)mastership level will be reached, *ceteris paribus*. If the impression be sound – it is no more than an impression – it might in part explain the current belief that child marvels never develop into real greatness. In its generality such a belief is certainly false. However, if it is true, first, that the capabilities of child wonders are apt to be highly specific – whether by nature or by nurture – and second, that 'real greatness' requires a more comprehensive giftedness, the belief may hold some statistical validity. But all this remains hypothetical. We cannot now enter upon the ways in which these hypotheses can be tested.

Section 65: Factors of Chess Talent

The experiments described in this book do not include any experimentation in diagnostic testing. The experimental study of the thought process combined with the data and analyses produced in the earlier sections of this chapter do, however, provide, a basis for a solid *arm-chair analysis of the factors of chess talent*. By such an analysis the groundwork can be laid for the formation of hypotheses to be tested later by means of specific diagnostic procedures. Again it is primarily the

statistically reliable decrement, is extremely slight relative to the range of skill among chess players generally...' More recently ERNEST RUBIN (1961, p. 42) adds the conjecture that 'increases beyond a certain average age (perhaps 40 years) decrease the quality of tournament chess play definitively.'

'core of mastership' that we shall consider, i.e., the highly differentiated system of thinking and playing methods.

1. The mental domain, in terms of the units of subject-matter from which chess thinking is built up, is a fairly specific one: *schematic, spatial* (two-dimensional) *possibilities for movement*. Roughly comparable in this respect, apart from other board games, are problems like getting to know one's way in a city or in a labyrinth, shunting problems either in a yard or in an analogous experimental situation or even in a test like Alexander's 'Pass Along' (ALEXANDER 1932). Correspondingly, chessmasters should be expected to score highly on tests in which a (dynamic) spatial factor is predominant.

2. This prediction can be generalized: *non-verbal intelligence* must be strong in chessmasters. In particular, chessmasters, if tested, can be expected to score highly in *spatial memory* and constructive *spatial 'imagination'* – already mentioned by BINET (cf. Section 2). The reader is warned anew not to confuse this rather abstract, schematic, spatial imaginative ability with concrete visualization and visual memory.

High *verbal intelligence*, let alone verbal fluency, does not appear to be an indispensable requirement. If we take into account the empirical finding that different types of intelligence tend to be rather highly correlated – the verbal-conceptual variant being the least likely to be neglected in our culture – there are, however, good reasons to expect a fairly high average for chessmasters' (verbal) intelligence quotients as well. (Prediction, based on a few testing experiments with chessmasters by J. T. BARENDREGT, 1951: mean IQ around 130).

3. Intelligence is not enough, of course. While the ability to solve quickly and adequately short-term, 'new' problems – test items – is emphasized in the current conception of intelligence, building up a complex system of interrelated operation dispositions (methods) in a particular field also requires a powerful *memory capacity*.²⁴ The system must be stored and adequately operated as well – which is where 'thinking discipline' (DE GROOT 1938) enters the picture.

4. Maybe even more important is the fact that the future master has

²⁴ The term 'memory' in the present context encompasses both knowledge (knowing that) and experience (know-how). The question in how far a subject's memory capacity depends on the kind of data to be stored is not considered here.

to build up his 'system of methods' all by himself. He must be able to learn progressively by experience, that is, to refine his operating rules constantly: to make new rules out of the exceptions to the old ones (cf. p. 300) by means of the mental operations of *abstraction and generalization*. In fact, the process of 'getting more and more experience' consists of a practically continuous alternation of setting up tentative hypotheses and testing and evaluating them (see point 5 below). It is true that these hypotheses may remain largely implicit, i.e., operative in the actual thought organization long before the subject, if asked, could explain them (would know about them); in other words, the system may largely remain one of intuitive experience. This does not, however, depreciate the existence and complexity of the system nor the requisiteness of such processes of abstraction and generalization for the system's very formation. The power to abstract tentative rules, operating principles, and analogies – within the domain of spatial dynamics – appears to be one of the most pronounced capabilities of the chessmaster.

5. The pendant of generating hypotheses is, of course, testing them. A chess player good in abstracting analogies and generating ever new hypotheses but lacking in the ability of objective reality testing could never become a master.²⁵ Apart from the fact that the game of chess probably develops both capacities, it would seem that the game also requires a considerable natural disposition for such continuous alternate functioning. The ability to drop quickly a hypothesis on disparate evidence – in order to quickly readjust, modify, or replace it by another – is highly important in the chessmaster's mental make-up. When we mentioned this feature of chess thinking in Section 62, we related it to another trait – again both an inborn disposition and a developed habit, in all likelihood – namely, the *relativism* that is so often found among top rate chess players.

6. Still another factor seems needed in order to explain how the developing chessmaster is able to build up his system of experience. Apart from being able continually to generate and modify hypotheses, ideas, rules, (board-)systems, and plans he must also be sharply

²⁵ From everyday experience we know that this combination of abilities is not a common one; the two may even be negatively correlated in some populations. In the social sciences, for instance, two kinds of scholars are often said to occur: those who abstract and generalize but do not test (sufficiently), and those who test but, regrettably, abstain from any large scale generalizations (relevant theories).

motivated to do so. This calls for an *affinity for active investigation*, for finding-out-by-yourself, by trying and analyzing – a mental need and/or attitude the chess player has in common with other personality types, e.g., scientists.

7. The underlying motivational factors are rather specific, however. Here we get back to the 'chess temperament' that we have come to know as a fusion of thinking, playing, and fighting passions. It is difficult – as it was with abilities and attitudes – to disentangle nature and nurture here, but it would seem that one indispensable requirement for reaching a high level in chess is a perceptible inborn disposition for becoming a passionate thinker-player-fighter. Apparently such a disposition is not found in all scientists; many, while investigators and thinkers by nature, react coolly to the fascinations of a fighting game.

8. Finally, questions might be raised that transcend the 'core of mastership': What are the personality factors underlying the differences between masters and grandmasters and the different playing strengths among grandmasters? And, terminally, what makes a world champion?

In the present study we have scarcely any empirical facts upon which an answer to these questions can be based. It is possible, however, to extrapolate from some of the previous findings and, furthermore, to make some tentative generalizations on the basis of biographical evidence.

First, there are degrees of instrumental talent, that is, degrees of strength in the cognitive disposition factors mentioned in points 1 through 5 above. From the life histories of chessmasters it is clear that Morphy and Capablanca reached through play – literally – an enormous virtuosity in chess, while it took Alekhine somewhat more time and effort and Steinitz and Euwe much more. Grandmastership is out of reach to many masters due simply to not enough talent in the cognitive sense.²⁶ The most important factor in determining a player's ceiling, in general, appears to be the scope of his 'memory capacity' (cf. 3 above).

Within groups of players of the same (instrumental talent) class, however, there are still remarkable differences in tournament and

²⁶ Nowadays, as a result of the numerical increase in the ranks of grandmasters, there are many 'weak' grandmasters who, for the same reason, will never become world champion candidates.

match achievements, differences that seem to correspond to the personality structures. In the biographies of the greatest world champions, who have maintained themselves throughout a number of years, conspicuous is the enormous *concentration on the one goal to win and on the strategies thereof*; for example: don't play too much, study your opponent's weaknesses, regulate your life habits in accord with the maintenance of an optimal condition, never agree to any arrangement that could diminish your chances, etc. In everyday language: They are 'fighters,' they have an enormous 'will power' and a correspondingly strong 'belief in themselves.' No chessmaster of the nineteenth century was such a hard, relentless fighter as Steinitz. No other master of his era developed such a tough, universally valid fighting philosophy as did Lasker. Scarcely any other grandmaster built up his chess career with such an iron consistency, goal-directedness, and self-discipline as Alekhine.²⁷ Alekhine's absolute renunciation of what would normally be called a serious addiction to both smoking and drinking, until he regained the world championship which he had lost to Max Euwe in 1935, is a well-known case in point. In general, the 'fighter' type appears to be in a better position to gain the supreme title than does the 'player' type – albeit neither Capablanca nor Euwe were especially pronounced fighters.²⁸

Further, pure physical endurance must be a factor of importance, considering the arduous demands of tournament and match play. Indeed, this factor can be subsumed under the general requirement of a sthenic personality structure – including a high degree of tolerance for frustration.

In rough outline the above armchair results agree with the 'psychogram' of the chessmasters fashioned by DJAKOW, RUDIK, and PETROWSKY (1927) – if we allow for differences in terminology. They are in agreement with BARENDREGT's unpublished 1951 findings as well (personal communication). It is clear, however, that more concrete formulations and testing of the above hypotheses – and of others, such as REUBEN FINE's (1956) hypothesis on the chess player's repressed homosexuality – can only result from systematic psycho-

27 Mikhail Botvinnik can now be added to this list. Among the younger grandmasters the most signal case seems to be Robert Fischer whose one-goal-'will-power' governs his *raison d'être*.

28 Among the top players of today Mikhail Tal appears to be an extremely gifted 'player,' somebody to whom chess is more a source of pleasure and fascination than a vehicle for ambition. Outwardly, at least, players seem to be happier with their addiction than do fighters.

diagnostic experimental investigations, which have not yet been undertaken.

Finally, not all of the factors that determine who will and who will not become a world champion or a candidate for world champion are to be found in the player's personality structure.²⁹

Cultural influences have already been mentioned: apart from stimulating chess talent, some cultures strongly uphold and encourage competition, mental pugnacity, and the will-to-win, while others emphasize sportsmanship, 'remembering it's only a game,' and not getting too involved in personal victory. The reader is certainly acquainted with the contrast between Eastern European (particularly Russian) and the Anglo-Saxon (particularly British) climates. As for the Latin cultures, the principal drawback appears to be that chess is a slow, long-term, and non-verbal activity. In general, Western cultures have bred many highly gifted 'players' who, in the last resort, are not adequately sthenic, while Russia grows primarily 'fighters.'

Cultural influences aside, there are other, rather imponderable factors that may determine success. In match play, for instance, it is obvious that 'psychological factors' play an important role. We shall not now discuss this imponderable but limit ourselves to quoting Max Euwe who recently drew attention to the fact that in the whole of chess history there is not one example of a world championship match in which the incumbent champion was able to defeat a challenger of really equal class – if 'equal class' is determined by an evaluation of previous tournament results. The champion appears to be at a strong 'psychological disadvantage.'³⁰

Section 66: Extracurricular achievements of chessmasters

In the present section we shall briefly examine what chessmasters do with 'the rest of their lives.' Starting from some biographical data we shall attempt to draw a few additional conclusions about (1) chess talent and mathematical talent and (2) personality types among chessmasters.

29 The question whether race factors are influential, either dispositionally or culturally, is a highly interesting one in chess, considering the proportionally enormous number of Jews (some 70%) among the world's best players: Steinitz, Lasker, Fine, Botvinnik, Najdorf, Bronstein, Tal, Reshevsky, Petrosian, Fischer and many others.

30 In the twentieth century: Lasker drew with Schlechter, lost to Capablanca; Capablanca lost to Alekhine; Alekhine lost to Euwe; Euwe lost to Alekhine; Botvinnik drew with Bronstein, lost to Smyslov; Smyslov lost to Botvinnik; Botvinnik lost to Tal; Tal lost to Botvinnik; Botvinnik lost to Petrosian!

Table 15 presents data on the formal training and occupation of 55 grandmasters. The table must be read with some reserve, however, especially with regard to the players' professional standings. The chess world does not bother itself very much about distinguishing professionals from amateurs so that it is often hard to decide whether a master is a professional who does other things for a living or a 'journalist' or 'official' who supplements his income by (playing) chess. As to the data on formal, academic training the reader must keep in mind that standards differ widely from country to country and from one educational institution to another.

TABLE 15: *Chess status, training, and profession of 55 grandmasters*³¹

Name	Birth-Death	Chess Status	Training and Profession
1. Philidor, André Danican	1726-1795	first in Western Europe	musician, opera composer
2. Deschapelles, Alexandre Louis Honoré Lebreton	1780-1847	first in Europe	nobleman, soldier (general)
3. Labourdonnais, Louis Charles Mahé de	1795-1840	first in Europe	nobleman; later professional
4. McDonnell, Alexander	1798-1834	rival of Labourdonnais	colonial merchant
5. Staunton, Howard	1810-1874	first in Europe	literary critic, chess journalist
6. Anderssen, Adolf	1818-1879	first in Europe	mathematician, high school teacher
7. Paulsen, Louis	1833-1891	grandmaster	tobacco broker
8. Steinitz, Wilhelm	1836-1900	world champion	studied engineering; professional
9. Morphy, Paul	1837-1884	first in the world	trained as a lawyer, never practiced
10. Kolisch, Ignaz von	1837-1889	grandmaster	banker after his chess career
11. Neumann, Gustav Richard	1838-1881	grandmaster	studied medicine, short-term military doctor; later professional

³¹ Thanks are due to Hans Kmoch and Dr. Max Euwe for their fact finding help in the original Dutch edition and to H. J. Slavekoorde and J. E. Armstrong for collecting and making available data on some of the post World War II chessmasters. The table includes world champions and grandmasters on candidate level plus some younger grandmasters who are likely to attain candidate level.

Name	Birth-Death	Chess Status	Training and Profession
12. Zukertort, Johann Hermann	1842-1888	grandmaster	studied medicine, short-term military doctor; later professional
13. Blackburne, Joseph	1842-1921	grandmaster	no specific training; professional
14. Tchigorin, Michael Ivanovitch	1851-1908	candidate for world champion	civil servant; professional
15. Gunsberg, Isidor	1854-1930	candidate for world champion	chess journalist; professional
16. Tarrasch, Siegbert	1862-1934	candidate for world champion	M.D., chess journalist
17. Janowski, David	1868-1927	candidate for world champion	no specific training; professional
18. Lasker, Emanuel	1868-1940	world champion	Ph.D. in mathematics, author; professional
19. Maróczy, Géza	1870-1951	grandmaster	engineer, insurance broker
20. Pillsbury, Henry Nelson	1871-1906	grandmaster	business training; professional
21. Schlechter, Carl	1874-1918	candidate for world champion	no specific training, chess journalist; professional
22. Marshall, Frank	1877-1945	grandmaster	no specific training; professional
23. Rubinstein, Akiba	1882-1961	grandmaster	no specific training; professional
24. Bernstein, Ossip	1882-1962	grandmaster	Jur. D., lawyer
25. Spielman, Rudolph	1885-1942	grandmaster	no specific training; professional
26. Vidmar, Milan	1885-1962	grandmaster	Ph.D., professor of physics
27. Nimzovich, Aron	1887-1935	grandmaster	no specific training; professional
28. Tartakower, Savielly	1887-1956	grandmaster	Jur. D., author, chess journalist
29. Capablanca, José Raúl	1888-1942	world champion	civil servant; professional
30. Réti, Richard	1889-1929	grandmaster	studied mathematics; professional
31. Bogoljubov, Ewfim	1889-1952	candidate for world champion	studied engineering; professional

Name	Birth-Death	Chess Status	Training and Profession
32. Alekhine, Alexander	1892-1946	world champion	Jur. D.,; professional
33. Euwe, Max	1901-	world champion	Ph.D., mathematician
34. Flohr, Salo	1908-	grandmaster	no specific training; professional
35. Najdorf, Miguel	1910-	grandmaster	merchant
36. Botvinnik, Michael	1911-	world champion	electrical engineer
37. Reshevsky, Samuel	1911-	candidate for world champion	accountant
38. Kotov, Alexander	1913-	grandmaster	technical engineer
39. Fine, Reuben	1914-	candidate for world champion	Ph.D., psychologist following his chess career
40. Pilnik, Hermann	1914-	grandmaster	civil servant
41. Keres, Paul	1916-	candidate for world champion	studied mathematics; chess journalist
42. Szabó, Laszlo	1917-	grandmaster	chess journalist
43. Smyslov, Vassily	1921-	world champion	opera singer, chess journalist
44. Gligoric, Svetosar	1923-	grandmaster	journalist
45. Bronstein, David	1924-	candidate for world champion	engineer
46. Geller, Ewfim	1925-	grandmaster	economist
47. Taimanov, Mark	1926-	grandmaster	concert pianist
48. Benkó, Pal	1928-	grandmaster	investment broker
49. Filip, Miroslav	1928-	grandmaster	Jur. D., jurist
50. Petrosian, Tigran	1929-	world champion	chess journalist; professional
51. Korchnoi, Viktor	1931-	grandmaster	historian
52. Panno, Oskar	1935-	grandmaster	engineer
53. Tal, Michael	1936-	world champion	philologist; journalist
54. Spassky, Boris	1937-	grandmaster	journalist
55. Fischer, Robert	1943-	grandmaster	no specific training; professional

Inspection of Table 15 shows that more than one third of the grandmasters listed are *professional chess players*. If we add to the palpable professionals those players who are government appointed officials with highly pliable schedules and journalists who write largely on chess, in short: the grandmasters who devote or have devoted most of their lives to chess, the professionals make up at least two-thirds or three-quarters of the total group. Grandmasters like Dr. Milan Vidmar and Dr. Max Euwe, who let their other profession prevail throughout their chess careers, or like Dr. Ossip Bernstein and Dr. Reuben Fine, who after years of outstanding achievements let other careers supersede chess, are exceptions rather than the rule. In view of the mental and physical exigencies of chess tournaments and matches the rule is hardly surprising: one must maintain a good fighting condition, theoretical knowledge must be kept up-to-date, and so forth. Apart from a few exceptionally gifted, the highest ranks can only be reached by the devotees.

The professions and occupations of the amateurs diverge widely: composer, general, merchant, engineer, literator, mathematics or physics professor, physician, lawyer, psychologist, economist, journalist, government official, to mention about half. Understandably, it is mainly *the professions and other higher occupations* that are able to compete with the enticements of the game. We know from Section 63 that the lures are difficult to resist, particularly when world fame appears to be within reach – or has already been attained at an early age.

If we survey *the fields* of training and occupation, is there any evidence to support the popular belief that interest and capabilities in *mathematics and science* correlate with high achievement in chess?

The belief appears to be primarily based on the observation that, among students, chess enthusiasts are more likely to be found in the departments of mathematics, physics, and engineering than in the humanities, for instance. In fact, Table 15 reflects such a tendency: Nos. 6, 8, 18, 19, 26, 30, 31, 33, 36, 38, 41, 45 and 52, that is 13 out of 55, were either temporarily or throughout their lives affiliated with mathematics and/or the exact sciences. In addition, there are three students of medicine, two of them unsuccessful (Nos. 11 and 12), one successful (No. 16). The group of mathematicians and scientists is larger than any other comparable group. It is by no means a majority, however. If we extend our sample to include a larger group of masters – the author's original documentation contained over 100 pre-war names – the ratio does not appear to increase. The large group of 'real' amateurs in the set of masters (and near-

masters), while containing a substantial number of mathematicians and scientists, again shows a wide variety of occupations ranging over multiple fields. There is no question of the mathematical minds having a majority, let alone a monopoly.

On the other hand, it can hardly be denied that certain types of mathematical thinking are akin to chess thinking in many respects: non-verbal (spatial) reasoning – with regard to movements and relationships on the board that can be mathematically described – certain thinking methods, e.g., the systematic grouping (of possibilities), kinds of problem transformations (cf. Section 39), methods of proof (direct and indirect), the high degree of mental flexibility that is required, etc. Thus the question can be put reversely, namely, how the relative weakness of the correlation between achievements in the two fields can be explained. If it can be taken for granted that this relative independence results, partially at least, from differences in intellectual and emotional personality requirements, what are these differences?

Mathematicians are not so much required – and are possibly even quite reluctant – *to make decisions on the basis of incomplete data*; chess players have to do this all the time. Consequently, such methods as mental trying out and intuitive completion are less preponderant in mathematical thinking, which is generally less inductive and less ‘relativistic’ and opportunistic than chess thinking. To the mathematician it is not so much the outcome that counts, let alone the decision, but rather the strictly logical way of getting there. This means that the mathematician must explicate his reasoning; even if his result (‘move’) is obviously strong he will not accept a ‘subjectively convincing argument.’ If the mathematician’s argumentation is to be of any value, he must formulate it and write it down; the chess player has no such obligation. Mathematical thinking is largely conceptual thinking, on the basis of strict definitions.³² As a result, real productivity in mathematics beyond the solving of problems and mathematical puzzles requires a *wider spectrum of aptitudes* than do the achievements in chess.³³ As stated in Section 62, an illiterate might become a master

32 On the other hand, the flexible, seemingly arbitrary way in which mathematicians are apt to set up or modify the definitions of their concepts in an imperious independence from the particulars of language usage and physical phenomena often lends a strong flavor of ‘relativism’ to their thinking.

33 The fact that the two fields are, of course, not comparable in their respective scope, variability, richness, and social utility does not concern us here – even though it is related to the band width of aptitude requirements.

chess player, but an illiterate mathematician is a *contradictio in adiecto*. In principle, the chessmaster’s ‘system of experience’ need not be verbally and conceptually anchored, which means, among other things, that *systematic book study and learning* is not an absolute requirement as in mathematics. On the other hand, chess achievements do require a continuously active mental productivity and a readiness for intuitive judgments and decisions that cannot be deductively or systematically derived.

With the latter observations we again approach the field of emotional and motivational personality differences. It is probably here that the differences between the chess player’s and the mathematician’s personalities are most crucial. The chess player’s passionate drive for productive thinking, decision making, playing, and fighting – all so characteristically blended in chess – finds no counterpart in the mathematician’s personality make-up. Correspondingly, the fact that so many chessmasters fail to complete their academic studies, be it mathematics or any other field, cannot be imputed to some general ‘weakness of will power’ – hardly an epithet apropos for men like Wilhelm Steinitz – it can rather be ascribed to a strong *aversion* or even inability *to study receptively and passively from books* in a situation where no productive goal is within sight. In a study situation the typical chess player’s need for a mental productivity of his own is strongly frustrated, while he can satisfy it to his heart’s content by playing chess. There is evidence that good chess players have the capacities to become productive scientific workers as soon as they find a way of getting through the receptive stage of learning that is often so frustrating to them. Nevertheless, chess activity still provides more direct opportunities for satisfying fighting and playing needs as well as short-term ambitions. If such needs are part of a person’s basic personality, chess may be enormously attractive to him; if they are not – as with many mathematicians and scientists they are not – chess is just an uninteresting game. Whether inborn or learned, it is clear that the characteristic drive patterns for the two groups differ systematically in these respects.

Is it possible to group the personalities of chessmasters into some tentative typology?

Let us make two assumptions to start with: (1) Great achievements in chess do not require per se an exceptionally high verbal-conceptual intelligence disposition; and (2) the course which the rest of the chess player’s life takes depends largely on whether or not he was,

educationally and environmentally, in a position to avail himself of a formal training without too much concomitant frustration. Thus two factors are introduced: What becomes of the personality of a gifted chessplayer is considered to depend, first, on the *band width* of his natural intellectual capabilities and interests (verbal intelligence especially, as a general vehicle), and, second, on the *opportunities* for 'easy' avail of proffered formal training. It is these two factors and their interaction that appear to be of decisive importance for the resulting general level of erudition and personality, in terms of breadth of interests, intellectual (or artistic) achievements in other fields, etc.

First, there is a group of masters who are hardly interested in anything but chess, do not do anything special with 'the rest of their lives,' and do not evince any brilliance of mind. They are chessmasters *only*, one-sided not by preference but by natural (and nurtural) limitations. In conversation they create an impression of mediocrity – apart from an ever present humor peculiar to chess – rather than of high intelligence. This group is relatively small (perhaps 10%).

Second, there is a large 'intermediate group' of chessmasters who lead another life but do not distinguish themselves by any conspicuous versatility or accomplishments. Their chess talent can best be viewed as a conspicuous top of a profile of capabilities, an above average but in no way protuberant profile. This is a frequent type among professionals from a background of a simple home, who did not have – or did not use – the opportunity for a good formal education. Characteristically, the level of their verbal or other non-chess achievements is lower than their verbal intelligence which is, again, much lower than their 'chess intelligence.' This group may comprise one-third to one-half of the whole (grand)master population.³⁴

Third, there is the group of (grand)masters who have shown that they are rather highly gifted in other respects too. This group contains, in the first place, the amateurs with a high and intellectually demanding social position (e.g., von Kolisch, Bernstein, Capablanca, Vidmar, Euwe, and others). Furthermore, masters who have distinguished themselves by specific achievements in other fields, such as Philidor (a composer), Staunton (a Shakespearian scholar), Lasker (a mathematician and philosopher), Tartakower (a literate writer). Finally, there are those without an engaging social position or

34 Hopefully, the reader realizes how highly hypothetical this whole classification is, particularly the numerical estimates. Both are based on an unavoidably subjective evaluation of biographical data, communication with others, and personal experiences in the chess world.

specific achievements in another field to their credit but who evidence a high general level of intelligence and erudition.

In the lower ranks of masters and near-masters many more examples of the third type are found, some of them outstanding in another field as well. The well-known English historian, N. T. Buckle (1821-1862) was one of the strongest chess players in Europe in his time. Von der Lasa (1818-1899) was of almost the same playing strength and was an important figure in the German diplomacy of the nineteenth century – e.g., ambassador at Copenhagen in 1862. The famous Swiss histologist, O. Naegeli, represented his country for many years as the first board player on the national team. Geniuses in two fields – one of them chess – do not appear in the collection; but these are, of course, very rare.

A special category could be formed by those who in their lifetime showed evidence of quite remarkable other talents – music, mathematics, literature, etc. – but only in the chess world managed to acquire some fame and to be remembered. A striking example of divergent activities was Lionel Kieseritsky (1805-1853), who studied philosophy and law, made a living by giving private lessons in mathematics, wrote poems, and invented, played, and propagated the so-called three-dimensional chess. He was described by his contemporaries as a witty, highly musical, literarily gifted person with a stray talent for acting, an original, lively, passionately active mind, and a real artist by nature. He must have been an extremely interesting personality and versatilely gifted, indeed, but it is only in the chess world that he lives on – mainly because he lost the 'immortal game' to Adolf Anderssen!

An even more picturesque personality from the same era was the Hungarian, Vincenz Grimm (died 1869), whose name is recorded in the chess annals as one of the Budapest master-triumvirate (Löwenthal-Szen-Grimm) who won the (first) chess match by correspondence against a Parisian team in 1843-1846. Apart from chess, Grimm was an excellent pianist, a prominent linguist, a crack whist player and billiards master – and as such well-known throughout the Austro-Hungarian Empire – and a highly gifted drawer. Originally he had an art business in the city of Pest, later a lithographic studio. In 1848 he played a rather crucial political role as a member of Kossuth's revolutionary government; he printed the notorious 'Kossuth'-banknotes in his studio. After the regime fell, he had to flee: he went to Turkey, established himself as a Moslem in Istanbul, and there earned his living by making drawings for the Turkish general staff and by

giving lessons in chess and drawing in the garret where he lived. His room overlooked the Bosphorus – a magnificent view, according to visitors, in strong contrast to the unspeakable chaos inside.

Many other cases, historical as well as contemporary, could be given to illustrate this type of chess player: numerous talents, but divergent and not well co-ordinated; great promise but an imbalance, checking high level achievements – except in chess where to a high degree one can make do without constancy and learn and develop ‘in play.’

If a high, rather general giftedness or an array of other talents is given by nature to a chess player, what will become of him depends, apart from environmental influences, largely on his power of self-integration (‘will power’): be it a strong amateur well settled in his social position, a successful professional player with erudition and at least some residue of other interests, a man who manages to maintain more than one field of productive activity and to succeed in both, or an adventurously artistic character without much goal-directedness in life, who to a certain extent becomes the victim of his divergent talents. It is not by accident that this last type is rarely found among the great chessmasters; top achievements in chess, as in other fields, demand constancy and concentration – in other words: a more bourgeois way of life.

EPILOGUE 1963

Section 67: The use of introspective methods

Throughout this study methods that can be called introspective have been freely used. While this is in accord with a still living European tradition, introspection is known to be rather suspect in the eyes of most – if not all – scientifically minded American psychologists. We shall not now unearth all the pros and cons of introspection, let alone offer an ‘apology’ to behavioristically inclined readers. It does seem in order, though, to begin the epilogue of the English edition with enough comment to clarify the author’s standpoint.

1. Introspective reporting is behaving

In his book on thought and judgment Johnson remarks that the work of the Würzburg introspectionists was never refuted or laid aside as unimportant; it was just discontinued (cf. JOHNSON 1955, p. 6). In much the same way the use of introspection as an experimental or observational tool was never definitively condemned; it just went out of fashion. In the United States in particular, the wave of behaviorism overtook the remnants of introspectionist methodologies. The defeat was primarily a social one, but the aftermath was nearly the same as what would have ensued had the otiosity of the method been shown: introspection became suspect, regardless of *how it might be used*.

What was really objectionable in the older investigations was the *use* that was made of the method and the introspective data. The emphasis on *content* rather than on the process rapidly grew theoretically sterile; such an emphasis could not but lead to descriptive distinctions and refinements that were hopelessly unverifiable in the last analysis (cf. Sections 22 and 23). *Process introspection* – the only variety we are concerned with here – was rarely employed in pre-Selzian days. When it was, the protocols were all too often supposed to reflect, directly and reliably, the ‘real’ course of psychic (or mental) events. The implicit claims were, first, that such a real – and unique – course of psychic events did ‘exist,’ second, that it could be unearthed

by means of introspective methods and, third, that this endeavor was the royal road for psychology to tread. Of course, these pretensions have been thoroughly refuted by now, but it so happens that the baby, viz., introspection as a method, was thrown away with the bath water.

A little argument is all that is required to restore the respectability of introspective methods. The first statement is self-evident: *If a person introspects he behaves*. That is, protocols produced by experimental or observational methods of the introspective type are registrations of behavior. In the case of 'systematic introspection' or 'thinking aloud' procedures we are even dealing with behavior that is experimentally controlled, to a certain extent. In general, it is true that introspective behavior is not readily measurable, but in this it is joined by many other types of human behavior that are regularly studied in psychological research. Nothing, in fact, prevents the modern psychologist from making use of behavioral data that are generated by introspective methods.

2. Many forms of 'introspective behavior' are highly reliable

'Introspective behavior' is the behavior of a person or subject who – in the latter case on request – introspects (or retrospects) in one way or another. It belongs to a specific subclass of verbal behavior which is elicited whenever a person tries to give a truthful answer to the question of *what he thinks he is doing*, or *how*, or *why he thinks he is doing it*. It is used especially – but not exclusively – when what he is 'doing' is a mental activity such as thinking, perceiving, or remembering; that is, an 'activity' which we all acknowledge though in itself it is not observable behavior.

It is extremely difficult, if not impossible, within this subclass of verbal behavior to draw a defensible borderline between what is generally meant by introspection (in the specialized sense) and the more common cases of answering what, why, and how questions on one's own (mental) activities and experiences. It can easily be maintained that everyday questions like: 'What do you see right now?' 'How do you feel about...?' 'What do you think of that plan?' 'What are your motives for choosing this line of action?' or 'How do (did) you solve the problem?' require some introspective effort from the respondent, provided that he is ready to take the question seriously and sincerely. In fact, these are essentially the same questions a subject is requested to answer in an experimental setting requiring introspection proper. Apparently, there exists a continuum of 'intro-

spective effort' or 'depth,' but just where on this continuum 'introspection proper' starts is not clear. In defining the universe of 'introspective behavior,' we shall therefore include the everyday cases.

Contrary to current opinion in some schools of psychological thought, the information yielded by such introspective behavior is generally quite reliable, in the sense that we actually do rely on it in daily communication and interaction with others. Only if we have reason to doubt a person's *seriousness* or *sincerity*¹ – the two conditions mentioned above – do we question the reliability of his answers.

3. Experimental controls improve the reliability of introspective behavior

In introspection both seriousness and sincerity can generally be controlled in an experimental setting; that is, we can make sure of 'co-operative' subjects. There is then no a priori reason to disbelieve that subjects feel, perceive, think, etc., as they indicate. There are, of course, limitations and problems of experimentation and interpretation. There is, for instance, *the limitation of the subject's ability for verbal expression*; the danger of generating insincerity by *suggestive* questions or questions that are too specific; the general problem of *the influence of the experiment(er) on the process*. But none of these is peculiar to introspective procedures in the traditional sense. They are common problems of experimentation in the behavioral sciences. We know how to handle them partially, but we also know that they will continue to induce some distortions and disturbances – although we may be able to minimize or even eliminate their influence.

One problem is rather peculiar to introspective experimentation, namely, that of *partly losing track of* (forgetting) what one is or was doing – respectively, how or why one is doing something – either before or during the introspective reporting. The first operating rule to control forgetting can be derived from our multiple experiences in everyday life (and, for that matter, in psychological experimentation): the longer ago the mental activities, the less the subject remembers, thus the less informative and/or reliable his reporting is. Another general methodological rule says that specific questions, about recent events, are more likely to be answered reliably than general questions. The two rules together, both corroborated time and time again, tender

¹ A person may be consciously sincere but 'unconsciously insincere,' e.g., he may rationalize his motives. That is, there are instances where a sincere attitude goes with an answer that, by other means of analysis, can be shown to be biased, quite incomplete, or fabricated. But, then again, we generally have 'good reason to doubt his sincerity.'

the case for experimentation. Asking a composer in an interview or survey, out of context, about his inspiration, or a chess player about the number of moves he usually sees ahead, is not likely to lead to reliable results: the introspective reporting should be as close as possible to specific events. Experimental techniques such as 'systematic retrospection' and 'thinking aloud' achieve this end in different manners – with corresponding pros and cons (cf. Sections 23 and 68).

In general, it should be clear that *sound experimental procedures can get all of the aforementioned disturbing factors under control to some degree and thereby improve the reliability of the subject's report.*

In the latter sentence 'reliability' means, in the first place: reliability as a report of what the subject *thinks* he does, or how, or why he *thinks* he does it. In the second place, it refers to the fact that we tend to rely on such information in daily life (cf. point 2 above), i.e., we mostly consider it 'true,' we believe that the report roughly reflects what the subject 'really' does or has done mentally, or how he 'really' did it. Now the question remaining is: of what importance is this practical reliability – truth, dependability – from the point of view of scientific truth. In other words: In what sense, if any, and in how far can an introspective report (protocol) be considered to reflect 'what really went on in the subject's mind'?

4. *Introspection can benefit the development of theory*

In modern eyes, 'what really goes on in our mind' is, of course, not a reality that must and can be unearthed by introspection but rather a hypothetic process that we should like to understand by describing it in the parsimonious terms of a theoretical concept.

Theories can be constructed on various levels. In the case of the study of thinking, for instance, we can at least conceive of neurophysiological theories, on the one hand, and of psychological theories, on the other. Within the latter group – the former is not of concern here – 'levels,' or degrees of detail in explaining the facts, can again be distinguished: micro versus macro psychological theories, molecular versus molar concepts, microstructure versus macrostructure analysis. In the present still rather embryonic state of knowledge of thought processes, there do not appear to exist a priori reasons to prefer one level of explanation over another. Rather the important criteria are whether (1) the theory is logically well constructed, (2) it more or less covers the field in a descriptive sense, (3) it produces strictly derived and testable hypotheses, (4) the predictions derived from the hypotheses come true and, finally (5) the theory can be fruitfully

applied. In this sense, a good theory is useful regardless of level of explanation; and we cannot but hope that macro and micro theories will, in the last analysis, fit together – in an as yet unknown way – and complement each other as they do, for instance, in physics and, somewhat less convincingly, in economics.

In what ways can introspective procedures be instrumental in developing good theories?

First, ideas in psychological theorizing frequently originate from some sort of introspection. In most cases, the source is just the theorizer's armchair introspection in combination with his reflections on experiences with the behavior of others. Although this unorganized, intuitive kind of introspection is likely to remain of some importance for theory and concept formation in psychology, experimentally organized forms with other persons as subjects are certain to enlarge the fund of data, to improve the reliability of the factual bases, to prevent overly subjective theorizing, and to further fruitful generalization. For that matter, the contents of this book provide an elaborate demonstration of *the value of introspective methods for hypothesis and theory formation.*

Second, in its vital theses a good theory on thinking should not be incompatible with consistent findings of introspective experimentation. True, for a good macro theoretical model to emerge, much of the detail of some introspective protocols must necessarily be disregarded, but the assertions of the model should not contravene experimental evidence. If, for instance, the model predicts an A-B sequence for two subprocesses while introspective protocols show a majority of B-A sequences, the model should be modified, to say the least. In this respect, introspective findings are just a subclass of a much larger class of oft-times neglected laboratory findings. Particularly in cybernetics theoretical models have now and then been constructed that were already definitely refuted at the time of their publication, sometimes by much older experimental findings of which the author was presumably unaware. To quote a recent warning by Kolmogorov: 'It is often forgotten that analysis of higher forms of human activity began a long time ago and has made considerable progress. And although this analysis is being carried on in non-cybernetic terms, it remains essentially objective, is much-needed, and must be studied and utilized' (KOLMOGOROV 1962, p. 52).

The statement above, that the vital consequences of a theory should not run counter to reliable and consistent introspective evidence, means, phrased somewhat differently, that *introspective experimentation*

can be utilized for testing theories. The possible uses of introspective methods are by no means limited to the preparatory stages of theory development. Strictly designed introspective experimentation aimed at the testing of specific hypotheses is quite possible and can even become crucial.

5. *Simulation: a new look for introspection*

A disadvantage of most of the older theories of thinking is their relative vagueness. The concepts employed are just verbally descriptive and/or loosely explanatory while their logical relationships remain rather ambiguous. Even the more empirical concepts are hard to operationalize. As a result, it is difficult to derive testable hypotheses and crucial predictions from these older verbal models – if models they can be called.

The theory of Selz, as it was expounded, modified, and applied in this book, is certainly stricter and more comprehensive than were most other early attempts at theorizing on the thought process. The present author has tried, with some success it is hoped, to operationalize a few of the central concepts; besides, he has been able to show, particularly in later work (DE GROOT 1954 and 1956) that it is quite feasible to derive from the theory a few crucial hypotheses and applications by means of which the theory can be tested. But even so, in proportion to the scope and pretensions of the Selz-de Groot theoretical system, the openings for empirical testing and improvements were, until recently, unsatisfactorily few. Even if vaguenesses and ambiguities are weeded out, the 'problem of complexity' (NEWELL, SHAW and SIMON 1958b) remains: the hypothesized mechanisms and variables are so many and so interdependent that the classic experimental strategy of testing one hypothesis at a time necessarily falls short of providing the empirical information needed. It should be possible to test the model as a whole (cf. DE GROOT 1954, p. 117), but until recently this was hardly practicable in an efficient manner.

The situation has changed radically, however, since the genesis of computer simulation of human thinking. With the advent of the high-speed digital computer it has become possible to design rational decision and problem solving programs of various kinds – from highly specific to rather general tasks – and to try them out on the machine. If this is carried out by starting from a given set of theoretical statements which are then objectively elaborated and programmed for a computer, it is in fact the theory which is worked out and put to the test. The test is in the machine output after new problem solving

or decision tasks are given to it: its results are compared to those of humans doing the same tasks (cf. NEWELL, SHAW and SIMON 1958a; GREEN 1961).

It is here that introspection re-enters the picture. For it is not sufficient to compare the *achievements*, the end results of the computer program, with those of the human. If these are on a par – or if the computer is superior – this does not guarantee equivalence or even comparability of the underlying mechanisms. Therefore, a second criterion is found in the comparison of human and machine *processes*. But in the case of theories of human thinking the only way to get sufficiently detailed information on the process is by employing introspective methods. It is, of course, a difficult methodological problem to decide just how far the requirements of equivalence or resemblance between human and machine protocols should extend, but there is no room for any doubt that for a comparison on any set of established criteria human introspection protocols are needed. They are now needed for testing and improving theories on thinking that are explicitly written in program language.

Thus far only few investigators – among them NEWELL, SIMON and co-workers (1958a, 1958b, 1961); see also LAUGHERY and GREGG 1962 – have taken the task seriously enough to develop a theory by means of simulation, by producing introspective protocols of human problem solvers under adequate experimental conditions. Apart from the fact that thus far few psychologists have understood and accepted this idea, the suspicion that behaviorism cast on all introspective investigations, as JOHNSON put it (1955, p. 6), has not yet been overcome. The present situation is certainly ripe for a *renaissance of introspection as an experimental method* – in a new light.

Section 68: On introspective techniques

In an experimental setting introspection can be utilized in a variety of ways. It may be used as the *sole source* of information on the process as in most of the experiments described in this book; or it may be *supported by other data on the subject's behavior*, his manipulation of materials, trying out of solution proposals, taking of intermediate steps (e.g., pushing buttons), and the like. In the latter case, introspective data may still be of primary importance or alternatively may serve as *commentary* by means of which observable behavior can be better interpreted. Further the subject may be requested to report (or comment) *during* the process, as in problem solving experiments when

thinking aloud is used; or he may have to report *after* carrying out the main task, as with systematic introspection (retrospection). The experimenter may now and then *interrupt* the subject for specific questions, maybe even break the process off in order by interrogation to get hold of the subject's total goal conception;² or he may think it preferable to let the process run its natural course in as far as that is possible. Finally, each of the standard specific problems of experimentation – e.g., problems of instruction, registration, checks and controls, protocol interpretation and coding – can be solved in a variety of ways.

Is it possible to formulate technical recommendations for introspective experimentation?

Obviously, which technique is most adequate depends on a number of other factors: the type of task and process under study (e.g., creative thinking, rational choice, practical problems, perception tasks); the scope and pretensions of the theory or program that is being developed (e.g., specific versus general problem solving, developing one person's program or a general theory) and, perhaps most important, whether the research goal is primarily theory formation or theory testing. Even so, some general remarks can be made. We shall present them in line with the standard experimental problems just mentioned.

1. *Instruction.* As we know from Chapter II of this book (cf. Section 16) Otto Selz was the first to consider consistently a directed thought process as a *sequence of operations*.³ Consequently, he read introspective protocols in terms of operations, i.e., in terms of what the subject was 'doing' mentally – in accord with the definition of introspection given in the preceding section (p. 372). In his time this rather revolu-

2 In the present study this technique was not applied but was, and is, recommended. It may become particularly important in simulation research, if used to set up criteria for the equivalence of human and machine processes: the machine should be able at any time on interrogation to produce its total goal conception.

3 Selz did not need to distinguish between 'operations' and 'solving methods' in his system, since it was meant only to cover directed thinking (reproductive or productive) in which every operation can be viewed as the application of a solving method. Aside from the possible desirability of maintaining some distinction between the two concepts within the theory of thinking, their equation is certainly untenable in other fields of human mental activity. In perception, for instance, where no problems need to be solved, *operations* can be observed or hypothesized. If perceptual processes are simulated, perceptual operations are supposed to result from *more generally applicable instructions in the program* – which is equivalent to Selz's derivation of thought operations from solving methods.

tionary way of interpreting protocols was little understood and hotly criticized, whereas nowadays it has become self-evident. In simulation research, at least, the possibility of conceiving of a process of thinking – or perception, for that matter – as a sequence of operations is an indispensable working hypothesis; and it has already proved to be a very fruitful one.

Since the search is for what the subject is 'doing' mentally or for how (and why) he is doing it, it is only natural to phrase the instruction in a way such that the subject will, in fact, focus his attention on his successive mental activities.

With *systematic retrospection* focusing is particularly essential. If a subject understands his task well, the introspective report may contain very valuable indications of his operations and their sequence, but if he indulges in describing the peculiar qualities of some of his inner experiences – as (European) students who have been 'trained in introspection' are sometimes prone to do – reporting on the process may be quite obscured. 'Process introspection' must be induced by a somewhat directive instruction; compare the instructions reported on pp. 78, 96 and 323 in this book. Moreover, it must be learned, like looking through a microscope must be learned. For the subject the main problem is to maintain a specific set by means of which he can select from his inner experiences that which is relevant to the process. Instructions should, therefore, stress the *what-have-you-been-doing aspect and*, in addition, *the chronology* of these 'doings.'

In case the technique of *thinking aloud* is used, the problem of making the subject select what is relevant to the process – focusing on mental activities – is much less critical because he hardly has an alternative. The main difficulty here is rather one of activating the subject to report at all. With some subjects gaps and pauses in reporting are frequent and of such a long duration that they cannot be assumed to result from actual pauses in thinking. He may just temporarily forget his second task (to think aloud), or he may not be able to verbalize adequately what he is or has been doing mentally. The technical question in point is whether or not, during such pauses, the experimenter should activate the subject by asking specific questions.

From personal experience the author is not in favor of asking questions during thinking aloud processes: they may be disturbing at the moment and have the additional effect of making the subject feel tense and ill at ease. The occasional gentle prod during the experiment not to forget to think aloud is about the allowable maximum. The problem of the silent subject should rather be solved

in a different manner, namely, by selecting subjects who verbalize easily – such as Dr. Max Euwe and Nico Cortlever in the present study – or possibly by training subjects. It should be noted that no serious objections can be raised against such measures provided the research goal is *theory* (or hypothesis) *formation* – program construction included. Only if introspective techniques are used for the *testing* of hypotheses derived from a theory on some human process in general, (i.e., not one person's program) are both selection and training of subjects objectionable, from the point of view of generalization. Thus far, however, strict experiments of the latter type have rarely, if ever, been executed (cf. Section 67, p. 377).

2. *Registration.* On several occasions in this book it has been remarked that transcriptions of tape recordings would have been preferable to the necessarily incomplete protocols produced by the experimenter's labors. The tape recorder – now standard equipment in introspective experimentation – does not solve all registration problems, however. It does guarantee the completeness of a protocol with regard to all audible subject behavior, it is true, but it does not guarantee the *understandability of a protocol*. On the contrary, if a subject is reporting 'to a tape' and not directly to an experimenter who is attentively trying to grasp his meaning, he is likely to spend less effort in making himself understood. Correspondingly, the experimenter, in his confidence that 'all behavior is registered,' may be inclined to underestimate the problems of protocol reading, interpretation, and coding that will arise when he reads the transcription. For this reason, tape protocols are often less clear and less informative of the process than hand written protocols.

This is not to argue against the use of a tape, of course. The relevant technical recommendation is rather to take another step: make a tape recording but *play and re-examine it immediately after the experiment proper*, in a co-operative effort by experimenter and subject to improve its understandability. In this way many obscurities can be resolved – such as simply determining what an expression like 'that is no good' exactly refers to – and a number of gaps in the subject's reporting can be filled in. The main advantage tape recording has over writing is, in fact, that it is so easy to go over the whole process again. The results of this inquiry in turn should be registered, either on a second tape or via the experimenter's notes.

'Understandability of a protocol' appears to be a somewhat global but essentially sound criterion (cf. Section 23). It implies, roughly,

that the experimenter should be in a position to understand to which objects in the environment (e.g. pieces, position characteristics), to which abstractions (findings, results), and/or to which operations every statement in a protocol refers. The protocol as a whole should be an understandable overview of *what* the subject has, consecutively, been 'doing.' The question of *why* he has been doing it in this particular way or sequence, in terms of his system of methods or personal program (cf. LAUGHERY and GREGG 1962) is a matter of later analysis and hypothesizing, however.

3. *Checks and controls.* In this subsection only one question will be raised: Is it desirable to design introspective experiments in such a way that the protocol and its interpretation can be checked by means of a *parallel record of subject behavior of a more tangible type*?

In their article on simulation of human problem solving behavior, LAUGHERY and GREGG (1962) analyzed and constructed models for the search behavior of their subjects on the basis of a double series of data: consecutive settings of switches as operated by the subjects and a corresponding series of introspective remarks produced by thinking aloud. It is clear that the two synchronous and interdependent series of behavior data supported each other. The analysis of the subjects' search behavior was thereby facilitated and the likelihood of constructing an adequate model increased. The relevant methodological questions are, first, whether the investigation of processes of combined thinking and acting that naturally produce parallel records is to be preferred over the investigation of 'pure' thinking (or perception) and, second, if pure processes are analyzed by an introspective technique, whether parallel records of neurophysiological or, for instance, intermediate choosing behavior should be artificially produced.

To the first question: Acceptance of introspection as a legitimate experimental tool implies that there is no reason to avoid the investigation of 'pure' processes if such processes are interesting and considered important on other grounds – as for instance the thought process of the chess player.

The second question cannot be answered quite so categorically; it depends, again, on the researcher's goals. For example, the perception experiments reported in Section 61 of this book served mainly to show the large difference in achievement between masters and non-masters; the introspective record was only used to support (with some new evidence) a pre-stated interpretation of this (predicted)

difference. To this end the technique of 'pure' systematic introspection was sufficient. If, however, the research goal had been to produce a detailed analysis of the perception process, in terms of sequences of operations, perceptual units on the board, etc., it would have been useful to register eye movements and to try in the interpretation to synchronize them with the subject's retrospective information on the chronology of his mental 'doings.' Analogously, for the purpose of a more refined (hypothesis testing) analysis of chess thinking, it might be useful to register physiological variables such as EEG-curves, etc. (cf. recent investigations of sleeping and dreaming, e.g. KLEITMAN, 1960). One might also consider providing handles and/or buttons to a subject who thinks aloud in a problem solving task – whether chess or another – by means of which he can indicate which solution proposal he is currently considering and/or how his preferences and evaluations stand. A parallel record of such data might be quite helpful in analyzing the process.

The danger to be avoided is that the additional task, the interruptions needed, or just the paraphernalia of the experiment may disturb the natural course of the process. It must also be possible to post-synchronize the introspective records with the other behavior records in a reliable way. Thus, artificial parallel behavior registration can only be recommended with rather strong reservations. In any case it does not make sense to avoid 'pure' introspective experimentation due to a prejudice that introspection would not be 'real' behavior or would not be respectable.

4. *Protocol interpretation and coding.* The question of how a given introspective protocol should be read, interpreted, possibly polished (if published), fragmented, and coded, presents a number of difficult problems.

Again, both the character and the weight of these problems depend largely on whether the experimenter's goal is theory *formation* or theory (hypothesis) *testing*. In the former case the investigator has a certain freedom to use the materials as he chooses, i.e., as he considers most fruitful; he can make up his own rules. But rules there should be, especially if protocols are to be published and used to justify, support, or explain the investigator's way of theorizing. The advantages of systematic experimentation (cf. Section 67, point 3) can be fully exploited only if inconsistency and arbitrariness in handling the data are excluded. But it is largely up to the investigator to decide how he is going to realize this exclusion; his responsibility is much less

heavy than it is in the latter case. If theory or hypothesis testing is the research goal, *ad hoc* interpretations are taboo: predictions on introspective phenomena, like predictions on any other kind of phenomena, presuppose objective operationalizations. The procedures for protocol coding, in particular, must be objectively established in advance – consonant with the hypothesis to be tested.

We shall restrict our discussion to the former case, however; we assume that introspective protocols are used solely for purposes of theory formation. This implies that nothing more than recommendations can be given, not strict prescriptions.

Let us first consider the case of *protocol publication*. We assume the original (verbal) tape record has become 'understandable' by the joint effort of subject and experimenter. Understandability of the text to the experimenter, however, does not necessarily imply understandability to a reader of the tape transcription. If it is to be published, in whole or in part, it is general practice to make certain that a reader who has neither attended the experiment nor the inquiry can get the same meanings the experimenter has grasped. This can be achieved with small changes in the text: by indicating in a more complete, unambiguous manner objects, abstractions, and operations the subject has referred to (e.g. chess notation, cf. p. 83); by extending elliptic sentences a little; by introducing adequate punctuation; by starting new paragraphs wherever it is clear that the subject starts a new phase. These techniques are acceptable, provided text changes are restricted to the minimum and never go beyond what the experimenter has really understood. Unwarranted interpretations can best be prevented by *asking for the subject's approval* of the changed, typed out text as soon after the experimental (and inquiry) session as possible.

According to these rules the protocols in the Appendix of this book have been somewhat 'padded,' although the rules were not always applied as strictly as the author would now deem desirable if he were to do the experiments again.

Much more difficult problems arise in the *structural and dynamic interpretation* of a possibly padded protocol. In the present study the research goal was primarily to derive from the experimental data a general, explanatory, verbal *description* of chess thinking. It was of vital importance, therefore, to interpret protocols as thoroughly and as objectively as possible: the 'general description' or, in other words, the design of a verbal theory on chess thinking, had to be based solely on empirical findings, with a minimum of arbitrary assumptions and

decisions. As it was impossible to completely objectify the criteria for interpretive decisions – on matters of, e.g., phase structure (protocol fragmenting), hierarchies of subphases, interstitial processes that were not reported but assumed to occur, the dynamics of transitions and, finally, the ‘system of methods’ (program) that was hypothesized to underlie those dynamics – much *ad hoc* effort was spent on avoiding subjective arbitrariness and on interpreting as consistently as possible. In working through the previous chapters the reader has certainly noticed this: the text seethes with discussion of categorization and other interpretation problems.

From the point of view of methodology (anno 1963) the first important question is whether the effort required for such meticulous protocol categorization and analysis is well spent. Taking into account that the research goal is, in fact, solely theory formation and that at best a limited number of verbal and somewhat vague theoretical assertions can emerge from the process, should we not rather take the descriptive, interpretative part less seriously and try to construct a program model on a less detailed basis in order to improve it later by objectively testing it as a whole?

The question amounts to asking whether an investigation such as is reported in this book would still make sense in the present situation. The author’s answer is yes – with the qualification that the modern introspective experimenter should analyze his data more directly with a view to its utilization in simulation research. Revolutionary as the simulation (or cybernetic) approach may be, it has after all only reinforced *the need for solid and detailed descriptive studies*. The safest way to prevent a program model from running counter to reliable experimental evidence (Section 67) is to base it on the main findings of already available studies. In many fields of research on cognitive processes, however, the relevant introspective evidence is either insufficiently known or highly incomplete if not non-existent. As a result, many a now current simulation model is likely to run demonstrably against rather simple future experimental evidence. The only effective way to improve the fit is to work steadily and co-operatively on both sides: the descriptive-interpretative with the inductive-constructive.

The second important issue is that of *the limits of justifiable interpretation*. We cannot now enter upon an analysis of this formidable methodological problem (cf. DE GROOT 1961, pp. 326-343 and DE GROOT 1963); suffice it to say that in protocol interpretation this problem will remain ‘hot’ regardless of the introspective technique

used. Where no completely objective coding system exists ‘borderline cases’ and corresponding *interpretative uncertainties* are bound to arise. In Chapters IV through VII of this book they have been amply and variously discussed. Interpretative uncertainties occur even with regard to the application of the most fundamental descriptive concepts, e.g., whether a given protocol statement must or must not be considered a ‘solution proposal,’ or, where in a given protocol the ‘first Phase’ ends. These uncertainties can only be solved by somewhat arbitrary decisions. Often it is partly on such decisions that the resulting picture of, for instance, the hierarchical problem structure or the dynamic problem development in a thought process rests (cf. Chapters V and VI, respectively). If the research goal is theory formation, there is no harm in this situation, provided that the investigator knows and explains what he is doing and tries to limit arbitrariness and subjectivity to a minimum.

There is one specific angle to this issue, however, that deserves special attention: ‘uncertainty’ in the interpretation of a protocol element may reflect a real *ambiguity in the thought process* itself. Since the implications of this possibility are of basic importance for any theory of human thought, whether in program form or not, they will be discussed in more detail in the next section.

Section 69: Ambiguity and multifunctional operations

Probably the most frequently voiced early criticism of Selz’s theory of thinking was that it was ‘mechanistic,’ i.e., the human mind was hypothesized to operate ‘like a machine.’ Such a statement may still be expressive of an emotional dislike among some of the more tender-minded psychologists but, as a rational criticism, it is definitely outdated. Even so, one of the underlying uneasinesses many of the older psychologists experienced when confronted with Selz’s theory is still worthy of serious examination; to wit, that it makes a thought process look much more rational, rigid, and explicit than we feel our thinking ‘really is.’

More recently this point of view was taken up and elaborated in detail by VAN PARREREN (1952 and 1953). According to him Selz’s theory is highly ‘rationalistic’; particularly unwarranted is the way in which both Selz and De Groot analyzed their protocols to make them fit their preconceived idea of how a thought process should operate. The expression ‘preconceived idea’ refers, of course, to the set of postulates on which the Selz-De Groot model – and, for that matter, every

program model – is based. In this model the thought process is conceived as a linear series of distinct operations, each of which can be viewed as the application of a distinct, specific solving method. A specific method, in its turn, becomes operative because of its being strictly linked to a distinct, specific type of problem development situation. Furthermore, since each solving method is ‘determined,’ it is supposed to aim at a specific, clear-cut subgoal that is either attained or not attained, dependent on the correspondence between the specific, distinct results (outcomes) of the operations in question and their equally specific and distinct anticipations (expectations). The problem raised by Van Parreren is whether all this is as ‘distinct’ as it is presupposed to be; are not many of these things vague, flexible, uncertain, non-determinate, *ambiguous*?

As a matter of fact, we get a much vaguer picture of what goes on in thinking from introspective evidence. Separate operations are not always so clearly distinguishable; the subject is often not aware of applying ‘methods’; the subgoals aimed at are not so clear as they are postulated in the analysis; both expectations (anticipations) and outcomes (results) often appear to be only fragmentary and uncertain, so that the question of whether the operation has just failed or succeeded is hard to answer. As a result it may be difficult to decide whether the next linking is cumulative or subsidiary – except by methods of interpretation that *make* explicit and distinct what is not explicitly and distinctly given. Or, to give a more concrete example: In order to explain in terms of playing methods (cf. Section 57) the chessmaster’s highly adequate, near-immediate reactions to a newly presented chess position, we hypothesized an implicit ‘typology’ or classification of chess positions or of parts thereof that is stored in his memory; but, if we would make him classify positions and explain how he does it the resulting picture would appear to be rather different, less rigidly ‘systematic.’ The chessmaster is certainly able to make a great many distinctions between positions, but the borderlines between ‘classes’ seem to be flexible, or indistinct, ambiguous; the whole system appears fluid rather than rigid, ‘soft-ware’ rather than ‘hard-ware.’

The facts are indisputable, indeed. In many respects the Selz-De Groot model postulates a fixed discreteness of distinguishable cases or classes; it postulates clear-cut differentiations where introspection yields ambiguous data only. Obviously, this is true for program models of human thinking as well, to an incomparably higher degree. The problem is in the evaluation of the facts, however: is it objectionable

to over-differentiate or to over-clarify the thought process in comparison with what appears in introspection?

The answer to this question can be brief: It would only be objectionable if the theory or model would pretend to simulate consciousness – but this is not (yet) the theoretical goal, neither of Selz nor of the modern machine model constructors. The pretension is solely to explain, respectively to simulate, the way the mind operates in directed thinking, in terms of (1) the end results reached (the *achievement*) and (2) the general characteristics of the subject’s intermediate search behavior (phases, chronology of trials, various approaches, solution proposals, problem transformations, problem development, in brief: the *process*). It is assumed that the process indications that can be inferred from introspective data are not to be neglected; they are supposed to be relevant and rather reliable – but they are also supposed to be highly fragmentary.⁴ They are fragmentary with regard to the unknown, hypothetical ‘real processes’ for which the theory provides a design. The fact that a theory or program model operates with a much more detailed and differentiated apparatus than we find in introspective protocols is in itself by no means objectionable. If we succeed in making theoretically predicted achievement and process characteristics fit satisfactorily with those fragmentary introspective (or other behavior) indications we possess, we can say we have a *possible theory on the ‘real processes’* in the – largely but not wholly – black box of the human mind.⁵

Can we conclude that the phenomenon of ambiguity has no value as evidence in itself? That it can be disregarded? It would be very unwise to do so. Ambiguities, i.e., non-determinable cases with regard to some theoretical distinction, may indicate that something is wrong with the

⁴ The working hypothesis, that introspective protocols yield only fragmentary data is well supported by the empirical fact that the detail of introspective data depend, considerably on the technique used (e.g., systematic retrospection may yield more protocol lines per second than thinking aloud yields per minute). It is no more than a matter of extrapolation if we suppose that the underlying ‘real processes’ are again much more complicated, differentiated, and detailed than the best available introspective technique is able to bring to light.

⁵ The restriction that such a theory would only be ‘possible,’ that is, not necessarily true, is a relative one. In a way every theory is no more than ‘possible’; witness the fact that we often try to select or construct ‘the best possible’ theoretical solution by means of principles such as that of parsimony. Most important is that a theory can be improved by continued research, i.e., made less and less arbitrary (the only possible) and more and more necessary. This can be done here as well as in any other field.

model requiring the distinction. It is here we get to a real *theoretical problem of ambiguity* that we shall discuss in some detail.

If a mental phenomenon – either ‘observed’ by introspection or inferred – is called ‘ambiguous’ this means that the *speaker* has the *intention* to determine what the phenomenon really is or will (appear to) be, in terms of *distinctions* that are current in his *conceptual system*, but finds that this cannot be done. Thus the adult finds ‘ambiguity’ (or ‘undifferentiatedness,’ WERNER 1957) in the child’s concept formation, the analyst in the neurotic’s motivation, the perceiver in (his perception of) an ‘ambiguous stimulus.’⁶ In the case of protocol interpretation the speaker is the investigator; both the conceptual system (theory) and the distinction(s) in question are his, while he has made the subject share his intention to classify or code the phenomenon.

On the basis of this brief analysis the heuristic (cue-)value of ambiguities can be specified as follows. When, in protocol interpretation, experimenter and subject agree on the ambiguity of phenomena with regard to a particular distinction, this may still be due to the shortcomings (fragmentariness) of introspection, it is true, but it may also be due to a shortcoming of the conceptual-theoretical model. In applying the model the distinction on which decisions are required may be inadequate – and possibly unnecessary or corrigible. In the latter case it must, by definition, either result in relative inefficiency of the theoretical model as compared to human thinking (the achievement criterion, cf. p. 387) or in poor process correspondence with human protocols (the process criterion, p. 387) or in both. It is wise, therefore, to pay attention to consistent ambiguities as cues for possible weaknesses in the model.

This important aspect of the problem of ambiguity can be illustrated by some examples taken from the conceptual framework of the present study.

In chess thinking examining a move and its consequences is a very frequent macro-operation. According to the conceptual system used in this book it may, however, result from the application of a variety of solving methods, each of which serves a different *goal*. For instance, we may contrast neutral *exploration* of consequences with (*proof-*)*directed investigation* of a move. If the goal is exploration the subject tries primarily to learn ‘what sort of things’ may happen after move Z_1 . The test to which he puts Z_1 (to see what it is worth) is still rather non-committal; as yet he has not much preference for (or against) Z_1 . If

⁶ Obviously the stimulus itself is called ambiguous if all or most perceivers experience ambiguity in the above sense.

the goal is direct investigation, however, Z_1 is the favorite and the subject tries in his analysis to make sure that Z_1 meets his expectancy. For another example, consider the goal of *positive proof* as against *negative proof* – both frequent in the later stages of the thought process. In the former case the goal is to prove that the move under examination is good enough (up to the level of expectancy or better than any other move) and to *prepare to play* it; in the latter case the goal is to prove that it does not satisfy the expectancy or that it cannot stand the comparison with some other move and thus can be *eliminated*. Both goal distinctions have been amply discussed and illustrated in this book; there can be no doubt that they are descriptively relevant. The questions now to be answered are:

(1) In how far are these goal distinctions theoretically relevant, i.e., in how far are different goals theoretically supposed to entail different processes, different sequences of operations, different strategies of thought organization (or correspond to different routines)?

(2) In protocol interpretation are there (many) cases of ambiguity with regard to these goal distinctions?

(3) If so, to which possible weaknesses in the conceptual model do they point?

(4) Can we indicate ways in which such weaknesses in the model can be eliminated?

Re 1: If in examining a move or variation the goal is *exploration*, only a few continuations are generated (often only one) at each point in the calculation; there is no striving for complete coverage; the final outcomes of variations are only used as sample results. On the other hand, the dynamic possibilities hit upon are well retained in memory for later use in generating possible first moves and possible own and opponent continuations.

If, however, the goal is *directed investigation*, say, of a favorite move Z_1 , many more continuations are generated; there is a striving for completeness in the examination of counter-branchings in particular; outcomes are seriously measured against a fixed expectancy; moreover, it is no longer necessary to retain the moves of those counter continuations that are satisfactorily refuted: they are simply eliminated.

Apparently, two different strategies of thought organization have been described. The fact that most differences were in terms of ‘more’ and ‘less,’ thus relative rather than absolute, does not diminish the number of variables on which the two processes differ.

For the second example, if the goal is *positive proof* (‘To prove: Z_1 is

good enough to be played) the subject explicitly strives for complete coverage of counter-branchings; own-branchings occur only here and there as a result of attempts at strengthening. If the goal is *negative proof* (To prove: Z_2 is bad enough to be eliminated) the situation is reversed. Finding that the move under examination (Z_2) is not good is a success, so the subject will proceed with a *cumulatively* linked operation: the next, and possibly final part of his proof plan for Z_1 (such as recapitulating its results to make sure of its superiority). In the positive proof case, on the contrary, finding that the move under examination (Z_1) is not good means failure. The next step is now determined by one of the *subsidiary* linkings listed in Section 51. (After such a late debacle, a conflict analysis in a transitional phase, with a crisis of expectations to boot, is not unlikely.)

Obviously, the two kinds of proof goal-settings entail completely different, even reverse strategies. The present distinction appears to be no less important theoretically than the one between exploration and directed investigation. In a program model different routines would be needed for the different goals. This means that both distinctions are theoretically relevant.

Re 2: We now know that for protocol interpretation according to theory there must be a decision made on each of the two distinctions taken up in our examples. Is it always possible to make an objectively defensible decision? To what extent does ambiguity occur?

In the analyses of protocols in previous chapters of this book the reader's attention has on several occasions been drawn to the 'relative arbitrariness of borderlines' between Phases. This implies that it is often hard to decide whether a particular examination (calculation) of a move and its consequences is carried out as an *exploration* or as a (*proof-*)*directed investigation*. Descriptively, such dubious cases can be said 'to serve both purposes simultaneously' or to be 'intermediate' or 'mixed' cases. Or, the protocol text and the calculational branches may suggest a 'gradual change' in goal-setting: the subject may start to calculate in an explorative sense but in the process 'gradually' develop a preference for the move in question; so that the end of one and the same elaborative phase is (proof-)directed investigation of a now favorite move. The present theoretical conceptual model, however, does not provide any procedure for handling 'mixed cases' of double functional goals or for handling gradual goal changes with corresponding changes in thinking methods or routines. The model requires a dichotomous classification: the goal should either be ex-

ploration or directed investigation. The result: frequent ambiguity in protocol interpretation.

At first sight, the distinction between *positive proof* and *negative proof* or *elimination* might appear easier to maintain. But, if there are no clear goal statements in a protocol text nor in the subject's additional remarks during the inquiry, it is often possible to interpret the available evidence in two ways. Own-branchings may represent attempts to complete the negative proof or attempts at strengthening, aimed at finding the one expected and sought after positive proof variation. Conversely, counter-branchings may represent attempts to establish the one expected and sought after refuting variation (negative proof) or attempts to cover every relevant counter-variation in order to complete the positive proof. If the result of Z_1 is negative, it may be impossible from the evidence to decide whether the subject is happy about the successful elimination or unhappy about his failure to prove (definitively) what he wanted to prove. The protocol and inquiry evidence on the ensuing transitional phase, if any, may also be inconclusive. In the inquiry, the subject may react to direct questions about his goal by answering: 'I don't really know,' or 'I wanted to eliminate Z_1 , to be sure, but on the other hand, I still had some hopes that it might prove playable,' or even 'I started trying to prove Z_1 was no good, but in the process my goal did an about-face.'

It appears that mixed and intermediate cases between positive and negative proof do occur in protocol interpretation. As a result, the question of whether the ensuing transition is cumulative or subsidiary may be indecisive as well. This means that frequent ambiguities occur even with regard to this most fundamental distinction in Selz's conceptual system. These cases can of course be resolved by *forced* decisions in every instance, but from the point of view of theory formation it is wiser to regard them as cues for desirable theory (or program) improvements.

Re 3: The problem raised in the previous paragraphs is that of *goal ambiguity*. In both examples the conceptual system presupposes a dichotomy of cases⁷: the goal must either be A or B since the ensuing investigatory strategy is defined for clear-cut cases only. The introspective evidence, however, suggests that *this forced distinction is too*

⁷ Goal ambiguities may involve more than just two possibilities; moreover, other ambiguities – *result* and *method* ambiguities, for instance – frequently puzzle the protocol interpreter. For the sake of simplicity, the following formal discussion is restricted to the case where a *dichotomy* of goals is required.

rigid and too simple to provide an adequate model for human processes. In particular, the evidence points to two possible interpretations of the phenomenon of goal ambiguity:

- (a) It results from a *dual goal*, i.e., from some combination of A and B;
- (b) It results from a *changing goal*, $A \rightarrow B$, or $(A \text{ and } B) \rightarrow A$, etc.

In the descriptive analyses of this study, both cases have on various occasions been mentioned. For another kindred example of *multi-functional procedures*, i.e., procedures that serve more than one purpose, the reader is reminded of the discussion of *sample variations* (cf. p. 203 and 294). The function of examining a sample variation (Z_1) is at least threefold, namely: (i) to see what sort of things one can do or must avoid or prevent; (ii) to see what the move Z_1 is worth; and (iii) to see what the position as a whole is worth. The second and third goals combine rather smoothly, it is true, since they are both primarily based on the outcomes of the Z_1 subvariations that are considered in the sample variation. Both the second and third are *investigative* (and evaluative) goals; goal iii can be viewed as a generalization of goal ii. The first goal, however, amounts to trying to *generate* possibly considerable other first moves, own continuations and counter continuations, and/or possibly considerable plans.

In most conceptions of thinking and thought simulation generating new possibilities and investigating the merits of given possibilities are conceived as separate processes. From introspective evidence it would seem, however, that they can be carried out simultaneously (dual goal) or, perhaps, successively or alternately (goal change) within one relatively brief, rounded part of the process – in our case: the calculation of one barely branched sample variation.

In other words: Introspective evidence with regard to goal ambiguity points to specific rigidity of the model as a possible weakness. It is very difficult though, to incorporate into Selzian theory (or into current simulation models, for that matter) mechanisms which allow *multiple functions* to occur.

Re 4: The answer to the question of how the model can be improved upon partially follows from the preceding analysis: the theory (program) should explain (simulate) multiple functions. This can be done in a variety of ways. The problem is far too complex to be treated systematically here, but we would like to make at least a few remarks on some of the principal possible modifications.

- (a) In order to incorporate 'the multiple functioning of the human

mind,' would it not be wise to do away with *the postulate of linearity*? Should we think of multi-functions as resulting from truly simultaneous processes occurring in different 'layers' of the human mind? In the calculation of a sample variation, for instance, should we think of two parallel processes, one (in the 'upper,' 'conscious layer') aimed at investigating the move Z_1 and evaluating its results and the whole position, the other (in a 'lower,' 'less conscious layer') governed by more 'autonomous processes' (VAN PARREREN 1951, 1952, and 1953) resulting in the generation of ideas for other ways to solve the choice-of-move-problem?

Such a conception would certainly find favor with those students of human thought for whom the distinction among various layers of functioning appears indispensable. (Apart from psychoanalysts, VAN PARREREN, for example, finds the 'autonomy-intentionality dimension' fundamental to a sound theoretical description of both learning and thinking processes.) The present author does not favor such a solution, though. Quite the contrary: One of the main advantages of Selz's theoretical conception of thinking is that he eschews 'layers' and maintains the postulate that thinking is a linear series of operations. It is an advantage for three reasons: (i) The distinction between 'truly simultaneous' and 'rapidly consecutive' processes in the human mind is hardly testable; (ii) as a result, it is generally possible to explain (simulate) the supposedly simultaneous processes by means of the model of consecutive, possibly alternating processes; (iii) the postulate of a single (linear) series of operations is much more parsimonious and simpler and, consequently, theoretically more fertile.

We had better not do away with linearity.

- (b) If parting layers is not necessary for theorizing, in the same vein is it not possible to *dispense with the distinction* between A and B with regard to which we find ourselves puzzled by ambiguity?

In principle, this may be a good solution. In case of goal ambiguity it amounts to changing the conceptual system in such a way that first, a goal AB, superordinate to both A and B be found and, second, the ensuing strategy of thought organization be made to operate in such a way that the A and B strategies emerge as somewhat extreme special cases of the general AB strategy.

This solution may be feasible for the distinction between positive (A) and negative proof (B). The superordinate goal would thus be *proof*: 'Recapitulate and complete the (subjectively convincing) argument.' One way to realize this would be to think of the proof part of the thought process as a *comparative choice* between a very restricted

number of moves (two or three) with small trees and few branches – the restrictions having resulted from previous analyses through eliminations and result generalizations. Within this stunted tree the *mini-max* principle and some priorities for determining the sequence for investigating moves and variations would then suffice to define a general AB proof procedure. In a given case, the actual process might contain one or more ‘pure’ A (or B) parts or even consist solely of A (positive proof) or solely of B (negative proof).

There is another way to define a superordinate goal AB and a corresponding AB strategy. Assume that the very *existence* of a move which measures up to a *precisely set expectancy level* is up for proof. Again, only few moves and branches will have remained ‘considerable.’ There may be preferences, but there is no one pronounced favorite whose superiority must be demonstrated. Since at this stage all outcomes are definitive, each must either satisfy or not satisfy the expectancy; therefore, a corresponding AB procedure can be easily defined. In this case the first move found that satisfies the pre-set level is approved and played. Although the goal here can be said to remain ‘positive’ throughout, the structure and implication of branches resulting from the investigation of a move that is rejected may have most of the features of ‘negative proof,’ B.

We cannot now enter upon a detailed discussion of the AB procedures just sketched, nor can we pass judgment on their tenability in view of empirical (introspective) evidence. They were delineated solely for the purpose of illustrating the possible dispensability of an ambiguous distinction.

(c) Finally, it is possible to conceive of *compound goals and corresponding compound processes*. This amounts to assuming a superordinate goal: ‘A and B’ and establishing a set of rules for how A and B procedures should succeed each other and possibly alternate.

As an example, let us again take up the distinction between exploration and directed investigation or, in particular, the ‘multi-functions’ within a sample variation. We have seen that at least two goals are relevant: first tentatively investigating and evaluating a particular move (variation) Z_1 and a few of its sub-variations (A; cf. goals ii and iii, p. 392), and, second, generating new ideas for different solution proposals (B; goal i, p. 392). The theory should now specify how and when consequences of Z_1 are scanned and screened (A), respectively how and when contributions to a ‘list of possible new ideas (solution proposals)’ are looked for, found, and stored (B). The compound strategy should normally consist of alternating A and B

subphases. Furthermore, the conditions for *goal changes*, from ‘A and B’ to A or to B, must be specified. One relatively simple case of a goal change to A has borne discussion in the previous chapters: If in an A-subphase of a ‘sample variation’ process, an unexpectedly favorable Z_1 -continuation is found, the goal changes to directed investigation of that possibility. The change may be sudden so that the exploration ends here and now; but a mechanism for gradual change (more A, less B effort) could also be specified. In any case the conditions for such changes, in particular for what is ‘unexpectedly favorable,’ must be defined to make the theory work. Conversely, in a B-subphase of a sample variation, an ‘unexpectedly good, new idea’ may be hit upon with the result that the subject shifts to the exploration – and possibly the investigation right afterward – of a new solution proposal. This change in goal appears to have occurred in protocol (G₄; A) (cf. the discussion on p. 294 and protocol (G₄; A) itself in the Appendix.) Here again, the conditions for such a transition (What is an ‘unexpectedly good, new idea’?) must be made explicit.

Hopefully, these few remarks are sufficient to show that ‘multiple functions’ can be theoretically worked into compound goal terms resulting in a process of alternating subphases. It will be clear, however, that the increase in the complexity of the theory is relatively large. If we want to go beyond primitive attempts – such as maintaining a ‘killer list’ while investigating attacking combinations⁸ – it is rather difficult to handle the B-part of the exploration. ‘New ideas’ found by exploring a few possibilities – or, in general, qualitative board ideas as found in the first Phase or in transitional phases – may have a great variety of forms. It is not easy to define objectively what they are and to design a strict routine for their generation and storage without losing too much that is significant.

In general, the procedures of tentative induction and hypothesis formation, of generating ideas, and of exploration are more difficult to get hold of than, respectively, those of deduction and hypothesis testing, of analyzing ideas, and of directed investigation. The resistance of the former to adequate theorizing may well be due to the fact that *perceptual and abstractive processes* play an important part in them – in line with the finding of the present study that chess perception is much more puzzling than chess thinking in the operational, discursive sense.

8 On a ‘killer list’ (McCarthy, 1962) moves and continuations are stored that have obtained some degree of priority by the fact that they appeared to be ‘killers’ in a certain number of previously examined variations.

Section 70. Remarks on chess playing programs

The last question to be raised is whether from the findings of the present study any inferences can be drawn with a view to chess programming. Only a few remarks will be made.⁹

Persistent rumors to the contrary, the level of chess programming has not yet measured up to the achievement criterion (cf. p. 387). Thus far, machine programs are poor players. At best they are rather narrow specialists of mediocre ability, such as the NEWELL, SHAW and SIMON (1958) program that is at least able to play a sound opening game. The ideal, a program that plays master chess, still appears to be quite remote.

Disappointing as this state of affairs may be, it has – apart from gratification to chessmasters and machine antagonists – one important advantage: namely, it may bring about a *rapprochement* between researchers working on chess programming for different purposes. In particular, those who use chess mainly as a test case or as a field of exercise for developing complex techniques of non-numerical programming are likely to look more and more for heuristics – basically human heuristics – in order to improve their programs. That is, their focus of interest is likely to approach the focus of investigators who explicitly want to develop heuristic programming and/or simulate human thinking. Given the current situation both groups should at least be interested in what the investigation of human thinking strategies yields.

Let us start with a recapitulation of some characteristics of human heuristics that have implications for existing machine programs.

1. From the analysis of protocols and from the additional experiments on chess perception we have learned that there is a *first Phase of problem formation*.

The *process* in the first Phase is characterized by a perceptive and receptive, rather than actively organizing, attitude on the part of the subject:

- first, perceptive integration, later orientation to and analysis of the whole position rather than investigation of specific solution proposals;
- three ‘moments’ to which the subject regularly gives his attention: the static moment (‘how things stand’), the dynamic moment (‘what

⁹ This is not the proper place for a more systematic discussion of the subject. The reader is referred to the existing literature (e.g. SHANNON 1950; BERNSTEIN *et al.* 1958a, 1958b; NEWELL, SHAW and SIMON 1958b; SAMUEL 1960).

sorts of things may happen’) and the evaluative moment (‘what the position is worth’).

The *function* of the first Phase is obviously the formation of a specific problem conception from which more actively organized investigations of the main part are inaugurated. The *result* of the first Phase is, in fact, a highly specific conception of the board problem to be solved. This problem conception generally includes: a highly selective set of possible board plans (seldom more than two in number, if that many); a corresponding set of considerable moves (again a very limited number) each aimed at attaining a specific board goal; preferences within this set of moves, possibly a favorite already; a rather precise, general level of expectancy (for the position); various other qualitative anticipations and quantitative expectations with regard to what a thorough analysis of the position would yield. In part these plans, considerable moves, preferences, expectancies, and anticipated solution attributes are definitive and certain; in part they are tentative and hypothetical. In fact, to some extent the subject’s total conception of the board problem is a (working) hypothesis. But then, among master subjects in particular, it is a surprisingly specific one. Objectively, moreover, the specific conceptions of master subjects are usually quite adequate – onwards from the very first seconds of the perception of a new position.¹⁰

On the basis of the specific problem conception produced in the first Phase, the main part of the thought process of an experienced chess player can be steered by a highly specific search ‘program.’ It amounts largely to *trying to answer (by calculation) a number of specific questions about the position*. One important result of this specificity is that the program of the human player can be highly selective in its branching.

To date, none of the existing chess programs produces a problem or goal conception with anything like the adequate specificity we find in the human player.

2. *The transitional phases of problem transformation* pose analogous problems for simulators. By means of program loops and recursions the

¹⁰ Truly, all this stems from ‘experience.’ Such specificity will not be found in experimental thought processes when subjects have to solve problems generically new to them. But if we are to simulate thinking – chess or any other kind – at an expert level incorporating experience into the program, in one form or another, is not only indispensable but is the very core of the simulation problem. The most vexing problems are how to store – or how to generate by a learning program – and how to retrieve efficiently an extensive body of largely intuitive information like the chess-master’s ‘experience.’

'return to more general problems' can be adequately simulated, but the problem is, again, that transitional phases include perceptive and abstractive processes. For the reformulation (transformation) of the board and search problems, the subject utilizes, apart from (quantitative) outcomes of variations, other (qualitative) information from fresh perceptions and qualitative *abstractions: things 'seen,' remembered, and inferred from previous explorations and investigations*. On the basis of such data the total goal conception is revised – including plans, the set of considerable moves, preferences, expectancies, and the anticipated solution attributes. As a result, revised operations-goals direct the ensuing elaborations. These elaborations serve to answer different and even more specific questions than during and immediately after the first Phase.

3. What has been referred to as *the omnipresence of anticipations* is a special aspect. Practically every operation in the thought process serves to *find out whether* things specifically anticipated are or are not confirmed. Again: Every operation is directed towards a highly specific goal of the nature of a specific question to be answered, a specific hypothesis to be tested. Even exploratory trying out appears to be largely 'trying if....' Apart from the anticipative role of the quantitative expectancy, a host of qualitative goal-delimiting anticipations have been inferred from the protocols: intuitive preferences, gradual favorite formations, anticipations of combinations or the attainability of other board goals (see Index of Subjects). As such anticipations develop during the thought process they contribute to the growing specificity and to the consecutive modifications of the subject's total problem conception and corresponding operations-goals.

4. The findings related under points 2 and 3 above have been summarized as follows: In the thought process a practically *continuous problem development* takes place. It is true that chess programs can also be said to go through continuous problem developments inasmuch as every rejected variation, for instance, changes (delimits) the problem, but like the problem formation of the first Phase, the problem development in the human subject is much more pronounced and much more specific in influencing the process.

The disparity comes out in contrasting human solutions to the choice-of-move-problem with still standard conceptions of machine solutions.¹¹ According to this conception the choice process must be based on subroutines or operating rules that:

- A. set a goal (board goal(s) and ensuing operations-goal(s));
- B. generate or select a set of considerable moves;
- C. calculate variations following rules that generate continuations (and their priorities);
- D. in each variation, apply a stop rule;
- E. evaluate the final positions by means of an evaluation function;
- F. choose the move to be played according to a selection procedure, such as minimax.

This standard conception of the process (first formulated by SHANNON 1950) corresponds roughly to the form of *the end result* of what chess players call an 'analysis' of a position (cf. Section 9), but it does *not* correspond to an actual thought process. The crucial difference is that in this schema (and in a completed analysis) we have to do with one goal or set of subgoals, one set of consecutively examined considerable moves, one pre-established system for generating moves and continuations, one stop rule, one evaluation function, and one choice procedure, while in the human process the continuous problem development (cf. Sections 44 and 46–49) makes for continuous changes in each. In fact, the names proposed for the Phases of longer and more difficult decision processes actually designate *different goals*: viz., 'exploration,' 'investigation,' and 'proof.' Correspondingly, the rules for generating moves and continuations, for stopping, for evaluating envisaged end positions, and for making decisions, vary according to Phase. They vary much more subtly – following anticipations, etc. – than this macroscopic Phase structuring suggests.

But all this has been amply discussed in the previous chapters.

5. Structurally the most striking evidence of the human subject's developing goal conception is *the phenomenon of progressive deepening*. Repeated investigation of the consequences of the same move would not make sense unless it was done for different reasons on each occasion. From the analysis of protocols, however, it is quite clear that it does 'make sense.' It has also become clear how re-investigations spring from changes in the subject's conception of the problem and in his operations-goals. It has even been asserted that the phenomenon

¹¹ This conception stems from programmers' perception of the problem: it is impossible to exhaust *all* legal moves and variations, so the crucial problem is to *cut down* the number of branches, in depth (using stop rules and evaluating) and in breadth (by judicious goal-setting). Had evaluative and goal-setting techniques enabled programmers to construct a satisfactory chess playing program, their interest in (other) heuristics would have remained slight.

of progressive deepening is expressive of a highly important economic principle in human thought (cf. p. 273).

6. The search process of the chess player who tries to decide on a good – possibly the best – move has been likened to *the empirical process of applied scientific research* (cf. Section 62). The resemblance is striking, indeed. The importance of complex perceptual and abstractive processes and the role of ‘hunches’ (anticipations) in guiding the process are apparent, both in the ‘first Phase’ – when the researcher starts a new problem – and in ‘transitional phases.’ Here, too, empirical orientation, exploration, directed investigation, and striving for (scientific) proof can be distinguished: *each a different research goal* with corresponding (and different) research procedures. The total process – not one specific investigation but the whole history of a particular research problem – may evidence a long, laborious problem development with many pronounced problem transformations; tentative hypotheses may become gradually more specific or may be modified, changed, or even reversed – like favorite formations in chess thinking.

Finally, it is not uncommon to find in such a process the same ideas or solution proposals taken up more than once, not as sheer repetition but each time within the context of a modified or radically changed ‘total problem conception.’ Such recurrence is by no means uneconomical; on the contrary, it is a direct consequence of the time-honored and experience-validated spiral development of human knowledge in science (cf. DE GROOT, 1961).

Such analogy has the virtue of casting some light on one facet of the acquisition of knowledge. Like the scientist, the chess player must acquire knowledge about a small, particular part of the world – the given position – in order to make good move choices. He does so step by step, by progressive deepening.

If we now take a closer look at chess programming, what are the weaknesses of current machine programs compared to human chess players and by what measures can such weaknesses be repaired or reduced?

As a matter of course, improvements are conceivable in each part of the cycle described on page 399; i.e., there are refinements in procedures for: goal-setting (A), move and continuation selection (and priority) rules (B and C), stop rules (D), evaluation function(s) (E), and choice rules (F). Besides, in an advanced program these various procedures are apt to be interdependent and interlocked in ways that

may also be open to improvement. The present argument, however, leads to the position that *really fundamental improvement hinges on provisions for problem development*, i.e., for goal feedback and goal changes by means of progressive deepening. The program should allow for a series of more and more specific problem cycles.¹² This is certainly true from the simulation point of view and it may quite well be true from the efficiency point of view – but let us first explore some other possibilities.

If for some reason we could not have problem development in our program and were forced to choose the most ‘promising’ of the A through F procedures, we would not hesitate in deciding on A, along with corresponding selection or generation techniques, B and C. Stop rules (D) and the final choice procedure (F) can be well adjusted, with some ingenuity, to the rationale of the program; while general experience with evaluation functions (E) tends to indicate that the added payoff in accuracy diminishes rapidly – and soon vanishes – as they are refined beyond the obvious. Techniques for differential goal-setting, however, can certainly be made more sensitive and adequate. This can be done, first, by storing book knowledge in the program’s memory. Apart from specific opening and endgame theory, in certain types of positions general knowledge of chess strategy and tactics may serve to make more adequate the program’s initial board goal conception. Second, this measure can be extended by constructing and storing an exhaustive typology of positions and of parts thereof – each type corresponding to specific routines for generating moves and continuations. In this way the master’s non-codified experience in classifying a position and in evoking a corresponding specific goal conception can be simulated. In more general terms: The program can be made, so to speak, more perceptive, abstractive, and inductive. That is, sub-routines can be developed for educating and processing qualitative information on features of a position and on solution attributes of the board problem that can be used in the setting of more specific board and operations-goals. There is no room for any doubt that such measures could be effective.

¹² The possibility of a learning program is not considered. However, a multicyclic program as will be sketched in the following lines has some of the features of a learning program. It does not ‘learn to play chess by experience,’ it is true, but by means of goal feedback loops it does learn more and more about the problems posed by a particular position. – At a later stage, the question might be asked whether the program could learn by experience how most effectively to learn about a given position.

Even so and quite apart from the question of whether such measures would be fertile from a research point of view,¹³ the optimal result would be a still dubious and in many respects inadequate classification of positions. If such a classification were used for the purpose of determining a definitive – and not provisional as in the chessmaster – board goal conception with corresponding search routines for each class, the gain would still be relatively small. First of all because it would be very hard to equal or even to approach the level of a master's perceptual and abstractive achievements; second, because there would be no means for correcting a possibly wrong board goal conception.

Therefore, if improvements of goal-setting (and move selection) procedures are to be fruitful *a feedback mechanism for goal correction* – analogous to the problem development in the human – appears to be indispensable. This means that *the crucial principle for the improvement of chess playing programs is to simulate the development of the human subject's total goal (problem) conception by employing the strategy of cyclic, progressive deepening.*

This can best be done *from the very start of the thought process.* Although the concept of progressive deepening has been primarily introduced to describe the 'deepening' of consecutive investigations in the main part – possibly of the same move(s)¹⁴ – it can easily be extended to include the position investigation of the first Phase. It is only logical to consider the initial problem formation (orientation) as the first deepening phase of the total problem development. Perhaps allowance should even be made for a series of problem cycles in the 'first Phase' of the program. Even with much effort spent on the improvement of the initial goal-setting routine (cf. p. 397 above) the total problem conception with which a machine program starts off is likely to remain much less

13 Since developing a machine that plays a masterly game of chess can hardly be considered a valuable research goal in itself, the research gain must either be methodological or theoretical. That is, 'promise' must be measured in terms of possible results that are conducive to better insights and skills in non-numerical, heuristic programming or are relevant to the development of theories on thought processes. Results should have a definite 'generalization value,' beyond the field of chess. It is preferable, for instance, to lower somewhat the level of the achievement criterion if its maintenance would require a large investment in programming activities that are useful to chess alone. (In the author's opinion, creating a master – the original ideal – would in fact require too large a specific investment, if such a creation is possible at all).

14 From the programming point of view acceptance of progressive deepening means *waiving a restraint*: there is no longer reason to exclude the possibility that the same solving proposition (move) be taken up and investigated more than once.

adequate than that of a master. This means that the machine is in greater need of completion and corrections. These must result from feedback: quantitative and qualitative results from orientative examinations of the position and of a few possible moves (dynamic moment). We cannot (yet) expect a machine program to 'see' dynamic relationships immediately as does the chessmaster; but it may be able to compensate for this lack of vision by one or more supplementary feedback cycles, i.e., by mechanisms of progressive deepening *within the first Phase.*

The implication of the latter proposal is that perceptual processes are simulated by largely investigative machine operations (calculation of move sequences): does this not run counter introspective evidence? Presumably it does not. What the master reports as (dynamically) perceived may actually be based on greatly abbreviated 'rapid calculations.' If a possible move is 'seen' to be 'absurd' – e.g., a capture which, if carried out, would immediately lead to material loss – the 'dynamic perception' may be based on some sort of elliptic, high speed calculation. Psychologically and physiologically, the possibility cannot be summarily excluded,¹⁵ to say the least. It will be remembered from the perception experiments that integrating a position – into a 'problem,' in fact – took a matter of seconds for a master subject. After the perceptual integration, his thinking does not start with separate data or parts that are carefully put together but with a *total* problem conception, however preliminary and hypothetical. Witness the early appearance of such total goal determinants as an evaluation of the position (functioning as an expectancy) and the early occurrence of 'move impulses,' i.e., 'abortive solving propositions.'

15 In a discussion within the Euratom group on the remarkably high *selectivity* of the human chess thinker (cf. Tables 4 and 8–10). Dr. REUBEN FINE, who was attending the meeting as a guest, rightly remarked that the figures in the tables reflect solely what has been reported by the subject. He put forward the hypothesis that a master carries out a considerable number of rapid calculations in his actual thought process which he has no time to report or even to remember in the inquiry. We should allow for this possibility. The author does not agree with Dr. Fine that this would detract much from the selectivity of the human subject. Explicit denials by subjects such as: 'No, I did not look at that move (variation) at all,' would seem credible. It seems quite possible, however, that a number of things reported by the subject as immediately 'seen' – for instance, that some Pawn cannot be taken (since it is sufficiently defended) – have actually been derived from 'rapid calculations.' It is hard, if not impossible, to distinguish between 'dynamic perception' and such rapid calculations. Consequently, there are no psychological objections against simulating human 'dynamic perception' by means of some extremely fast calculations on the machine.

In general, few findings have been so often reported and emphasized by students of thinking as the 'total goal' which the human subject has in his mind from a very early stage – the *Gesamtaufgabe* or schematic anticipation of the completed total task to use SELZ's terminology. In conclusion, it would appear that provisions for progressive deepening – from the very start on, first Phase included – cannot but improve the simulation qualities and are likely to raise the level of achievement of chess playing programs.

Let us try to strengthen these ideas somewhat by describing the macro-structure of a not too simple move decision process.

The process would consist of a *series of macro-cycles* in each of which the following macro-operations (subroutines) would be consecutively carried out:

(1) collect information on the position, i.e., on the board problem, by retrieval of relevant stored information, board perceptions, result abstractions, and generalizations ('things remembered, seen, and inferred');

(2) correspondingly, formulate the (revised) board problem conception, including the setting of partly tentative and hypothetical board goals, both quantitative (expectancy) and qualitative (considerable plans and corresponding moves);

(3) correspondingly, determine (set the parameters for) the investigative strategy to be followed: in particular, determine the next operations-goal(s);

(4) correspondingly, carry out the investigative routines and collect relevant results, including: (a) hypotheses confirmed? that is, board goal(s) attainable or not? (b) outcomes of optimal variations? (c) qualitative results, such as repeatedly effective 'killer' or 'refuting' moves?;

(5) correspondingly, integrate this new information (from (4)) with old information, for the purpose of establishing the next, revised conception of the board problem; now we have returned to (1).¹⁶

The *number of consecutive macro-cycles*, i.e., cycles with explicitly programmed problem revisions would, of course, have to depend on

¹⁶ Cf. Section 37, p. 131 ff. In that description of an assimilative cycle the first perceptual phase was lacking, while the fourth macro-operation above was divided into two parts; for the rest, the two descriptions are equivalent.

The present five phase schema corresponds perfectly to the well-known cycle of (empirical) scientific inferential procedure, namely: (1) observation, (2) induction, (3) deduction, (4) testing, and (5) evaluation (cf. DE GROOT 1961).

the particular position. Like the human, the program should be able to skip routines whenever a sufficiently precise problem conception (and an adequate move decision) can be reached by a short cut. Normally the number of cycles should not run too high. One could envision some six deepening phases for a difficult choice problem: e.g., two 'orientative' cycles (first Phase simulation), one 'explorative,' two 'investigative' cycles, and one 'proof' cycle as a maximum.

Since the program would aim at getting to know the board problem as much as at solving it, the resulting process can be described as one of *collecting and integrating information on the position* by means of a *series of questions*, each of which aids in determining the true nature of the problem. Some of these questions – not all of them – could only be answered by means of a dynamic ('empirical') analysis. For example 'Can I (he) immediately gain material?' might require a quite complicated subprogram in itself. Following point (4) above, the results would not consist solely of the answer to the question (hypothesis confirmed or not? (a)), but would also include the most important differential information on the results of moves and variations (b) and, maybe, some newly abstracted ideas for solution (c).

One question the program should always be in a position to answer on request, is the 'interruption question' (cf. p. 185 and 187): *What do you think of the problem now?* or: Please specify your actual problem conception. To this end a special working memory for 'relevant problem data' might be useful.

It should be obvious that the process resulting from such a program is no longer just a matter of choosing among a given set of legal alternatives – in a somewhat heuristically adjusted way. The process would really be one of *problem solving*. At the start of the process all the ingredients of the problem situation are given, it is true, but the problem itself is *not*: it must be posed and developed in the process. The alternatives are not 'given' once and for all either: in each cycle they must be generated according to a board goal (hypothesis). Nor is the position really 'given' at the start: it must still be typologically determined and individually analyzed by means of a programmed sequence of relevant questions and search routines.

Presenting a particular position to a machine could be likened to asking an experienced player to play a move in a position which he is not allowed to see! The player may ask any well defined question about the position. Binary questions such as: 'Material equal?' 'King in check?' 'Queens present?' or quantitative questions: 'How many pieces (of White) have been taken?' 'How many moves have been

played?' etc. The primary task for the problem solver is *to give shape to the board problem* through an economically programmed series of questions, that is, to try to classify the position accurately enough to set up the first board goal hypothesis. In fact, as soon as questions are posed that can be answered solely by dynamic analysis, they gain the status of hypothetical board goals; e.g., 'Can I immediately gain material?'

The analogy between a 'blind' human player and a machine program should not be stretched too far. In later stages of the process the human player will have the advantage due to his ability to envisage the position. For the first stage of problem formation, however, the analogy appears rather illuminating, in particular inasmuch as it demonstrates that, at the start, there is no problem to be solved as yet.

The above exposition remained sketchy and rather vague, it is readily admitted. Such relative vagueness may have the advantage, however, of helping the reader to see that the underlying ideas are by no means restricted to the simulation of chess thinking. The author believes, in fact, that *the simulation of problem solving or rational choice processes hinges on adequate provisions for problem development, quantitative and qualitative goal feedback, and as a result, progressive deepening*, regardless of subject matter or field. As soon as problems become 'difficult' one-cycle-procedures are no longer adequate from a simulation point of view. It is hypothesized that *they are no longer efficient either*, provided the field is complex enough to force the programmer to utilize heuristics.

In all likelihood, chess is not the most suitable field to test the latter hypothesis. But it can hardly be doubted that under proper conditions the hypothesis is testable and worth testing. It might serve as a focus for further research both by computer specialists and experimentalists in the field of human thought and choice.

APPENDIX I

THE GAMES FROM WHICH POSITIONS A, B, AND C WERE TAKEN

White: A. D. DE GROOT, Black: C. SCHOLTENS, Event: PRELIMINARIES, DUTCH NATIONAL CHAMPIONSHIP; date: APRIL 10th, 1936

1. P-Q4	P-Q4	14. N-K5	R-B1
2. P-QB4	P-K3	15. B-R2	B-B3
3. N-QB3	N-KB3	16. QR-B1	Q-N3?
4. B-N5	B-K2		
5. P-K3	O-O	Position A	
6. N-B3	P×P	17. B×N/5	P×B
7. B×P	P-B4	18. Q-B3	Q-Q1
8. B-Q3	N-B3	19. KR-K1	K-N2
9. O-O	P×P	20. N-N4	N×N
10. P×P	N-QN5	21. B×B	Q-Q2
11. B-N1	B-Q2	22. B×Rch	R×B
12. P-QR3	N/5-Q4	23. Q-B4 and White won.	
13. Q-Q3	P-KN3		

White: J. M. A. WIND, Black: A. D. DE GROOT, Event: CLUB COMPETITION, date: SPRING 1935

1. P-K4	P-K4	21. Q-N4	Q-K4?
2. N-KB3	P-Q4	22. B-K3	B-B5
3. P×P	P-K5	23. B-Q4	Q×B
4. B-N5ch?	P-B3	24. Q×R	P-N3
5. P×P	P×P	25. Q-B5	Q-Q2
6. B-R4	P×N	26. Q×N	B×N
7. Q×P	N-B3	27. Q-N4	Q×Q
8. O-O	B-K2	28. P×Q	B-B3
9. B×Pch	N×B	29. KR-K1	
10. Q×Nch	B-Q2		
11. Q-KB3	O-O	Position B	
12. P-Q3	Q-B2	29. . . .	R-N1
13. N-B3	B-Q3	30. P-N3	R-N4
14. P-KR3	B-B3	31. R-K7	R-N4
15. Q-K2	KR-K1	32. P-KB3	P-KR4
16. Q-Q2	N-R4	33. P×P	R×Pch
17. Q-N5	B-R7ch	34. K-N1	B-Q3
18. K-R1	R-K4	35. R/7-K1	B-B4ch
19. Q-R4	R-KB4	36. K-B1	R-R8ch
20. N-K4	B-N6	37. K-K2	R-R7

38. R-KN1	B×R	46. P-N5	B-B4
39. R×B	K-B1	47. R-Q2	R-R8
40. P-Q4	K-K2	48. R-N2	R-QN8
41. P-QB4	P-N4	49. R-Q2	R-N5
42. R-K1	K-Q3	50. P-B5ch	K-Q4
43. K-B2	P-B3	51. P-N6	P×P
44. P-N4	R-R5	52. P×P	R×NP
45. R-Q1	B-Q2	53. Resigns	

White: DR. J. H. PANNEKOEK, Black: A. D. DE GROOT, Event: XII MATCH GAME;
date, SUMMER 1934

1. P-QB4	P-K3	17. ...	P-K4
2. P-Q4	B-N5ch	18. B×B	QR×B
3. B-Q2	B×Bch	19. N-R7	Q-K5
4. Q×B	P-KB4	20. Q×Q	N×Q
5. N-QB3	N-KB3	21. N×R	R×N/6
6. P-KN3	P-Q3	22. P×P	R×BP
7. B-N2	Q-K2	23. N-K7ch	K-R1
8. O-O-O	QN-Q2	24. N-Q5	N-R7ch
9. P-K4	P×P	25. K-N1	P×P
10. N×P	O-O	26. QR-K1	N/7-B6ch
11. N-QB3	N-N3	27. N×N	N×Nch
12. Q-K2	Q-Q2	28. K-B1	
13. N-B3	Q-B3		
14. P-N3	P-QR4		
15. P-QR4	QN-Q4		
16. N-QN5	N-QN5		
17. B-R3			

Position C

Drawn

APPENDIX II

COLLECTION OF PROTOCOLS

(G1; A) S: PAUL KERES E: DE GROOT T = 6 MINUTES NOVEMBER 9, 1938

- First let's have a look at the position. Well, it is quite a position! Who is actually better off? Difficult.
- Let's first have a look at what can be taken; are there any immediate attacks? 1. B-R6 and 2. N×BP – not sufficient.
1. N×B maybe? He must take back with the Pawn; with the Rook costs a Pawn, and with the Queen will not be possible either – indeed – so
1. N×B, P×N; 2. B×N/5, BP×B; 3. Q-B3. Pfu! That seems to win a piece. (Pause)
- The Pawn at QN2 is attacked.
- Right off**
1. B×N/5 maybe? Nothing special. Let's calculate it though: 1... B×B; 2. B×N and 3. N-Q7 – possibly even 3. N×B first. 1... N×B costs a piece. 1... P×B; 2. Q-B3 maybe? Let's check 2. Q-B3. What should Black do? 2... K-N2; then, for instance, 3. N-N4 – will probably win.
- Well, White will win after 1. B×N/5.

(G2; A) S: ALEXANDER ALEKHINE E: DE GROOT T = 9 MINUTES NOVEMBER 28, 1938

- At first sight there is a dark memory of a tournament game Botvinnik-Vidmar (Nottingham). There's a certain resemblance: the same Queen position on Q3.
- Opening type: Queen's Gambit Accepted. In half an hour I should be able to logically reconstruct the moves up to this position. Is the Pawn at QN2 really attacked? Or isn't that essential?
- See what counter possibilities White has. Which moves are worth considering? 1. B×N/5 – to calculate: 1... B×B is no good; 1... N×B is no good; so 1... P×B. And then the Knight at B6 is getting a little weak. 2. N-N4 or 2. R-K1 might follow or also 2. N×B and 3. R-K1; or possibly 2. Q-B3. Well, in any case 1. B×N/5 is the one enticing move.
- But now let's have a look at other possibilities: 1. N-N4 deserves consideration, but doesn't look so good.
1. N×N maybe; 1... N×N; 2. B×N, B×B/N with many capturing possibilities but at first sight not convincing. 1. N×B, at first sight 1... P×N would follow and then Q5 gets stronger, that's a pity. In any case White has by far the best of it. I would be very happy if I had such a position in a tournament game.
- Are there any other moves, apart from 1. B×N/5, that are decisive? 1. B×N/5 is very strong, that much is certain.
1. N×B to calculate; 1... P×N; pressure on QB6, the pair of Bishops: also very agreeable, but the position promises more. Check over other sorts of things again. Well, in case of time pressure I would play 1. B×N/5.

(G3; A) S: SALO FLOHR E: DE GROOT T = 10 MINUTES NOVEMBER 28, 1938

- It's been a Queen's Gambit Accepted or Caro-Kann. KB6 is weak, but Q5 is a strong point; the Pawn on Q4 is isolated and, as such, weak. White's chances are in the attack, Black's in the endgame. 1.N×N or perhaps 1.B×N/5 deserve consideration.
- 1.B×N/5 looks good. Let's calculate it: Does it give a decisive advantage? No other move would win immediately in any case. One must first see if there is anything decisive in the position.
- 1.B×N/5 and now only 1...P×B is possible, and then one plays for instance: 2.Q-B3 or 2.R-K1. No direct decision.
- Maybe first neutralize the Bishop at QB6?
- 1.N×B, then 1...P×N. Then for instance proceed positionally with 2.N-R4. Also possible is 1.N×B; 2.B×N/5 and 3.Q-B3. 1.N×B is a good move in any case; if there is nothing decisive, then one should hold on to a small advantage. The pair of Bishops and the weak Pawns for Black. But wait, can't you win the exchange? Yes:
- 1.N×B, P×N - forced, 1...R×N doesn't hold - 2.B×N/5, BP×B; 3.B×N and 4.N-Q7 (sic).*
- Yes, 1.N×B looks good; I play that.

(G4; A) S: REUBEN FINE E: DE GROOT T = 8 MINUTES NOVEMBER 30, 1938

- It has been a Caro-Kann or a Queen's Gambit Accepted. Obviously Black has had to defend himself against an attack along the diagonal with P-KN3.
- White has to play for the attack; otherwise he has nothing. Must he go for KB7? Or not? Black has few threats. Q×NP is probably impossible. Or is it? Let's try
- 1.N-K4 - take it away.
- 1.Q-Q2 - then Black exchanges on my QB3 and plays 2...N-K5, so that won't work.
- 1.N×N - not satisfactory.
- 1.KR-K1 for instance: 1...Q×NP; 2.B×N/5, N×B; 3.B×B, N×B; 4.R-N1, Q×RP; 5.N-Q5, Q-Q3 - there's nothing there. Maybe another way:
- 1.KR-K1, Q×NP; 2.B×N/5, N×B - 2...B×B is no good - 3.N×N, B×B; 4.R-N1, Q-R7 - or 4...Q-Q7? No, then 5.N-K7ch - and then ...Q-Q7. White has nothing.
- 1.KR-K1, Q×NP and now something else: also unsatisfactory. (Short Pause)
- But what about 1.B×N/5? If 1...B×B, the exchange is lost; if 1...N×B, he loses a piece, so Black must play 1...P×B. Then maybe 2.KR-K1 or even better 2.Q-B3 or 2.N-N4. Yes, 1.B×N/5 is the best move.

(G5; A) S: MAX EUWE E: DE GROOT T = 15 MINUTES DECEMBER 15, 1938

(Italics are for textual clarity and do not represent G5's emphases.)

- First impression: an isolated Pawn; White has more freedom of movement. Black threatens Q×NP. Is it worthwhile to parry that? It probably is; if he takes, then QR3 is also attacked. White then take advantage of the open file? Does not look like it. Still again: 2.N×B and then by exchange the Pawn

* Subject G3 discovered his error later but probably hoped that it would slip by the experimenter!

- at QR3 is defended by the Queen. Indirectly in connection with the hanging position of the Knight at KB6 and possibly because of the overburdening of the Bishop at K7. But wait a moment: no, Q×NP is rather unpleasant after all because the Bishop at R2 is undefended. Can I do something myself? Investigate that first: the pieces on KB6 and Q5 are both somewhat tied down.
- Let us look at the consequences of some specific moves.
- 1.N×N, possibly preceded by 1.N×B. Then 1...R×N is probably impossible because of taking on Q5. Black has a number of forced moves, there may be a possibility to take advantage of that. It's not yet quite clear.
- Let us look at other attacks:
- 1.B-R6 in connection with KB7 - but I don't really see how to get at it.
- 1.P-QN4 in order to parry the threat - but then exchange on QB3 will give some difficulties in connection with 2...B-N4 - oh, no, that is not correct, one can take back with the Queen.
- So far a somewhat disorderly preliminary investigation. Now, let's look in some more detail at the possibilities for exchange: 1.N×B or 1.N×N or maybe 1.B×N/5 or maybe first 1.B×N/6.
- 1.N×B, R×N; 2.take on Q5; for instance 2.N×N, P×N; wins a Pawn, but there may be compensation for Black on QN2. But better is 2...N×N; then 3.B×N, R×R is nearly forced, no, it is not, he can play 3...B×B as well. I see no immediate advantage. 1...P×N is not forced therefore; and even if it were forced you couldn't be quite sure of winning. It's happened before that such a position proved less favorable than it seemed to be. The point Q5 is reinforced by it, that is a disadvantage. 1.taking on Q5.
- 1.N×B at any rate gives the pair of Bishops; if I don't find anything better, I can always do this.
- 1.N×N, B×N; is that possible? Q7 is free then. 2.B×N, B×B/3; 3.N-Q7, Q-Q1 can then be done.
- 1.N×N, B×N; 2.B×N, B×B/3 will probably yield something. 1...N×N is also possible; maybe better. Then 2.B×N, B×B/N and now there are the possibilities to take on QB6, or to play something like P-B4; once again:
- 1.N×N, N×N; 2.B×N, B×B/N - no, nothing then, 3.R×B does not help any; it is a cute move but at the end of it all everything remains hanging. Something else: 2.B×B - he just takes back. 1...P×N is very favorable; he won't do that, it needn't be investigated.
- 1.N×N, N×N remains. 2.B×N, B×B/N; 3.B×B, B×R is then possible. No, can find no way to make anything out of this. 1...N×N; 2.B-R6, KR-Q1; 3.Q-KB3 with some threats; if Black now has to play his Bishop back to K1, then one gets a good position.
- 1.B×N/5: this must be looked into. Does that make any difference? 1.B×N/5, B×B is again impossible because of 2.N-Q7. That is to say, we will have to look out for 2...B-B5, but that we can probably cope with: the worst that can happen to me is that he regains the exchange, but then I have in any case some gain of time. 1.B×N/5, N×B; 2.Same difficulties as just before. No, that is now impossible: 2.N×N wins a piece.
- 1.B×N/5, B×B; 2.B×N, B×B; 3.N-Q7, Q-Q1. Let's have a closer look at that: 4.N×B/5, P×N and I'm an exchange to the good: very strong. 1.B×N/5, P×B is therefore forced. But that's good for White. The Knight on KB6 is weak, the Bishop at K6 hangs - and the Bishop on QB3 stands badly. On positional grounds one could already decide on 1.B×N/5. Is there some immediate gain?
- 1.B×N/5, P×B; it looks bad for Black. Probably some more accidents will soon happen. Much is still up in the air. One plays, for instance, 2.Q-B3. Defending the Knight on KB6 is not so easy; 2...K-N2 looks very unpleasant. Yes, I play 1.B×N/5.

(M1; A) S: LODEWIJK PRINS E: DE GROOT T = 18 MINUTES OCTOBER 29, 1938

- The first thing that strikes me is the weakness of the Black King's wing, particularly the weakness at KB6. Only after that a general picture of the position. Finally, the complications in the center are rather striking: possibilities for exchange in connection with the loose Bishop on K7. Still later: my Pawn on QN2 is *en prise*.
- Now work out the complications:
1.N-K4 perhaps?
Rather
1.B×N/5, or perchance
1.B-R6 in order to weaken KB7.
- Let's work through the possibilities in the position; the counterchances of Black? If the Bishop on N5 goes away, for example, in case of B(on R6)×R, then ...N-B5 and my KNP is attacked: I don't feel quite safe.
- Now let's calculate
1.B×N/5; that is after all the most plausible: 1.B×N/5; if Black now retakes with the Bishop or with the Pawn, then 2.N-N4, and if Black takes, then 3.B×B and White is better off.
- Go over
1.B×N/5 again...
- Is there a calm move to prepare the attack? Maybe
1.Q-R3 with the threat of 2.Q-R4. It looks very strong. But now a drawback appears: on 1.Q-R3, Q×QP follows. So it doesn't hold up.
- 1.B×N/5 again. 1...B×B; 2.N-N4; then the *Zwischenzug* 2...B-B5! Complicated, difficult to calculate.
- Perhaps another move is better after all?
- 1.P-QN4 with the threat of driving the Bishop away and then taking on Q5? No, then he plays 1...N×N, and if then 2.R×N, then 2...B-N4. Rather disagreeable.
- 1.B×N/5 again; 1...B×B; 2.N-N4, B-B5. Is that really serious? Wait a minute: White can play 3.B×N with a mate threat. So that's refuted. Now to look over everything again. It's becoming clearer.
- 1.B×N/5 is the move.
- (M1; A)* *The instruction to this protocol, the first one of the series, was somewhat deficient. Subject M1 was told that he would have a chance to fill in his thought process immediately after his choice of move. Protocol (M1; A)* presents these 'after thoughts.' In all subsequent experiments, however, the subject was instructed to express everything in the first place, while he searched for his move.*
- 1.N-K4 'feeling': personal preference: Knight to the King's wing, besides it takes advantage of the weakness on KB6 and of the hanging position of the Bishop on K7. (Results in nothing; from there probably to) Take on Q5? (the intention is to win the Bishop on K7 or some such thing. That doesn't succeed after 1.N×N, N×N; 2.B×N, B×B/N! Therefore): postpone N×N (with the gain of tempo) until the second move; thus
1.B×N/5.
- Now begin with the probably bad countermove – in order to be able to eliminate those – so 1.B×N/5, N×B? then 2.N×N, etc. 1...P×B looks bad; White not only has 2.N-N4 then but also 2.Q-B3 at his command. (1.B×N/5), B×B; 2.N-N4 would be very strong. White stands better; temporarily let it be.
- A stupid move perhaps?
1.B-R6; if that should prove even stronger, then I don't have to figure out all the variants of 1.B×N/5. Or
1.P-B4 to 'quietly' strengthen the attack? After an easygoing move, must take into account the counterchances of Black, notably along the diagonal from my KB1 to QN5, or on KB4 if the White Bishop disappears.
- Now a better calculation of
1.B×N/5. 1...B×B; 2.N-N4. Now further: 2...K-N2; 3.B-R6ch, etc.

- 2...Q-Q1 cannot be good because after N×B and R×R the Queen must retake on B8: such as 3.B×N, 4.N×QB, and 5.R×R winning a piece on KB6.
- 2...N×N; 3.B×B, KR-K1; 4.B-N5 and White has the pair of Bishops, at least one very good Bishop. His Knight on N5 is in trouble; weakened position. Calm move?
- 1.Q-R3-R4? In order to make use of KB6 and KR6? No, the Pawn on Q4. Then something else will go wrong.
- Once again.
1.B×N/5. 1...B×B; 2.N-N4 (Now Black has a choice) 2...B-B5! Troubles. Temporarily dismiss that.
- 1.P-QN4 threatens P-N5; then the Bishop must go to K1 – B-Q2 can't be good – that's not attractive for Black. So 1.P-QN4 – no, then comes 1...N×N and the Rook cannot retake on account of 2...B-N4. Well, it can for that matter, but it's not nice: the advantage is gone. Then here we go again with
1.B×N/5. On 1...P×B; 2.Q-B3 possibly followed by N-N4 which cannot be good for Black. 1...B×B; 2.N-N4. Oh, yes, now 2...B-B5. Is there nothing then? Ignore it? Yes, 3.B×B threatens N-R6 mate – then on 3...B×Q, follows 4.N-R6 mate. But in case of 2...B-B5; 3.B×N, B×B? Well, then something like 4.Q-B3, B×R (*sic*); 5.N×Bch, and 6.R×B; that wins. Well, 3.B×N is good.
- Now just a short recapitulation: 1.B×N/5. Not 1...N×B now, and in case of 1...P×B; 2.Q-B3 and White stands splendidly. So 1.B×N/5, B×B. Then ... look over everything quickly. Quite satisfies me. So:
1.B×N/5.

(M2; A) S: NICO CORTLEVER E: DE GROOT T = 18 MINUTES NOVEMBER 19, 1938

- Count Pawns, pieces. White's position is good, but for the endgame not so good. Will probably have to try an attack on the King. 1.B×N/6 and then 2.N×N – to eliminate my weak Pawn maybe; if 1...B×B; 2.N×N then 2...P×N and the weakness is already gone; and on 1...N×B maybe 2.P-Q5? 1.B-R6 maybe; not so very promising.
- 1.N×N, taking immediately on Q5? An annoying sort of position! With such a rotten Pawn (Q4)!
- Now let us calculate some:
1.B×N/6, B×B; 2.N×N, P×N;
3.N-N4, B×P – not so good.
- 1.P-KR4 is possible too; taking on KB6, P-R4-R5 and B-N1, etc., to attack. Does Black threaten anything? Not much to speak of.
- 1.N×N, then comes 1...N×N.
1.B×N/6 again; 1...N×B; 2.P-Q5, takes; 3.takes and then R-Q1 – bad. Let us look around for other possibilities.
- 1.P-KR4 followed by B-N1.
1.N-N4 immediately maybe, or:
- 1.P-QN4. Then 1...N×N; 2.R×N, B-Q4; and now 3.B×N and 4.B×B? Nothing, that's a draw. Just look at the position.
- 1.B-N1 maybe? Can he then take the Pawn on QN2? Yes, so
1.P-QN4 perchance? Don't like it.
- 1.N×N – calculate that somewhat better now: 1.N×N, N×N; 2.B×N, B×B/N; 3.B×B, B×R; 4.R×B, Q×NP; 5.R-N1 – something like that? Other variants of this: 1.N×N, N×N; 2.B×N, B×B/N; 3.P-B4, B×B; 4.P×B. No result. Something else to that move? 1.N×N, N×N; 2.B×N, B×B/N; 3.B×B, B×R; 4.R×B, P×B; 5.N-Q7, Q×NP; 6.R-N1, Q-R7; 7.N×R. The alternative: 1.N×N; 2.B×N, B×B/N; 3.B×B, B×R; 4.R×B, R×B; 5.N×R, P×N – somewhat better.
- Once more:
1.N×N, N×N; 2.B×N, B×B/N; 3.B×B, B×R. How is the material situation now? Three pieces against Bishop and Rook ... and Knight – so

- an exchange down. But then maybe immediately 4.N-Q7; no, then 4...Q×B follows, so it is better to take back on B1. 1.N×N, N×N; 2.B×N, B×B/N; 3.B×B, B×R; 4.R×B, P×B; 5.N-Q7, Q-Q1; 6.N×R works well. But Black can play differently: 5...Q×NP; now maybe 6.Q-K3, KR-Q1; 7.N-B6ch, K-N2; 8.N-N4 – that hardly can be correct: too crazy.
- Now again a general inspection; other moves.
- 1.N-K4: also a good move.
1.KR-K1 the same. The Knight on QB3 cannot move very well on account of B-N4. But:
1.N×B does not appeal to me.
- 1.N×N, N×N; 2.B×N, B×B/N; 3.P-B4, B×B; 4.P×B – does not appeal to me very much either. Perhaps common moves are better:
- 1.Q-N3 to R4; then Q4 is weak.
1.KR-Q1 to prepare it, 1...Q×NP. Then 2.N-B4, I win the Queen! Is there perhaps some counterchance for Black? No, indeed the Queen is caught.
1.KR-Q1 for Q-N3 and also against B-N4. If now 1...N×N then 2.P×N. Better for White.
1.KR-Q1, KR-Q1; then still follow the same recipe. 2.Q-R3 – stronger than 2.Q-N3 – and possibly a sacrifice on KB7? Then rather
1.KR-K1 – pleasant on the open King's file; 1...KR-Q1 for instance; 2.Q-R3, threatens N×BP. The other Rook can then defend Q4; it isn't doing much on the Queen Bishop's file anyhow. 1...Q×NP still doesn't work because of 2.N-B4. Pressure on KB7-K6. Pleasant.
1.KR-K1 is my move.

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(M3; A) S: S. LANDAU E: DE GROOT T = 15 MINUTES DECEMBER 8, 1938

- Let's have a look. White's position is superior in any case. To search for a combination in connection with KB6 and Q5.
- To figure out
1.B×N/5, with possibly N-Q7 afterwards, in case Black takes back with the Bishop. 1...B×B then later N-Q7; so 1...P×B is forced; but one doesn't get very far with that either.
- Let's look at
1.N×B. Also important. Calculate 2.N×N, to see if that provides any direct advantage. 1.N×B, R×N; 2.N×N, N×N; 3.B×N, R×R probably leads to nothing. 4.R×R, B×B; but wait a moment, also 4.B×R is possible. What then? Let's have another look:
1.N×B, R×N; 2.N×N, N×N; 3.B×N, R×R. Now something else: 4.B×B, R×Rch; 5.Q×R, R-K1; 6.B-B5 probably to the advantage of White. But it can be played otherwise:
5...P×B; 6.B×R – haven't achieved much then; leads indeed to nothing.
- Look for something else:
1.N×B, R×N; 2.N×N, N×N. Now something else? No, doesn't yield much.
1.N×N, N×N; what then? 2.B×B, N×B – nothing. There is no decisive combination. Then maybe an ordinary attacking move:
1.P-KR4 for instance; but what then on 1...Q×NP? Is there any compensation then for the Pawn? Probably not.
1.B-R6 maybe. See if anything comes out of that. (Short Pause)
Still, I keep looking at taking on Q5; there might be something in it. 1.N×N or 1.B×N/5. If 1.B×N/5, then 1...N×B is impossible. 1...B×B would be forced, therefore – but that has its drawbacks too: loss of the exchange. Yes,
1.B×N/5 is the move. With that White gets the advantage. 1...P×B is forced, and then a favorable position is reached. Play 2.P-B4 for instance. Maybe we can get even more out of it.
1.B×N/5, P×B; 2.N×B in order to win a Pawn possibly? No, that doesn't

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- work. Let's look at 2.Q-B3. Then 2...Q-Q1 is forced. What then?
- 1.B×N/5, P×B. A combination with 2.B×N, B×B; 3.take on Q5 and N-Q7 – insufficient.
1.B×N/5 followed by 2.P-B4 is probably the best. Maybe a better continuation? Yet possible perhaps to get material advantage? Search for something.
Once again:
1.N×N, N×N; 2.B×B, N×B. Does that yield anything? No, better is 1.B×N/5, P×B; and now? What further? In any case
1.B×N/5 is good.

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(M6; A) S: RAYMOND WEINSTEIN E: BAYLOR T = 2½ MINUTES AUGUST 28, 1961

- Well, I see my Queen isn't *en prise* anyway. Ah, yeah, diagonal is closed. 1.B-R6 puts the, ah, Rook in an odd position. Let's see. 1.Q-R3 – there's a move, but ah, might have some Q-R4 possibilities. My NP is *en prise* – got to watch the NP – yeah the NP is important.
- However,
1.B×... 1.B×N/5, N×B; 2.N×N, wins a piece. 1.B×N/5, B×B; 2.N×B. Ah wait a second, wait a second – there's some sort of, ah, some sort of idea there: 1.B×N/5, B×B; 2.N×B, P×N. I play 1.B×N/5. I don't see it all, but I see that 1.B×N/5...
- Oh, wait a second, there's another possibility ... hmmm ... yes, there's still another possibility:
1.B×N/5, P×B; there's a possibility of playing, ah, playing what? 1.B×N/5, P×B. I play 1.B×N/5 mainly because it gets rid of my weakness, and I have some Queen move with a tempo. For instance, I can play 2.Q-B3; if he plays 2...Q-Q1, I don't see that I've gained anything definite except his Rooks are unconnected now, and I still have the initiative.
All right, I play
1.B×N/5.

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(W1; A) S: MISS HEEMSKERK E: DE GROOT T = 7 MINUTES DECEMBER 6, 1938

- Both sides have equal material. The Knight on K5 is good for White. (Looks silently) 1.P-QN4 because the QN Pawn is attacked; but then my Knight on B3 becomes a bit 'weak.'
- Let's look at
1.N×N. 1...B×N is not good – then I take again and 2...P×B is not then possible because he loses the exchange. Indeed, he loses the exchange. 1...N×N; 2.B×N, B×B/N; and then what might turn up I don't know. No, 2.B×N is not good because after 2...B×B/N the Rook on QB1 is also under attack. Disadvantage. So rather 2.B×B, N×B.
Let's look at
1.N-K4 – that's not very good: ...Q×NP follows.
1.N×N, N×N; 2.B×B, N×B; 3.P-QN4 then probably.
(E: Who's better off?)
White's better off – I'm prejudiced.
Yes, my move is
1.N×N

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(W2; A) S: MRS. ROODZANT E: DE GROOT T = 28 MINUTES OCTOBER 17, 1939

- First have a look at the material. Count Pawns. White has an isolated Pawn. Look at how many pieces of White and of Black are under attack. The Queen at Q3 is undefended. Two White pieces are attacked. On the basis of the fact that White attacks more men in the Black position, he is better off. Still to see

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how many squares one controls. (Many short pauses in between here) The Knight on K5 exerts quite a pressure on Black's position. Moreover, KN7 is weakened; White's King's wing has not been weakened. Let's look for how White can best get on the attack. Also examine whether the hanging Bishop on K7 can be profited from. First by exchanging moves leaving it undefended and then to profit from that. Doubling the Rooks on the QB file is also possible; then P-QN4-N5 in order to attack his Rook on QB8 twice.

1.N×N is what I'm looking at. Can't get any decent advantage. 1...N×N; 2.B×B, N×B; then indeed I've gotten his Knight back on K7. But on the other hand, the diagonal of his Bishop on QB6 is open. No great advantage. Also look at:

1.B-R6.

Perhaps I would play here:

1.R-B2 and then see about continuing with P-QN4-N5.

(E: In a real tournament game would you make your move on the basis of all this?)

No, it's very difficult. I'm looking now at combinations in connection with 1.Q-R3. Exchange on Q5 often enough so that he has to retake with the Pawn and then his Rook on QB8 is once again attacked.

Perhaps

1.P-B4 is playable, in order to continue with P-B5. 1.P-B4 could indeed be played; then he cannot play 1...P-KR3, for I just take it; the Rook gets a free file (KB file). Then also the Queen comes in with it. In any case White will have to extract some profit from that weakness after all.

Yes, methinks that

1.P-QN4 is the best to play, with the intention of chasing the Bishop away

with P-N5. If 1...N×N, then 2.R×N and KR-B1. But wait, then he can play 2...B-N4, and then I have to exchange my Rook at B3 and his Rook will get on the open file and ... then I must lose the exchange. But I also can take back with the Queen, 2.Q×N; then after 2...B-N4, 3.Q×R gets two Rooks for the Queen. Doubling Rooks is O.K. then, but then the Pawn on Q4 gets lost. So on 1...N×N, 1.P-QN4 actually runs ashore.

1.Q-B3 should be looked at - but that is idiotic to go stand on the Bishop's diagonal. The intention was to exchange twice on Q5 and then have an attack on KB7.

Let me look at

1.N-K4, 1...N×N; 2.Q×N doesn't work, then the Bishop on N5 falls - and then, too, the Queen stands in the Bishop's line.

Let's look once if any attacks on the Black Queen are possible. Can we get her under the fire of light pieces? But what if I just take off the Bishop?

1.N×B; if P×N, then a weak Pawn; so 1...R×N, then 2.N-K4, N×N - or 2...R×R, but then my Rook gets the open file - and then 3.B×B. Let's see if all that's true:

1.N×B; Q×N is now out of the way: Queen stands in the line of the Rook; so 1...R×N; 2.N-K4, N×N; 3.B×B. ...Oh, wait a moment, that Knight (on Q5) is still here; the Bishop on K7 is covered. Then another way is 2.N×N, R×R, then 3.N×Bch wins a piece. 2...N×N, then 3.B×N and I attack the Queen (*sic*); he must answer that and I win something: the Bishop at K7. And if he plays 2...P×N, then a Pawn on a White square on the Bishop's diagonal is also favorable.

Yes, I play

1.N×B.

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(E1; A) E: DE GROOT T = 22 MINUTES DECEMBER 6, 1938

White has more room. Look further. The Pawn at QN2 is attacked. Can it be

taken? Quite probably; a threat. 1.P-QN4 is to be considered. Does it threaten anything? Hardly. KB6 is weak, and White has more room; he is probably somewhat better off. 1.P-QN4 simplifies too much; not so strong.

Look for a combination; maybe something there.

1.N×B, R×N; 2.B×N/6 and 3.P-Q5 maybe. How to make use of the weakness of KB6, and then to weaken K6? Thus 3.P-Q5, takes; takes, takes; general exchange, without advantage, rather disadvantage.

1.N×N does not lead to anything either.

1.B×N/5; is that something? To take back with the Bishop is impossible; 1...N×B doesn't work either, so 1...P×B is forced. And then maybe 2.B×N, B×B; 3.R×B (*sic*) and 4.N-Q7. But then comes simply 4...Q-Q1 - no advantage.

1.B×N/5 may still be good. Let's look at 1...P×B; 2.Q-B3. No, then comes 2...Q-Q1. It is a pity to waste the King's Bishop if nothing comes of it. Let's again calculate the variant with 3.R×B. So 1.B×N/5, P×B; 2.B×N, B×B; 3.R×B, takes back; 4.N-Q7, Q-Q1. Does this lead to anything? Apparently doesn't go any further.

1.B×N/5, P×B; 2.B×N, B×B; 3.R×B (*sic*), P×R; 4.N-Q7, Q-Q1. Get back the exchange, then probably nothing more: 5.N×R, K×N. What then? Pawn on Q5 is covered, Pawn on Q4 undefended. Is there anything to be done then? 6.Q-N5 for example? Then the Pawn on Q4 hangs - nothing special.

(E2; A) E: DE GROOT T = 12 MINUTES DECEMBER 6, 1938

Equal number of pieces. (Again silently looks at the position) 'By feel' White is somewhat better. KB6 and such are weak. Is there anything to get at, to pick out an immediate advantage? Take the Knight on Q5 perhaps?

1.N-K4 maybe, 1...N×N; 2.Q×N

Let's look at new variants. These are not clear. White stands better though, so there should be something. And 1.B×N/5 looks like a strong move - I don't know why I cling so to 1.B×N/5. 1.B×N/6 - I don't believe in that.

1.B×N/5 and 2.B×N I keep coming back to. 1.B×N/5 and 2.Q-B3 maybe. There must be something to that. But no, that is no good either. There follows 2...K-N2 or 2...Q-Q1. The Pawn at QN2 is attacked. Is that a threat I should be worried about?

1.B×N/5. 2.B×N and 3.P-KR4 perhaps, to assault the King's wing? Or maybe:

1.P-KR4. What to play after 1...Q×NP - on an indifferent move, 2.PR5 follows. But wait! 2.N-B4 costs a Queen. Yes, so

1.P-KR4 now deserves consideration. What will Black play? Probably prevent P-R5. If 1...N×N, then 2.P×N again threatening P-R5. Maybe 2...KR-K1 or something, to cover the Bishop on K7. He cannot play 2...B×RP because the Knight on B6 is hanging. Is there another move for Black? Once again 3.P-R5 is threatened; is this simple to parry?

Go on calculating

1.B×N/5 again.

1.B-N1 also comes into consideration. (E: Now you're gradually beginning to get into time pressure.)

Still look quickly at a few things; weigh things. Yes,

1.B-N1 is my move, later to be followed by P-KR4.

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- 15 Black play? Probably 1...KR-Q1 to consistently hit away at Q4. But then 2.P-R5 is perhaps already immediately possible. 2...N×P doesn't work, 2...P×P looks very bad. So it's probably
20 better not to play 1...KR-Q1 but to cover the King Bishop or move it away. For example, 1...KR-K1; then 2.P-R5
- is prevented. But then White can nicely play 2.KR-K1 in order then to play P-R5 again, eventually. Then N×P is again impossible because the Bishop on K7 hangs. Go over
25 1.P-KR4 again. ... Yes, I play 1.P-KR4. 29

(E3; A) E: DE GROOT T = 16 MINUTES DECEMBER 6, 1938

- Have a look at the position. White's better. Have a look. Does the capture on Q5 yield anything? 1.N×N followed by Q-B3 maybe? 1...B×N doesn't work
5 because of 2.B×N and loss of the exchange. 1.N×N, B×N indeed loses the exchange, but can Black maybe win it back; by 2...B-N4 (*sic*) or something like that? Probably not. How does Black
10 take back on Q5 then? With the Pawn thus. 1.N×N – let's calculate all the variants. How to exploit the weakness of KB6, perhaps simultaneously with a threat
15 on Q5. 1.N×N, P×N; 2.Q-KB3 or something. Pawn on Q4 hangs; that makes matters more difficult. Search for possibilities in order to reach something positive.
20 (Calculations) 1.Q-R3 maybe, or as a second move (after 1.N×N) 2.Q-R3. Then the Queen has two possibilities: attack on the King or pressure on KB6 (Q-R4 possibly). Search for forced moves for Black.
25 It's a difficult position; you quickly lose count. Maybe the move Q-R3 is also strong in connection with an indirect attack on the Rook on QB8.
- 1.N×N and then 2.Q-R3 is principally under consideration. First take on Q5 in order to prevent Black from making it easier for himself with N×N.
35 The Pawn on QN2 is also still attacked. Playing for the win of a Pawn is rather dangerous as the whole lot of White Pawns also hangs; Black will probably win back one of them easily.
40 The capture on Q4 is presumably not so dangerous (in the 1.N×N and 2.Q-KB3 variation) for White; he can easily sacrifice that Pawn and play R-Q1 with an attack.
45 1.N×N and then maybe 2.N-N4? Also an idea. Calm move – then he takes on QN2. Is that dangerous? It looks dismal for White to win back his Pawn with advantage. Take on Q5 and R-N1; or even 2.R-N1 immediately (*sic*). These variations do indeed appear to prevent Q×NP. It doesn't seem auspicious for Black to take on N2.
50 1.N-B4 comes into consideration. Doesn't elicit much; the Knight doesn't do much more there.
55 1.N×N and 2.KR-K1 is also possible. (Time pressure warning) Just go over the main points again.
60 1.N×N.

(E4; A) E: DE GROOT T = 7 MINUTES DECEMBER 7, 1938

- First impression: White is somewhat better off. 1.N×B looks good in order to exchange the strong Bishop, and next to exchange on Q5, to get a favorable
5 ending. Another possibility is to start a direct King's attack, possibly to be prepared by Q-N3 and N-N4 or something like that. But most promising

- seems the former system.
10 Let's calculate 1.N×B. 1...R×N; 2.B×N/5, N×B; 3.B×B, N×B; and then 4.N-K4 is an idea, in order to then plant a Knight on KB6 or on QB5. Meanwhile does
15 2.N×N or 2.B×N/5 produce any immediate advantage? Another method: 1.N×B and 2.P-QN4 – just to await what Black will do. It is not so easy for
20 Black then. Something else: 1.N×N and exchange twice, then ex-
- change on QB8 and play N×KBP. No, that is also nothing special. 1.B×N/5, B×B; 2.N×B and 3.B×N and 4.N-Q7. Wins the exchange. I would play 1.N×N. If 1...P×N, then a flank attack or something like that. Probably best then is 2.Q-KB3.
30 1.N×N, B×N; then 2.B×B to win the exchange; so he is forced to take back with the Pawn, and then I play 2.Q-KB3. Yes, 1.N×N. 35

(E5; A) E: DE GROOT T = 7 MINUTES DECEMBER 7, 1938

- Let's see. What kind of opening has it been? Queen's Gambit, with ...P-Q4 and ...P-QB4. Not sure. Black has a hole in his King's wing. So search for a
5 good move. The isolated Pawn is blockaded. And QN2 is under attack. White must play on the attack. Look for a combination. 1.B×N/6, N×B; 2.N×NP, RP×N; 3.B×P and 4.Q×P perhaps? Or taking
10 on Q5? 1.N×N, then two, actually even three ways to retake. 1...N×N; then 2.B×N. Then he plays 2...B×B/N. How does that go on? Hold on: 3.P-B4, Bishop
15 away; 4.B×B, P×B; 5.N-Q7 wins an exchange! White must aim for there: to weaken the Q7 square; get rid of the Bishop on QB6 and the Knight on KB6. Just go over the
20 1.N×N variation: yes, that's good! I play 1.N×N. 23

UNTRANSLATED

(C1; A) S: H. W. OUWEELEN E. DE GROOT T = 23 MINUTES NOVEMBER 3, 1938

(C2; A) E: DE GROOT T = 16 MINUTES DECEMBER 22, 1938

- The first thing that strikes me is the Rook on the open QB file; the Queen on Q3 – not unpleasant. This is not an
5 easy position! Let me see. There's no threat, is there? The Pawn on Q4 is attacked but defended. The Pawn on QN2 is attacked. 1.B-R6 is possible, to chase away the Rook. How solidly the pieces fit together! Everything is
10 defended. Let's look at exchanges; what will remain then? 1.N×N, B×N; 2.B×B; then he takes back – with the Pawn. Then play 3.R-K1 maybe? Then his Knight is no longer defended (by his Knight at Q5), and then the Bishop on K7 isn't defended so well. Should ...B-Q1 have to be done, then the Rooks are separated. It seems to be a system after all.
20 1.N×N, and if then 1...N×N – then there's still nothing to it since his Bishop on K2 is defended. 1.N×N, N×N; – the situation is not

- 25 inconvenient for Black. Now let us tackle it from another angle. As the thing stands now, Black cannot do much either. Oh, yes, he can take the Pawn at QN2.
- 30 1. P-KR4 maybe. 1...N-R4; then 2. B×B, N×B may follow – that also comes to a dead end again. 3. P-KN4 then? No, not best: the Knight gets to KB4. Calmly doubling the Rooks is an idea too:
- 35 1. R-B2, then the Pawn on N2 is defended as well. What can Black do then? Maybe 1...B-Q3 – then it has a somewhat better position. Then 2. N×B and double the Rooks. But Black can do that too.
- 40 Is there really no feasible plan for an attack? The Bishop (on R2) stands on the diagonal – it reminds me of a game of my own where sacrifices on K6 were possible. Aren't there any possibilities for sacrifices on KN6 or K6? But then it would first be necessary to get the Knight on Q5 out of the way. That does succeed, e.g.:
- 50 1. N×N, N×N; 2. B×B, N×B; 3. N×NP? No, for now his Knight on K2 once again defends his N3. Let's start over with:
- 55 1. B-R6. Another little plan: 1...Rook goes; 2. N-K4, N×N; 3. Q×N – what have you then?
- 60 1. P-KR4 may be in order to still play to break open the King's side. 1...N-R4, then exchange on Q5. Yes, but this must then be preceded by
- 65 1. B-R6 probably. Wait a moment, if N×N, then ...B-N4 – unappetizing. Have a look:
- 70 1. B-R6, Rook moves; 2. N×N, B-N4 - doesn't work now because of taking on K7 with check and then N×NPch, and again N×NPch and Q×Pch (*sic*); we just let him have the Rook on KB1.
- 71 Yes, I play 1. B-R6.

UNTRANSLATED

(C3; A) E. DE GROOT T = 23 MINUTES DECEMBER 22, 1938

(C5; A) E. DE GROOT T = 28 MINUTES JULY 30, 1943

(Blind recount after 15 seconds of looking at the position. After that the normal thinking aloud procedure.)

- Began with a maze of pieces. After that: White has an attacking position – concentration of Queen, Bishop(s), and the Knight on K5 on the King's position, weakened through ...P-KN3. Then looked if there were also immediate chestnuts by Black to fear; didn't see any, idea that there aren't any there. Then: Now I'll have to start thinking of a combination, but I am afraid that there are no immediate winning possibilities. The stand: I have that in my head more in lines than on squares. Black: Castled position, Pawn on KN6;
- 5 Black Bishop bangs his head against the Pawn on K6; Black Queen and Rook
- 10 on the Queen's wing. White: Castled position, Queen and Bishop on long diagonal, Queen on Q3, Bishop on QR2, other Bishop on the QR1 – KR8 diagonal. Wait a minute, there is no place for it: the Knight's on QB3, Pawn's on Q4, Knight's on K5; no, it wasn't there. Where then? on KN5? Yes, it must have been there. Concerning moves I looked briefly at 1. B-R6, followed by a Knight move, sacrifice on KB7 or KN6, and then a Queen move.
- 15 (On being shown the position he was struck by: the Bishop on QB6 was otherwise; thought the QN file was open; the Knight on Q5 is very solidly defended!)
- Thinking aloud:
First the combinational stuff there with
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1. B-R6; doesn't appear so very favorable, still too well fortified. (Rather long pause) Had 1. Q-R3 in mind. No, that doesn't work, then he takes on Q4. No, the whole setup's a bit too fragile for that. Now to try and open up the King's side a bit more: exchange on Q5 (1. N×N); what then? 1...N×N and then you must exchange the Bishop on K7; then too much goes off and it doesn't help much. Maybe
- 40 1. B-R6 after all to chase away the Rook on KB8 (protecting KB7). 1. B-R6, KR-K1 – then is 2. N×BP not sufficient. Otherwise 1. B-R6 wasn't really so crazy, because if I exchange on Q5, then 1...N×N and the Bishop exchange, not so agreeable.
- 45 1. B-R6 in order to shun that and then afterwards take on Q5? I'm now looking at 1. N-K4. 1...N×N; no, because that doesn't help things. Exchanging is the simplest. Not good. The whole thing is a matter of maintaining the pressure.
- 50 1. N×N, N×N; 2. B×B, N×B. What to do then? Hold on, now we're coming to still quite another idea:
- 55 1. Q-R3, to start with a kind of pin of the Pawn on K6 – after Pawn takes on Q5 then the diagonal to QB8 opens up. 1. Q-R3 to be followed by taking on Q5; then after Pawn retakes on Q5, the diagonal's open. Oh, no, 1. Q-R3 doesn't go because of the Pawn on Q4.
- 60 Bah, the thing doesn't become any better from that.
- 65 (E: What are you thinking about?) Now about the Queen Pawn again. I saw in the beginning already that QN2 is attacked, but that didn't seem important to me; that's why didn't give it a further thought. We cannot let him take the Queen Pawn.
- 70 (On being asked:) Still no convergence to one or another particular move. I have the awkward feeling that if I want to start something I immediately get simplifications and then my attack is gone. So maybe I should think about a positional move, but when I make a
- 75 positional move, then Black can himself simplify. If I must play safely: 1. P-QN4 or some such thing; if on the attack: then
- 80 1. P-KR4, for example. What would the follow-up of 1. P-KR4 be? No, it doesn't suit me; gives too many counter-chances. Takes too long. Let's contemplate building up the attack with
- 85 1. B-N1 – but then the Queen's in front, that's wrong; that's overdoing it a bit for sacrificial combinations on KN6. If the attack shall be, then it must be somewhat more circumspect.
- 90 1. N-K2 maybe, to N3, P-KR4, and afterwards P-R5 – doesn't work, then the Queen gets in on QN2. If I want to keep hold of that attack, then
- 95 1. B-R6 really does come into consideration, in order to keep hold of the Pawns and the pressure. And then, for instance, make a safe move; P-QN4 and possibly N-K2.
- 100 1. B-R6 and 2. P-QN4 probably. (On being asked:) In a game I would still keep looking, but it would presumably come to that. (Long Pause) Let's see. (Time pressure remark) First
- 105 1. P-QR4 maybe, in order to prevent his Bishop from coming to my QN5.
- 110 If I chop off that Bishop some time, what then? Wait a minute, I can make him choose. I must get rid of that Bishop on QB6: too many menacing possibilities.
- 115 1. N×B as a first move to see how he'd take back. 1...Q×N, then P-QN4's superfluous. And on 1...R×N or 1...P×N, then he has lost the Bishop at any rate, then at least my Knight on QB3 can get into stride. Well, that's not such a completely crazy idea at all. Black can't start so much. (Pause) What remains after total simplification? Don't believe that that's favorable, then I
- 120 don't stand soundly.
- 125 1. N×B; possibly R-K1 after it if holes turn up on the King's file. Yes,
- 130 1. N×B.
- 134

(G5; B1) S: MAX EUWE E: DE GROOT T = 15 MINUTES NOVEMBER 21, 1939

*Retrospective report printed in Section 45, pp. 200-201**(Thinking aloud):*

Now, I ought to analyze that plan in some more detail:

1...R-N1-N4 - doesn't seem very promising. Then maybe try K-N2 and P-KR4; for instance:

1...P-KR4; 2.P×P, K-N2; 3.P×P, R-R1ch; 4.K-N1, R-R7 and then he cannot defend the Pawn at N2 very well. But he can play 5.R-K4, B×R; 6.P×B. Can I then take the Pawn on N3? No. No time for that since he threatens 7.P-KN3. Now let us count Pawns. That is unpleasant, he has too many of them. Maybe 4...B-R7ch instead of 4...R-R7. Somewhat unclear. But it may be possible to force a draw - I ought to be happy with a draw anyway.

I am looking for another way: R-N1, etc. If I play immediately:

1...K-N2 it will never get better. After that 2...R-R1 is possible but that takes more time. He may be able to shut off the whole thing by P-QB4 and P-Q4.

Again from the other side. It is good that my Bishops are practically impregnable: they command the King file almost entirely.

1...R-N1; 2.P-N3 will probably be his answer; then his Rook may have a difficult time. I could maneuver my Bishop to QB6, but that appears to become purely defensive.

1...R-N1; 2.P-N3, R-N4 is an idea of course. Not so bad. It may even be good. It threatens P-KR4. I have only to make one move, is that right?

(E: Yes, but after that, as you know, I shall make a reply and ask you to play the next move.)

Well, he will be obliged to play P-KB3 sometime. Then one Bishop is eliminated, but the other becomes stronger. Let me look again at the Rook file combination, if there's really nothing positive in it.

Yes, I have an idea:

1...P-KR4; 2...K-N2... but he is not obliged to take on N3. Yes, I have serious doubts whether that will work out. On the other hand my pieces come into play very quickly. (Pause)

1...R-N1 and then the Pawn sacrifice. 2...P-KR4 and 3...R-N4. Then can I go to the KN file? 1...R-N1; 2.P-N3, P-KR4; 3.P×P, R-N4; 4.P×P, R-N4 - and possibly he can first take on B2; so for instance 5.P×Pch and 6.P-KB3. Then it would be wonderful if I could get the King Bishop on the diagonal; otherwise there's nothing more than a check. I have the feeling if you don't proceed actively, you will slowly lose. Materially it is not so good. I have seen games, though, where you could not make any progress. But there the passed Pawns were restrained, here they aren't. 1...R-N1 will do no harm in any case; you do not lose a tempo. (Pause)

The move P-KB3 cannot be prevented in the long run, and then you can hardly organize a mating attack. After P-KB3, perhaps play B-B2 to go to N3. Yes, but there are also other possibilities (after P-KB3): B-R7ch; K-B1, B-N6 and forces the Rook to remain on the first rank. (Pause) If only I could force him to go to KB1, but after B-R7ch comes K-B2. That can never be prevented.

Were I in time pressure here, I would already have played 1...R-N1. But let's examine the consequences somewhat better.

1...R-N1, then 2...R-N4; 3.P-B4, R-N4 - what have I actually got then? No, that's not it. But wait a moment: I think I see something else though. Put the Bishop on QB6 and then try something with P-QR4. I begin to feel that I shall have to set up a defensive position. The King Rook file may then

be tackled later. (Pause)

(E: What do you think now?)

I don't think anything - that happens sometimes. (Short interruption for a discussion on the meaning of this phenomenon.)

The Rooks can do nothing, that's nice.

I play in any case

1...R-N1.

(E: Suppose now that the opponent plays 2.P-N3. What will you play next?)

(G5; B2) Second part, in the new position after 1...R-N1; 2.P-N3. T = 20 minutes

Well, I did not think any further, really. I have seen, of course,

2...K-B1. Also possible is

2...B-Q7, but a disadvantage is that after B-QB6 these two Bishops are on one file - in relation to a White Rook on the QB file. But let's look. If White were to play now, he would play P-QB4 followed by P-Q4. Then his position would be very much strengthened. I cannot let him do that. So

2...P-QR4. Then 3.P-Q4 - no 3.P-Q4 is not so good; then the Queen Pawn gets into trouble: B-Q7-QB6 and R-Q1.

A blockade is necessary; curiously enough nothing is to be done against the King's wing. If I attack the Pawn at KN5, then he plays P-KB3 and I make no progress. Maybe I can get into the QR Pawn's hair? No, taking Pawns is not quite possible. I must aim at a blockading defense.

After

2...K-B1 or some such move, 3.P-B4 is not so dangerous, then comes R-Q1 and R-Q5. After 3.P-Q4 it is more difficult. So is it, for instance, on 2...K-B1 in order to prevent R-K7. If I let him do it and then play 3...K-B1, what happens then? I am now wavering between 2...K-B1 and 2...P-QR4. (Pause) But what's the sense in 2...P-QR4? Nu ya; after 3.P-QR4 his Pawn at QB2 is more or less fixed.

2...B-Q7. Now, let's really start to calculate: 2...B-Q7; 3.R-K7, B-QB6; Rook goes some place. Then he sort of has the threat: R-B7. As soon as I have brought the Pawn on QR2 into safety, by P-QR4, he will do that. Then R-N3

is necessary. No, it's looks to me that I only provide more activity for his Rooks. If:

2...K-B1... The Bishop on KB5 has a fine position, but I will have to use it for blocking his Pawns. So let's look: 2...K-B1; 3.P-B4; R-Q1. (Pause) Then 4.KR-Q1; no, why not 4.QR-Q1; but then after R-Q5 he will make no progress either. (Interruption) So 3.P-B4 is nothing to be afraid of. Let's not be afraid of that.

2...K-B1; 3.P-Q4, B-Q7; 4.R-K5 (or K2), B-QB6. At K5 the Rook is not so well posted. Yes, it is, for after B-QB6 there is a threat: R-QB5. So we have to go about it more carefully. Those two Bishops hang; that makes it difficult.

2...K-B1; 3.P-Q4, R-Q1; 4.QR-Q1 and then White proceeds with P-QB4 and P-Q5. So P-Q4 is a threat, after all. All as a consequence of the accidental position of the two Bishops on the QB file. 2...K-B1 I don't believe in anymore. Yes, but what then? If immediately.

2...B-Q7, then he plays 3.R-K7, threatening R-B7 and R×RP. Yes, I foresee a grim future.

2...R-Q1 immediately prevents P-Q4 easily enough. But then the Rook gets in on the seventh rank. (Pause) I guess I am taking a little rest again to collect some ideas. It is a rotten position. Under these circumstances I am almost ready to look again to see if I can mate him.

2...R-N4... Yes, now I have an idea. 2...R-N4. Suppose he doesn't want that Rook. I see indeed a possibility to win the exchange. Against the loss of a Pawn, it is true, but still an improve-

ment. The maneuver is B-K4 to Q5, after his P-KB3, with mate threat. 85 2...P-KR4 or 2...R-N4? The order is not certain yet. But there is something nice in it. I don't know if it's good enough for a draw. Let's just continue roughly:

90 2...P-KR4; 3.P×P; R-N4; 4.P×P, for example. (Pause) But wait, we still have 4...R-R4ch and 5...R-KN4. That leads to a forced sacrifice – we have had that before. Then he has five Pawns 95 against my piece. But maybe I can improve that; or win something back immediately after? Then, he stands not so well after all. Perhaps first go to KN4 before checking on R4? It is also 100 possible. It might be possible to play P-B4 in between, to prevent his R-K4 – but that is a bit too liberal with tempi. 2...R-N4; 3.P-B4, R-N4; 4.P-B3. Is that forced? Of course. 4...P-KR4, 105 5.P×P, R×Pch; 6.K-N1, B-K4;

7.QR-Q1; B-Q5ch. Well, that's a drawing combination; that's nice. Is that so? No, 3.P-B4 is not necessary to begin with. But even so it's something. So 2...R-N4. He must be a bit careful. 110 He needn't play QR-Q1, but that does not make any difference. The White Pawn at KN2 cannot very well be defended.

Let us look at some other possibilities. 115 No, the Rook not only has to be on the KR file but also on the fourth rank, otherwise I never can play B-K4.

2...P-KR4 would be nice but after the sacrifice of the exchange (after R-K4) 120 he will win. What if he plays after (2...R-N4; 3.R-K7, in order to gain a Pawn? ... Well, I've got that other threat: 3...R-N4 and 4...P-KR4; 5.P×P, R×Pch; etc. Yes, I play in any 125 case

2...R-N4. 127

(M2; B) FIRST PART (M2; B1) S: NICO CORTLEVER E: DE GROOT T = 15 MINUTES

OCTOBER 17, 1939

(S counts the Pawns.) Difficult: this is my first impression. The second is that by actual numbers I should be badly off, but it is a pleasant position. I can do a whole lot of things – as usual. Get my Rook into it, at the Pawns. Nowhere 5 for his Rook to stand on the King's file, except on my K2. And that I can always prevent with K-B1. 1...K-B1 is impossible. If I, for example, play 10 1...R-N1; he 2.QR-N1, B-Q4 – then he can still get in. I might try to block his Pawns; then I can't win, but that won't be too easy anyway. Also seek 15 something to do along the Rook's file: P-N4 – P-KR4 – K-N2 – R-R1. Then he must play something like P-KB3. Exchange of Rooks in general not good; must avoid that.

20 The first move under consideration is 1...K-B1 to keep the Rook out. Then 2.R-K2 and double them; ... doesn't

help matters either then. But if he then advances with his Pawns: P-B4, P-N4, or like that; then hard to stop them. 25 Straight off, that won't work, after 2.P-B4 then R-Q1 and R-Q5 follow and I blockade him.

1...P-KR4, I don't like the idea of that very much. 30 If immediately 1...R-N1, then 2.R-K7 is annoying. I should play 1...K-B1, or perhaps 1...B-KN4. 35 Oh, no, 1...R-N1; 2.R-K7 won't do because of 2...R×P. Consequently he must reply 2.QR-N1 or 2.P-QN3. Then perhaps 2...B-Q4; but then comes 40 3.P-B4, so that is not so good. I do not stand well after all. He can always play P-QB3. Can I prevent that?

1...R-N1; 2.QR-N1 and now B-Q7-

45 QB6. Or perhaps 1...R-N1 and then 2...K-B1; then he still cannot play P-N4, so maybe 1...R-N1; 2.QR-N1, K-B1; 3.P-QB3. No, doesn't suit me though.

50 Maybe still something better; still on the King's wing: 1...B-KN4 or something like that, 2.R-K5, P-B3 – no, a touch of fantasy. 1...P-KR4 immediately, 2.P×P, P×P 55 or 2...K-N2; no, then 3.K-N1 – not worth much either. 1...R-Q1 and 2...R-Q5 – no good either. 1...R-N1; 2.QR-N1, P-KR4; 3.P×P, then 3...R-N4; 4.K-N1 – or 4.P×P – 60 R×RP; and 5...R-R7. Maybe not so crazy. 1...R-N1; 2.QR-N1, P-KR4; 3.P×P – 3.P-KB3 is not good, then the Rook

comes to KN4 – therefore 3.P×P, R-N4; 4.P×P. Am I doing anything to him then? I also can immediately play 65 2...R-N4 and then P-KR4. The Bishop, can that do anything? 1...B-Q7; (2.R-K7), B-N5. A little slow. And the Bishops are well placed. 70 Yes, a little slow. 1...P-QR4 – but that is nonsense. The Rook must do something; other moves are a bit too passive in that position. 1...P-QR4; 2.R-K7, R-K1 – Rook 75 exchange; no, that is nothing. Yes, I begin with 1...R-N1.

(E: Suppose now that the opponent plays 2.QR-N1 or 2.P-N3. Let's make it 2.QR-N1. What will you play next?) 81

(M2; B2) Second part, in the new position after 1...R-N1; 2.QR-N1. T = 10 minutes

Let's have a look: 2...B-Q7; 3.R-K7, B-QB6; 4.P-N3, R-N5. ... Ach, no, 3.R-K7 is bad. 2...P-KR4, what then? Or 5 2...R-N4. Then 3.R-K7, P-KR4; 4.P×P, R×Pch; 5.K-N1, B-R7ch and 6...B-Q3. Then 7.QR-K1 or something like that. If I do nothing, I have the idea that I am gradually going to lose. To 10 hold back the Pawns doesn't work so well. No, I must do something on the King's wing. Perhaps to play 2...R-N5 – doesn't seem so strong to me. The Bishop on B5 must then first 15 move. No, that stands well. No, look again: 2...R-N4; 3.P-B4; then 3...R-N4 and I win a Pawn; unless 4.P-B3. Then

4...P-KR4; 5.P×P, R×Pch and advance with the KN Pawn – not so 20 bad. 2...R-N4; 3.R-K7, P-KR4; 4.P×P, R×Pch; 5.K-N1, R-R7; 6.P-KB3, P-N4; 7.R×RP, P-N5; 8.R-KB1, P-N6, threatens mate! Not unpleasant. 25 If he now defends the Pawn instead of 4.P×P, thus 4.P-KB3, then take twice on KN5 and R-N4. 2...R-N4. Then also to offer a Pawn at KN4, in order to stop the attack (after P-KR4) – possible. But now I can take with the Rook; after P-KB3 I can then move the KR Pawn forward. That also gives good chances. 30 Yes, I should play 2...R-N4. 35 36

UNTRANSLATED

(M4; B) S: T. D. VAN SCHELTINGA E: DE GROOT T = 28 MINUTES OCTOBER 17, 1939

(W2; B1) S: MRS. ROODZANT E: DE GROOT T = 15 MINUTES OCTOBER 18, 1939

Let's count Pawns. Figure out who's better off. So materially I am behind.

White has quite a number of passed Pawns. Difficult. I have not one passed

- 5 Pawn. I could start with.... The Rooks are so situated that after a while I might be able to attack two Rooks with a Bishop.
- I can attack the Pawn with my Rook:
- 10 1...R-N1 - all said and done the Rook belongs on an open file. (Pause) But that is an endless task: with
- 1...P-QR4, I wanted to try to contain the Pawns. The White King is on the
- 15 Bishop's diagonal - but I can't extract anything from that either, of course, The King cannot get out, it's true, but that's about all then. (More pauses: Subject is encouraged to report.)
- 20 Well, I can attack his Pawn on KN4 with

(W2; B2) SECOND PART, IN THE NEW POSITION AFTER 1...R-N1; 2.P-N3 T = 7 MINUTES

- Well, if 2.P-N3 were played, ... then maybe
- 2...B-R3, in order to play B-KN2; or what actually comes to the same thing:
- 5 2...B-Q3. But I can also play
- 2...P-QR4. Only, by Jove, I don't see how I get anywhere with it! Well...

(C2; B1) E: DE GROOT T = 13 MINUTES JUNE 14, 1943

(Blind recount after ten seconds of looking at the position. After that the normal thinking aloud procedure.)

- The first thing that strikes me are the two Bishops, which have such magnificent positions. Next his Rook on the open file, set me to thinking of a back rank mate; no, that's not possible, there's a hole and besides there's my Rook on QR1. How to strengthen the position is the idea now, how to activate the Rook. That's the only piece that doesn't do anything yet. I looked for a moment at 1...P-KR4 in order to get the Rook to the KR file, after K-N2 that is, and further glanced at 1...R-N1 - probably simpler and better. I saw
- something like 1...R-N1; 2.P-N3, B-Q7 and 3...B-QB6. (In answer to the question:) Still no idea about who is better or worse off, I had no time for that.
- (The position is dictated correctly except for some uncertainty about a Black Pawn on the Queen or Queen Bishop file.)
- Thinking aloud:*
- The Bishop at KB5 is practically unassailable, neither P-KN3 nor R-K4 to attack it works. Material situation? A pair of Bishops against a Rook and three Pawns. Ugh. Of course I have to make a plan here. A King's side attack - P-KR4, for example - or try to win

- something, 1...R-N1 or something like that. I'm now thinking about the general strategy, which plan is best. I'm particularly looking at 1...P-KR4 and 2...K-N2. If White lets me take, then the possibilities are good. There's probably no mating attack, the King is still too safe for that.
- 1...P-KR4. Defending the Pawn doesn't work and if he captures - no, he can defend after all with 2.P-KB3, then comes 2...B-N6, prevents him from doubling Rooks and it ties the King down. (Pause) Let's have a look at White's chances. What if he just ignores me and plays 2.R-K7 in order to double the Rooks? ... White's counter-chances rest with his Pawns - next to the possibility of getting in with his Rooks. The question is, in fact, must anything concern Black so much that he cannot play 1...P-KR4? On principle I am inclined to play 1...P-KR4. Let's look at it a bit closer.
- If White starts his Pawns; let's see if
- Black's attack is strong enough before the Bishop on QB3 gets into troubles. Thus
- 1...P-KR4; 2.P-Q4, P×P; 3.P-B4, K-N2; 4.P-Q5; then R-Rch; and 5...R-R7; 6.P-B3? I have the feeling that White has the better of it - on the basis of his Pawn majority. The Bishop's diagonal will soon be closed. 1...P-KR4 doesn't offer so many prospects after all.
- 1...R-N1; then White plays the Pawn up. Then maybe B-Q7-QB6. But that plays into White's plans. Have also looked at the possibility of R-N1-N4 and then P-KR4 to get to the KR file that way - or possibly to KN4. That doesn't make much difference though, no improvement over 1...P-KR4. Then I rather play
- 1...K-N2.
- (E: Suppose now that you had played 1...R-N1 and your opponent answers with 2.P-N3. What will you play next?)

(C2; B2) SECOND PART, IN THE NEW POSITION AFTER 1...R-N1; 2.P-N3 T = A FEW SECONDS

- 1 2...B-Q7. I already saw that, in fact in the first ten seconds. I don't know anymore why I discarded that a few minutes ago.

UNTRANSLATED

(C5; B) E: DE GROOT T = 30 MINUTES JULY 30, 1943

(G5; C) S: MAX EUWE E: DE GROOT T = 7 MINUTES NOVEMBER 21, 1939

- (Blind recount after ten seconds of looking at the position. After that the normal thinking aloud procedure.)
- I saw that he threatens B×Pch, and that can be prevented with 1...P-Q4. But for the rest I didn't see much. Might try to break up his Pawn position, but how this should be done is not clear. I have a weak Pawn but his King's position is weakened. Ma-
- terially, both sides are even, I believe. (The position is dictated correctly except for uncertainty about the Knights on the KB file.) If I have a Knight on KB3, then he must have one somewhere too - on his KB3? No, that's no place for a Knight; or maybe it is? Well, probably neither side has one.

Thinking aloud:

Now (with the Knights on B3) the move 1...P-Q4 is rather beastly - no,

then I would rather sacrifice on K3. His Knight on B3 is sort of hanging. I might play 1...N-K5. Or have I to... No, I needn't fear P-Q5, not after B×Pch either. I'd just take the Queen Pawn off. 1...N-K5 is at first sight a nice little move. By capturing on K6 he only helps me to develop. For that matter I would only be pleased if he took. My Rook comes in with gain of tempo. But maybe he'll play N-N5 after B×Pch and Q×Bch - wins the exchange; that's going a little too far.

Let's look for a move. Can I start something there? I mean:

1...Q-K5. No, I must guard my Pawn on B2. Let me see: 2...N×Q if he takes mine; 3.N×BP, R×N. Can I then demolish him? Yes, that's quite good.

(G6; C) S: SAVIELLY TARTAKOWER E: DE GROOT T = 15 MINUTES OCTOBER 21, 1939

So, apparently Black has a position where he's put all of his eggs in one basket. He's played on the attack, it's been a sharp conflict. Black is on move? (E: Yes.)

Well, those are the general points of view. Black will have to defend on K3. 1...P-Q4 then comes... 2.N-K5, disagreeable; that doesn't go.

1...N-K5 is also possible - kind of masked attack. But temporarily let's be unassuming. Simply cover the Pawn. Yes, the White attack is completely blocked.

Numerically both sides are equal. Black will probably have to realize that he stands badly - but still, one can perhaps pull off a masterly stunt. Search for combinative finesse.

1...R-K1 has the objection that the Rook is off the KB file. Purely defensive. One can also try

1...Q-K5. Masked attack, with the main variation being (slowly): 1...Q-K5; 2.Q×Q, N×Q; 3.N×BP, R×N; 4.N×R, ... R-B6ch. Very strong, gives at least a draw. Possibly even 4...R×BP instead of 4...R-B6ch. Yes, that's

1...Q-K5 is not bad. But he plays 2.KR-K1, exchange; and then my Pawn on B2 is weak, and also the one on K3 - no, the endgame is not favorable.

1...N-K5 is not unattractive. The 2.N×BP capture, is such a combination possible? 2.N×BP, then 2...N-B6 wins a piece; yes, if 3.B×Pch, then 3...K-R1. So after 1...N-K5; 2.N×BP doesn't go.

1...N-K5 is a good little move for the time being. (Pause) I do not have many threats. Well, N×BP is threatened. Can he annoy me? 2.N×BP - no, cannot be done. The Queen is tied down. And the Knight cannot do anything. And if 2.P-Q5 I take it away. Yes, I play 1...N-K5.

favorable for Black, this variation. What else to play for White.

1...Q-K5; 2.KR-K1 - and everything of Black's remains hanging. Thus this variation is also nothing for Black. Nothing remains but to play the passive

1...R-K1.

(In between, the Subject tells the anecdote about Lasker, a kibitzer, and a Queen sacrifice.)

1...Q-K5 is very bad, the exchange on K7 only benefits White's game, has very unappetizing consequences.

Play

1...R-K1 - utterly passive.

1...P-Q4 doesn't work so well. Now look at that for the second time.

1...P-Q4; 2.N-K5, Q-K1; then 3.N×P. 2...Q-N3 then, but then comes 3.P-B5, doesn't hold. Or maybe after all: 3.P-B5, Q-R3 and now his Knight on N5 is pinned, so can't take on B7.

1...R-K1; 2.N-N5; then 2...P-K4 can be played.

1...R-K1 is really the normal defense. (Pause)

(E: Imagine now that you must make

a move, that you have this position in a tournament game (this was already said in the instruction, of course).)

1...R-K1 is playable, but I prefer 1...P-Q4. If 2.P-B5, then 2...P-QN3. 1...P-Q4; 2.N-K5, Q-N3; 3.P-B5, Q-R3; 4.KR-K1, N-K1 and then later P-QN3 (or first P-B3).

1...R-K1 is purely passive, doesn't suit me. No, I would play 1...P-Q4.

(M2; C) S: NICO CORTLEVER E: DE GROOT T = 30 MINUTES OCTOBER 18, 1939

One, two, three, ..., seven - counting Pawns. ... Black is perhaps a bit better off. White's King position is torn up.

1...N-K5 is the first impulse. What happens on that? ...N-B6 and ...N×BP threaten. He can play 2.P-Q5 but I just take it off. No, not so rotten. Is 2...N-B6 really a threat? (1...N-K5; 2.No Move), N-B6; 3.B×Pch, B×B; 4.Q×Bch, and 5.N-N5, so 5.N-N5, N×R; 6.N-B7ch; I cannot take it, yes, I can - no, 6.N-B7ch is not so good; 6.R×N is better.

Oh! The Q-B6 square is defended! But ...N×BP is still a threat after 1...N-K5. 1...N-K5; 2.KR-K1, N×BP; 3.B×Pch, B×B; 4.Q×Bch. This is quite acceptable. Can he do anything else instead of 2.KR-K1? 2.P-Q5, P×P; 3.N/5-Q4, Q-B4; 4.B×B, QR×B; 5.N-K6, Q×KBP; 6.Q×Q, N×Q; 7.N×R, K×N; 8.KR-B1, N×R - White sacrifices a Pawn and gets some attack. Let's have another look:

1...N-K5; 2.P-Q5, P×P; 3.N/3-Q4, Q-B4; 4.B×B, QR×B; 5.N-K6, Q×KBP; 6.Q×Q, N×Q; 7.N×R, R×N; not such a favorable endgame, weak position of the King.

1...N-K5. Let's look at other moves. For example 2.B×Pch, B×B; 3.P-Q5, *Q-B4; 4.Q×N, Q×KBP; 5.N-N5 threatens mate. No, then it's not so good. 1...N-K5; 2.B×Pch, B×B; 3.P-Q5. Can I do anything else then?

... No, that cannot be sound. I can, of course, play 1...P-Q4 myself - no, then I get a hole at K4. Maybe my position is not so good after all.

1...KN-Q4 - no, a spineless move. It threatens nothing in due time. Yes, I would really like to annoy his Knight on B3. Can't I get an attack?

1...N-K5. I really don't see much else. 1...N-K5; 2.B×Pch, B×B; 3.P-Q5 (Pause) 3.P-Q5, R×N. I am looking at a Queen sacrifice, but that is not worth much either. 1...N-K5; 2.B×Pch. Whether I take or not now, the answer is always the same: 3.P-Q5, 1...N-K5; 2.B×Pch, K-R1; 3.P-Q5, Q-B4; 4.Q×N, Q×KBP; 5.N-N5. A piece down and on the point of being mated. Of little worth.

I don't see many other moves.

1...N-R4 is nothing to write about. And the other pieces, the Rooks cannot do much.

1...Q-K5 perhaps; 2.Q×Q, N×Q; 3.N×BP, N×BP. No, not so good.

1...Q-K5; 2.Q×Q, N×Q; ... (mumbles). No,

1...N-K5 is much more attractive. The Q-K5 variations don't appeal to me at all.

1...Q-K5; 2.KR-K1; exchange of Queens, then my K3 is weak. 3...N-R4 then; no, rather 3...N-K1 to cover the Pawn on B2, and his Knight on B3 is

* Subject M2 never saw that the White Queen Pawn can just be taken (3...B×P wins). This is one of the many instances from which it appears that position C, with its pinned Pawns and other tactical peculiarities, leads more easily to miscalculations and plain errors than the other positions used. Also with Subjects G6 and M4; practically every C-protocol contains some evident errors or miscalculations.

attacked. If he moves it away, my position is as rotten as a mackerel. 1...Q-K5, 2.KR-K1, Q-N3; 3.B×Pch, 4.Q×Bch, K-R1; 5.N×BP, QR-B1; 6.N moves, back to QN5. ... Oh no, there's a mate on QB7. After 4...K-R1; mate on QB7 is threatened. So, e.g., 5.R-Q2. Not so crazy. Also 3.N-R4 directly after 2...Q-N3 is unpleasant. Let's get back to 1...N-K5; 2.B×Pch, B×B; 3.P-Q5, N×NP; 4.Q×Bch wins a piece. 1...N-K5; 2.B×Pch, B×B; 3.P-Q5, Q-Q2; 4.P×B, Q×P; 5.N×BP, then 5...Q-R3ch. That works out, not so crazy. 1...N-K5; 2.B×Pch, B×B; 3.P-Q5, Q-Q2; 4.Q×N, B-B4; then I've lost a Pawn but it's not so crazy. But wait, 4...B-N5 is also possible, already wins an exchange – no, he can play 5.N-N5 then. So: 1...N-K5; 2.B×Pch, B×B; 3.P-Q5, Q-Q2; 4.P×B, Q×P; then I always have Q-R3ch, and I get out of the pin. Has he anything against that? Yes, he can play 5.N-Q2, parries the check. 5...N-B6 then? No, then he takes my Queen with check. 5.N-Q2 – 5.N×BP is not good because of 5...Q-R3ch – 5...R×P; then 6.Q×N with the win of a piece. 5.N-Q2, Q-K4; in order to give check on QR8 or QB6 maybe. Again: 1...N-K5; 2.B×Pch, B×B; 3.P-Q5, Q-Q2; 4.P×B, Q×P; 5.N-Q2, Q-K4; 6.Q×N, then 6...Q-R8ch; 7.Q-N1. Q-B6ch and mate on the next move. Oh, no, 7...Q-B6ch is impossible. But ... on 7.Q-N1 I still have something: 7...N-Q6ch; 8.K-B2, N-N5ch with a draw. But after 7.N-N1 it's rather frugal. Maybe I can play 7...R-K1 then. I really have an idea that that's not so terrible. 1...N-K5; 2.B×Pch, B×B; 3.P-Q5, Q-Q2; 4.P×B, Q×P; 5.N-Q2, Q-K4; 6.N×N, now 6...Q-R8ch – but that's not necessary, first 6...R-K1 threatens to win back the piece. If he now takes on QB7, then I take on KB7. 1...N-K5; 2.B×Pch, B×B; 3.P-Q5, Q-Q2; and now for instance not 4.P×B but 4.Q×N. Then comes 4...B-B4. So, 1...N-K5 is playable. Now suppose he doesn't take on K3. Let's examine some other variations. (E: You are getting into some time pressure now.) Is it possible to do something else? 1...K-R1, 1...KN-Q4, or 1...N-R4? Or 1...Defend the King Pawn (1...R-K1); I really have even less faith in all that. Everything is going badly. Yes, I play 1...N-K5. In case he does nothing, then I play 2...N-B6 nonetheless. Then: 3.B×Pch, B×B; 4.Q×Bch, K-R1; and then on 5.N-N5 I probably play 5...QR-K1. Yes, I play 1...N-K5.

(M4; C) S: T. D. VAN SCHELTINGA E: DE GROOT T = 21 MINUTES OCTOBER 19, 1939

Well, that's a peculiar situation. Seemingly (interruption). ... Peculiar. The White's King's position is broken up. Yes, the first thing you look at is the possibility, 1...N-K5 to QB6. The Pawn on K3 is attacked, you must do something about that. Probably execute a counter combination. 1...N-K5 looks like a very good move to me; threatens N-B6. 2...N-B6 – but maybe it's not such a serious threat, then you take at K3. ... Oh, no, then comes K-R1 and then you have to lose something. (Pause) You can also ..., let's have a look. Is 1...P-K4 something? Just another look at 1...N-K5. There's always ... 1...N-K5 threatens 2...N-B6; can be parried with 2.K-N2 if need be. Perhaps some-

thing to do against the Knight on B6. Yes, 2...N×BP does it. Thus the double threat: 2...N-B6 and 2...N×BP. We must therefore look for a sharper defense. What other reply to 1...N-K5? Of course, he can ..., no nothing; N-N5 I thought, but it gets taken. 1...N-K5 looks very strong. Yes, a good move. I think 1...N-K5 is a good move. 1...P-Q4, then comes 2.N-K5. But 1...N-K5 is good – I really don't have to look into a passive move. How can White answer 1...N-K5? 1...N-K5; 2...N-B6; 3.B×Pch, and now can you also take; yes, can also take But then comes P-Q5. So for instance 1...N-K5; 2.R-K1, N-B6; 3.B×Pch, K-R1; 4.P-Q5. 1...N-K5; 2.QR-K1 parries both – but then perhaps P-Q5 doesn't go, after taking on K3 – yes, it does. It's not so easy. How fitting. ... 1...N-K5; 2.QR-K1, N×BP – also possible – 3.B×Pch, B×B; 4.Q×Bch, K-R1; 5.P-Q5 – the Knight on KB6 and the Rook on KR8 are attacked; then (after 5.P-Q5) to play 6.N-N5. Where does the Queen go? 5...Q-K1 then 6.N×BP and 5...Q-B4, then follows 6.N-N5, N×R; 7.N-B7ch and I'm not allowed to take that, yes I am: 7...R×N; 8.Q×R threatens R-K8. So there are objections – that counter-attack is strong. (Pause) 1...Q-K5 is also possible: 2.R-K1. ... Oh! What nonsense (discovers White Knight on QN5) 2...N-B6 is no threat after 1...N-K5! Now, of course, other moves require consideration. So after 1...N-K5, the only threat is 2...N×BP. 2.R-B1 and then that threat isn't so strong anymore either. Let's look at 1...N-R7ch. If he takes, then I take at KB6 with a good position. But it's not so much. If I play 1...P-K4, just look. 1...P-K4; 2.P×P, B×B; 3.P×N, R×P. Good position. Yes, 1...P-K4 suits me much better. 1...P-K4; 2.B-N2. What threat is there, not much. Then I play 2...B-N5. I cannot profit very much by the hanging Knight on KB6. 1...N-R4 – is no move. Then he takes on K3. 1...P-K4 is a relatively simple move. 1...P-K4; 2.P-Q5, Q-B4. If he exchanges on QB1, then I have a good position. Later ... P-B3 will follow. And on 2.B-N2, I play 2...B-N5 or 2...B-B4. Then you threaten ..., not even anything definite, but ... Maybe I can profit from the pin of the QB Pawn, but I don't see very well how. 1...N-Q4 is senseless. I'm not now thinking of anything definite, but I'm looking if I can strengthen the matter. No, I can't. So just play 1...P-K4, 1...P-K4; 2.P-Q5, Q-B4 and then quickly comes ... P-B3. For the rest I don't see any attack. In this position I should play 1...P-K4. Yes, I play 1...P-K4. (E: Do you think Black has a better position?) Yes, I rather think Black's position is better.

(W2; C) S: MRS. ROODZANT E: DE GROOT T = 20 MINUTES OCTOBER 19, 1939

Count pieces and Pawns – equal. (Many and rather long pauses) Peculiar, those two Knights. Black's Pawn on K3 is attacked twice and defended once... White's King position is more dangerously open than Black's... Let's look at the control of space. In that respect White must be somewhat better off... My Knight on QN5 is not dangerous; on N-R7ch; K-N2 is always possible. That's really a peculiar setup, there on that diagonal: Queen on QB3,

- White Knight on KB6, White Rook on KR8.
- 15 What about playing
1...P-K4? Then a Pawn gets lost it's true.
- 1...P-K4, then the Bishop is attacked.
1...P-K4; 2.B×B, P-K5 – doesn't work
- 20 because of 3.B-K6ch, I cannot sacrifice that Bishop. Otherwise a Pawn gets lost The King's on the same file as the Queen. Is there something to make out of that? Actually that Pawn on K3 doesn't matter so much. ...
- 25 Wait a moment I can also play
1...Q-K5. Then his Queen can't move. But he can exchange and then capture my Pawn on B2. But that will hardly
- 30 happen. 1...Q-K5. ... (Pause)
Wait just a moment, if my Knight on B3 moves away, then his Knight on B3 hangs.
- 1...N-K5 is also possible – then his
- 35 Knight is not attacked twice, it is true, but ...N×BP is threatened and, of course, ...N-B6 threatens to win the
- exchange. So
1...N-K5 threatens ...N-B6. Ah!
...N-B6 is trash! Completely impossible. 40
Well, in a fast game you'd play
1...R-K1 here, facing the Queen, or maybe
1...P-N4, to fork him – but that's not serious. 45
1...K-R1 is possible too.
(E: But what would you play in a tournament game?)
1...KN-Q4 can also be played, it is possible, isn't it? No, it's not after all because of 2.B×Pch and the Knight on Q4 falls. Well ... if I don't take the Bishop, I have to move over with the King anyway. 50
1...KN-Q4; 2.B×Pch, K-R1; 3.B×B – then I might check him at QR7 – no, that doesn't work, he takes it. 55
1...P-Q4? Then N-K5.
Move my King Knight away so as to profit from the position of his on B3.... 60
So let's play
1...Q-K5. 62

(M2; ...P-R5) S: NICO CORTLEVER E: DE GROOT T = 10 MINUTES APRIL 8, 1943

- Count pieces. Black's positionally better off but on the defensive on the King's wing. Trapping the Queen doesn't work here. He has no threats for the moment.
- 5 Counterattack with 1...P-R5? 2.N×P, N×P? 1...R-N7 to threaten the Pawn on KB7. 1...R-KB1 chases the Queen, 2.Q-R4; then he still doesn't threaten anything. (Pause)
- 10 No, at first sight:
1...P-R5's still the best. But if he just does nothing let's me take, there's not much to it. First lure the Knight away (1...R-N7) and then P-R5 or P-QN4?
- 15 1...R-N7 – a bit strange.
1...N-N5 doesn't appeal to me either – let's get a closer look: 1...N-N5; 2.R/6-R4, N×P; 3.Q×BP, R-KB1; not so crazy. Still I've got the idea that there's a hitch somewhere. 20
- 1...N-N5; 2.R×RP? No, doesn't work.
2.R/6-R3 – loses the exchange.
1...N-N5. Let's examine precisely what he must do. 2.R-K6 and after 2...N×P; 3.R-KB1, for example, and after the Knight goes away, take my Pawn on B6 – or 3...R-N7; 4.Q×BP.
- 25 First
1...R-KB1 after all and then N-N5?
1...R-KB1; 2.Q-R4, N-N5; 3.R-R5, R-B5 threatens to win the Queen; 4.K-N1. 30
First
1...R-N5 and then R-N7? I don't think so. 35
1...Q-N5 doesn't make much sense either. Maybe it's better after all to play
1...P-R5 first, then we can always get funny later. Yes, I play
1...P-R5. 40

(M2; K-N2) S: NICO CORTLEVER E: DE GROOT T = 12 MINUTES OCTOBER 24, 1939

- Well, we have been busy 'preventing.' Difficult.
(E: You are supposed to devise a plan, not so much one move.)
- 5 There is little to start on the Queen's wing. P-B4 doesn't give much either.
He cannot start much on the Queen's wing – thus do something like playing the King to QN2 for one thing, to be
- 10 able to attack on the King's wing then. Then the Bishop to K2, Knight to QN5 to make attacking with P-QN4 impossible on the Queen's wing. Rooks to the KR and KN files.
- 15 I can also go about it in another way: King to KR2 and the Rook to KN1 in order to play P-KN4. An objection is that the Pawn on KR4 hangs.
1.B-N5 is also possible, offering a Pawn
- 20 – that is rather dubious.
I can also bring the Knight to K3, that is also an idea. Well, there is really very little cooking in this position. (Pause)
The Bishop on B6 is bad and my Bishop
- 25 is good. My White Bishop on the other hand is bad, so I don't have to fear B-QB1.
I can also bring the Knight to KN3, but, well, then I have to play P-KN4 and the Pawn on KR4 then hangs. Then
- 30 prepare for that perhaps by bringing the Bishop to K1, playing P-B3, and only then go ahead with P-KN4: but that really is a rather sterile idea.
- 35 Or perhaps play
1.P-R5 and R6 for an attack on the King – but on P-R5, B-KN4 follows. Then he swops my beautiful Bishop. I can again prevent that, of course, by
- 40 putting my Queen on K3, but then in
- reply to P-R5 comes the move P-R3; disagreeable. It is really hard – furthermore he can always play P-N3 and B-N2. Indeed that probably is the best; can I stop that one way or another? By means of the looseness of the Bishop on B6 perhaps: some kick so that the Queen must go away, must withdraw? Probably not.
- I can also play
1.Q-N4 and then P-R5 and P-R6 – but then comes 1...Q-Q2, forcing a Queen exchange. Perhaps I can arrange it in such a way that I play P-R5 and R6 as soon as he plays P-N3.
- 55 1.Q-R1 maybe; looks a little bit queer. (Pause) Perhaps the best is still to play the King away.
1.Q-B5 – then P-N3 follows later – no, B-B1 wins a piece, not quite true; but still quite disagreeable, troublesome. 60
So let's play the King away and then bring the Rook to the KR file. Maybe still try to prevent the Bishop trade after B-B1? (Pause) No. He can also sacrifice on KR4, but I'm not really so afraid of that. 65
Maybe as preparation for my King walk I can first play P-B3 and bring the Queen to KB2, then R-KN1 and KR1. Which move should I play first now. Actually it seems rather devil-may-care. First maybe the Queen move.
1.Q-K3. At the same time that prevents P-KN4 for good. Of course, he can play N-N3; but a sacrifice on KR4 is nothing. Nor can he play P-B4, for the time being. Yes, I merely play
1.Q-K3. 70
75
79

(M2; P-B4) S: NICO CORTLEVER E: DE GROOT T = 8 MINUTES APRIL 8, 1943

- Count Pawns. Won for White, if only for positional reasons: the bad (Black) Bishop, passed (White) King Rook Pawn, wretched Black King position.
- 1.P-KR4 immediately, up to and including KR8! Not much to do against it.
1.P-KR4, P-R3; 2.P-R5, P×N; 3.P-R6, R-B2. No, that doesn't work; it
- 5

costs the Queen. 3...Q-B2 and so forth.
 10 No, doesn't come off so smoothly.
 1. N-Q4, Q-Q3; 2. Q-B6, K-Q2 - then
 the King's in the line of the Rook -
 3. N-N5, Q-B4 - nothing.

Immediately
 15 1. Q-B6, K-Q2 must follow - or go away
 with the Bishop, costs the Bishop or
 Queen Pawn, can't be done. So:
 1. Q-B6, K-Q2; 2. R×Pch doesn't
 work; 2. P-B4 maybe?

20 1. P-B4 is also a move. Taking with the
 Rook doesn't work, not with the Pawn
 either. The most wonderful thing is

that he can neither move his Rook nor
 his Queen away. Therefore there's time
 for all sorts of funny moves.

1. P-B4, B-Q2; then comes 2. R×P and
 it's about over.

1. P-KR4 is a little bit too stormy.
 1. N-Q4 blocks the Rook's line; too
 slow.

1. P-B4 looks rather nice; do you
 threaten still more? Yes. 2. P-B5 is
 threatened.

1. P-B4, P-R3; 2. P×P, P×N; 3. P×B
 pretty well does it. Yes,
 1. P-B4.

(M2; ...P-QR3) S: NICO CORTLEVER E: DE GROOT T = 3 MINUTES APRIL 8, 1943

Count pieces. Equal. Well, the Knight
 is attacked.

1...P-QR3; 2. R×N - maybe the Rook
 can be trapped. (Time = about 1
 5 minute)

(E: How do you arrive at that?)

The hanging position of the Rook gave
 me the idea; after 1...P-QR3, the
 Queen threatens to take a piece. Not
 10 until *after* that did I see that the Black
 Knight was attacked. Furthermore:

1...P-QR3; 2. R×N, P×N; 3. Q-B2 -
 3...Q-B3 must be stopped, can that be
 done in another way? No - 3. Q-B2,
 R-B1; and 4...Q-B3; so that won't do. 15
 (Pause) Let's look at 3. Q-Q1 immediately.
 No, then 3...Q-B3 right away. 3. Q-B2
 doesn't help either. It would help if the
 KN Pawn were on KN3. The idea is to
 play R×QP as soon as he plays Q-B3. 20
 Everything's checked. Yes, I play
 1...P-QR3. 22

(M2; ...N-K1) S: NICO CORTLEVER E: DE GROOT T = 8 MINUTES OCTOBER 24, 1939

Well, that is a kind of position as in a
 a game De Groot - Foerder. But it isn't
 it after all, no. One, two, ..., eight, one
 two, ..., eight. Count pieces. White
 5 stands better. Different ideas are possi-
 ble: either aim at P-QN4 or P-B4.
 White can perhaps play P-QR5 or
 P-QN4. Furthermore 1...B-B1 is play-
 able, to bring the Bishop to another
 10 diagonal. Black can try to plant his
 Bishops more pleasingly. Four White
 pieces hit QN4, I have but one. So that
 doesn't promise much. If we go through
 with P-B4, then perhaps White plays
 15 P-B4.

Yes, we should really make a plan. He
 can't start much on the Queen's wing;
 if he plays P-R5 and I take him, then
 my QR3 does become weak but so does

his QN3.

1...K-R1 followed by N-N1 to prepare
 P-B4, is an idea. Can also play
 1...N-K1.

1...P-N3 followed by N-R4 and N-N2
 is also possible. What can he do against 25
 it - against the P-B4 push at all. He can
 play P-QN4, but then his QB4 gets
 weak; the breakthroughs are not yet
 possible. If he plays P-B4, then N-Q2
 can follow. 30

An immediate

1...N-Q2 is also possible - but then
 B-R3 is annoying. 2. N-K4 is also there
 for White (probably meaning after
 P-B4, to play N-N5 and then to go to 35
 K6).

1...K-R1 is maybe bad after all, seeing
 that Bishop diagonal. Another look at

1...P-N3; what can he actually do?
 40 2. P-B4 doesn't hurt. Yes, 1...P-N3 and
 then later the Bishop to KN2, there it's
 not so badly placed. 1...P-N3 also
 prevents the Knight maneuver to KB4,

after P-K4 and P-KN4: N-K2-N3-B5.

For the moment he can't do anything to
 me. Well, so I play

1...P-N3.

(M4; ...N-B3) S: T.D. VAN SCHELTINGA E: DE GROOT T = 7 MINUTES NOVEMBER 15, 1939

Ah, a nice thing to give me! Is Black on
 move?

(E: Yes.)

Must I straighten up that rubbish of
 5 Black?

(E: Doesn't it appeal to you?)

Well, on closer inspection it might have
 been even worse. But there's something
 to his N×P; ... B×P; it would seem as
 10 though White intends a good deal, but
 it's not so bad after all. Let's see ... if
 there's something to be done. It would
 seem as though something must happen
 with those two Bishops.

15 I'm looking at something like

1...B×N and 2...N-N5. But that's not
 a very serious combination: he plays
 3. P-B3 on 2...N-N5.

Aha, if I just play

20 1...N-B3 and 2...B×Pch if he takes!
 3...N-N5ch; the King must go to N3 -
 yes, 4. K-N1, then 4...Q-R5 and it's
 over. So the King must go to N3; then
 I play 4...Q-N4. Is that so? Then
 25 5. P-B4, Q-N3. ... Yes, so, that's the
 well-known combination with B×
 RPch.

1...N-B3; 2. P-B4, then ... eh ... then
 play 2...N-QR4, that is not so nice for
 White. Well, if I had Black, I should
 30 play 1...N-B3 almost without hesitation.

1...N-B3. Now he can play 2. Q-N7,
 stopping both Bishop and Knight from
 taking on K4 - oh, no, that's chess
 blindness. I really don't know what
 35 could prevent me from playing 1...N-
 B3. Well, I would certainly do it. White
 will probably have to play 2. B-Q2, the
 Pawn must quickly be given back. Or
 2. N-QB3. Yes, I would certainly play
 40 1...N-B3. Still let's look at something
 that works even faster. (Verifies
 1...N-B3) It works fine. Everything
 goes with check, so the fact that the
 Queen on K2 is attacked (after 2. N×N)

45 doesn't matter. Again look at
 1...N-B3; 2. P-B3 - nu ya, then 2...N-
 QR4. 1...N-B3; 2. P-B3, N-QR4;
 3. Q-B3, P×P; and then the Bishop to
 Q4. Or first N-Q4 somewhere in
 50 between. (Subject calculates this varia-
 tion once more.) Yes, Black's position is
 enormous then. Indeed, I play
 1...N-B3. 54

(M4; ...Q-K2) S: T.D. VAN SCHELTINGA E: DE GROOT T = 21 MINUTES NOVEMBER 15, 1939

(E: Do you know this position?)

No, I don't know this position. Let's
 have a look. Count Pawns, pieces. Cer-
 tainly there's something to P-B5. But
 5 no, that can be sufficiently repelled.
 Nothing to be said immediately. Black
 stands somewhat nicer. King's wing is
 deadlocked, not much threatened there.
 How to forge ahead with the attack?

10 Let's look at something with

1...R-Q5. But then comes 2. Q-K2;
 not much in sight.

1...P-B5 or first perhaps prepare it
 with

1...R-R6. I'm looking at direct com- 15
 binations first, otherwise something like

1...N-N4 should be considered. But
 then comes R×P, yes 2. R×P, that is
 not much, then 2...B-B1, Rook to ...
 B6, no B3, no B2. ... The intention is
 to bring the Knight to Q5. I'm looking
 at

1...N-N4; 2. R×P, B-B1; 3. R away;
 oh, no, Rook to B4 is best. Then White

25 needn't worry. So that doesn't work very well.

Maybe

1...B-B1 to prepare for N-N4 - no; it doesn't go so quickly as I thought. Not so simple.

30 1...Q-R6 ... no. Indeed, I should ... (reverses himself again). Let's look.

1...B-B1. What can he do then? 2.N-B4, N×N; 3.B×N or 3.R×N - then you're not exactly better off either. It's not so easy to break through his position. White will be able to hold his own on the QB file.

Or something with

40 1...P-KB3, in order to play 2...B-R3 and in that way threaten something with P-B5.

1...P-KB3; 2.P×P, B-R3. Yes, hold on, that looks good. 2...B-R3. Does that threaten 3...P-B5 then? ... Indeed it does: 3...P-B5; 4.B×P, N×B; 5.R×N, B×Nch, etc. Well, so it threatens something. But, he can parry 1...P-KB3 with 2.P-KR4 - but I can take then, and I have a protected passed Pawn. I still don't know quite what I should play. (Pause) I keep on thinking of 1...P-KB3. I don't know anything else. But it doesn't yield so much, even if I carry out that combination.

55 1...B-B1, to get the Knight to N4 - but then 2.N-B4 is the retort; we've already seen that. Have a look, what all can White do anyway? Something like P-B4? (Short pause)

60 1...P-KB3; 2.P-B4 - no, that takes too long; 2.N-B4 is better for White. Something else to do. Let's have another look.

65 1...Q-R6 again. I'm looking at all kinds of possibilities. 1...Q-R6. ... Let's look again at pushing on B5.

1...P-KB3 again: 1...P-KB3; 2.P×P, B-R3. Now if he gets out of the bind with 3.K-Q1, for example, then - no that doesn't work: 3...P-B5 and then comes 4.N×P and if I take on Q2, 4.K×B; R-Q5; etc. 3.K-N1 instead; then take on Q7 and 4...P-B5 forks. Oh, no, the Queen takes back on Q7 and the Rook on N5 hangs. But that can probably be improved upon. Yes, first 4...Q-N4. That is probably not good for White.

1...P-KB3 rather suits me. Yes, I think that I shall play that. Look at other possibilities. White doesn't have to take, he can play 2.P-KR4; then I don't see how to proceed.

1...P-B4 right off; got something there? - but I don't see what good that is. (Short pause) Really the whole thing is to know what Black has been aiming at. The plan, I should like to know his last move.

I should play

1...P-KB3 in spite of everything. Look to see if White has nothing. (Pause) 2.Q-B3 maybe; something to do with the Queen.

1...P-KB3 and now maybe 2.N-B4 right away, going to attack the Pawn on B4 then. 2.N-B4, N×N; 3.R×N - nu ya, then you take on KN4. That doesn't work, costs a Pawn. Well, probably I'd play

1...P-KB3 here - then he probably answers 2.P-KR4. ...

(E: What do you play now?)

I play

1...P-KB3, yet I don't know how I shall proceed. The combination rests on his taking and especially B-R3 of course. And I thought of 1...Q-R6 in order to rule out P-KR4, to separate the Pawns.

1...R×Pch, 2...P-B4, 3...Q-R6, and 4...R-K3. Is it already mate then? Yes, 1...R×Pch.

(M5;...R×Pch) S: A. D. DE GROOT E: CORTLEVER T = ½ MINUTE APRIL 15, 1943

Curious position. Circles around the King: mate in three or four. 1...R×Pch or 1...P-B4ch - look for a combination of these moves and Q-R6.

(M5; R-Q1) S: A. D. DE GROOT E: PRINS T = 30 MINUTES AUGUST 2, 1939

At first glance the center formation Pawns on (KB3), K4, and Q5 and the Queen on Q4 are striking. White seems to be better off. After that discovered the Black pieces on the QB file. First I thought the Rook on QB4 was a Bishop. Black can play R-B8, counter possibility. So White has the extra center Pawn; that should be exploited in connection with a King's side attack. It would be quite nice to prevent the Rook exchange through R-B8, that probably increases the drawing chances. But still White stands quite a bit better even though the QB file is annoying. 1.P-K5 deserves consideration in order to immediately make a passed Pawn on the Queen file. Just looking at this superficially. Furthermore 1.P-B4 is to be looked at. In the future N-KB5 can perchance occur: sacrificial possibility for the sake of a King's side attack. Moreover 1.R-B2 or 1.K-N2 are possible, to escape the Rook exchange.

Let's look a bit closer:

1.P-B4. Then, for example, 1...R-B8; then perhaps 2.P-B5 anyway; what then? 2...R×Rch; 3.K×R and then Black can give check, but White still remains somewhat better off. After

1.P-B4, N-B5 immediately is also possible, probably better. How it goes on after that is not very clear. Perhaps 2.P-B5 anyway? No, that's not good, that leaves the K5 square free. Finally, still the possibility of

1.P-N4, but presumably not worth much either. 1.P-N4 threatens to exchange on R5 and N-B5, and on 1...P-R5 can follow 2.P-N5 and N-N4. But 1...Q-B5 is perhaps an objection, blocks the Bishop Pawn; oh no, that's not true: 2.P×P and 3.N-B5 with advantage.

1.P-N4, N-B5 can be tried. Then 2.P×P, N×N; 3.P-R6 threatens mate. 3...P-B3 is forced and White has won a Pawn. What's the good of it? It's good after all. - But wait, it can be much

simpler: 1...N-B5 is no good: 2.P×P, N×N; 3.Q×N, P×P; 4.Q-N5ch, etc. Let's search for something else; lots of possibilities. Once again:

1.P-N4, N-B5; 2.P×P, P×P; 3.N-B5. Maybe Black can defend himself. Play 3...N-K4 or so, there was a threat of mate on N7. (More variations) 3...Q-K4 doesn't go, costs a Pawn on K7. And 3...N-K4 probably doesn't go either, it's not so very good; indeed 4.P-B4 is unpleasant, it wins.

1.P-N4 is a nice move. He can just take on N4, can't he? Then it's a query whether to take back with the Pawn or with the Knight. With the Pawn: then the attack might peter out. With the Knight: then 2.N-R6ch is threatened, but he can play K-R2. Then 3.Q-K3 comes into consideration with the half threat of 4.Q-R6ch - doesn't yield much. So take back with the Pawn after all: 2.P×P. Then probably 2...N-B5 once again - then it's not so easy to withdraw the Knight from an exchange.

1.P-N4 is actually a bit warped; only makes sense if a sharp attack comes out of it. If not, discard it. In my opinion taking on N4 makes the question dubious.

Suppose I play

1.P-K5. Again 1...N-B5. (Long Pause) Perhaps 2.P-Q6 then, P×P; 3.N-Q5 or such.... Is presumably not good. He plays 3...P×P, attacks the Queen; doesn't suit me particularly. The thing is to keep the Knight from exchange. (Pause) Something must happen immediately; if Black can exchange pieces, White won't win so easily. The (Black) Knight threatens to come to B5, and R-B8 is also possible. Then it's not so easy anymore.

Once again:

1.P-B4 - doesn't succeed on account of 1...N-B5 (hesitation); 2.P-K5 doesn't work because he takes on K6.... Difficult position. I have the feeling that White

must follow up his advantage immediately. Then again maybe not.

Also look to

100 1. P-Q6. He takes with the Queen, and then I don't see it. Just now I didn't mention it, but I looked at a few other variations too, for example:

1. P-N4, P×P; 2. P×P, N-B5; (Pause)
105 3. N×N, R×N; (hesitation) 4. Q-B2 with a threat on B7. Then he plays 4...P-B3 – so it doesn't yield anything, at least nothing special. Then you can still play 5. P-N4. Let's see how it stands then; maybe worthwhile. No, it's not
110 after all, he plays 5...Q-K4 and if you then take on B6 – and you have nothing better – then a Rook ending ensues, presumably drawn.

115 The search is for a possibility to avoid the exchange of the White Knight, so that after N-B5, it doesn't have to be withdrawn. So I am looking either for a good square for the Knight or a threat such that after N-B5 I can leave the Knight because he cannot afford to exchange. (Pause) 1. P-N4 and 1. P-K5 are the most prominent considerations. Once again

125 1. P-K5, N-B5. Maybe there's just nothing to get out of it. Oh, yeah, another variation of just a minute ago that I forgot to mention: 1. P-K5, N-B5;

2. N×N, R×N; 3. P-Q6, R×Q; 4. P×Q, R-QB5 and Black wins a Pawn; I'd
130 already discarded it.

1. P-K5. (Pause) There always follows
1...N-B5. I don't believe in any other move for Black. (Long Pause) Then you should play 2. P-Q6, P×P; 3. N-Q5, P×P; then maybe a Queen move –
135 4. Q-Q2 doesn't go – 4. Q-Q3 or so, so that you can still make some attack with N-B6 or such. No, this has also been rejected. With that 1. P-K5 falls.
140 Then look at a calm move for a change,

1. K-N2 or something like it. (Pause) N-B5 always follows. (Pause) No, I believe in

1. P-N4 after all. Glance at the main variations: 1...P×P; only variations in which he gets something. 2. N×P. But now R-B5 will soon be disagreeable, very disagreeable. Still nothing. Well, remarkable position, it's not so good on
150 second thought. (Pause) Extremely difficult position. (Pause) (Interrupted by lunch bell) Interruption is not serious; I was as good as finished anyway. I think
155

1. R-Q1. Simple; only in order to have the Rook behind the future passed Pawn. Let me play just that. See nothing better. We shall play

1. R-Q1.

160

(M5; B-R7CH) S: A. D. DE GROOT E: PRINS T = 35 MINUTES DECEMBER 24, 1938

Let me get oriented. Are pieces equal? (There were difficulties because the pieces were not immediately recognizable; a miniature chess set with not very distinctive pieces was used.) Who's on
5 move?

(E: White.)

The Rook's attacked; material balance: he's two Pawns ahead, otherwise equal.
10 What to do? It must be possible to checkmate the Black King. If 1. B-R7ch, then 1...K-R1. The Bishop or Rook sacrifice on R6 must be looked at. Lots of possibilities will have to be calculated.

15 The Queen must get there too in one way

or another, but how? Bishop sacrifice? Wait a moment, a long variation – first go over it:

1. B×P, B×R; 2. B-R7ch, K-R1 – doesn't have to; I thought 2...K-R1
20 was forced, but he can get away now – 2...K-R1; 3. B×Pch, K×B; 4. Q-N6ch, K-R1; and then, for example, 5. Q-R6 with serious threats. But wait, then he can play 5...R-B2 or something
25 – no, that is probably, certainly not good.

Once more:

1. B-R7ch, then in any case 1...K-R1.
2. Q-N6 doesn't make so much sense
30

then. Black can defend himself all right then: 2...Q-K1 or something like that.

1. B×P looks quite tempting. On
35 1...B×R, then 2. B-R7ch, ... and then you've lost everything and all the fun's gone. So

1. R×RP. What then? Will have to take. Then 2. B-R7ch, K-R1 – really must – and then something like 3. Q-N6, takes rather long; can mate still be
40 enforced then? Quite probably; 3. Q-N6: how must he defend himself now? 3...R-KN1 maybe – then White doesn't have enough pieces left.

45 Finally

1. R-N3 is still possible as a first move, threatens B×P. Then 1...K-R1 for example – then there's still time for sacrificing. Not simple. In general, these
50 sorts of positions aren't up my alley.

Mate combinations.

1. B-R7ch, then 1...K-R1. The Queen should be in front of the Bishop! Can that be achieved? Doesn't look like it.

55 First put the Rook in a safe place:

1. R-N3, K-R1; 2. Q-K2 threatens K6. Then what? A dumb move for Black, even then do you have anything? If he covers the Bishop, then simply R-K1 and I win something. Yes? Yes. Not
60 much he can do against that. Can some use be made of the ungarded position of the Bishop on K6?

We might further look at, quite simply,

1. B-B5; takes; then 2. Q×B with a splendid position ... (manuscript unreadable:); Try out something violent again. From the looks of it that really
70 should succeed. Problem: How to get the Queen in play? (Pause)

1. B×P, B×R; 2. B-R7ch, there was something more in that, but it slipped my mind. Try to bring it back. Can I find it? Let's look again at

1. B×P, B×R; 2. B-R7ch, K-B2 – no
75 2...K-R1; 3. Q-N6 threatens mate, 3...P×B; 4. Q×RP, R-B2 and then it sort of peters out. Or doesn't it? 4...R-B2... Yes, it does. The Rook's lost and there's no mate within the
80

foreseeable future. Come on.

1. R×RP once again, take it away. Then? (after thinking) A whole Rook sacrificed but still a lot of pieces left to mate him with. 2. B-R7ch, K-R1 – not
85 many branches, that makes it easy. 3. Q-N6 doesn't get me much further. Well, I'm afraid I'm too dense. I'm quite convinced that in an actual game something like that can be examined
90 until the end.

1. R×RP again, takes; then it's annoying that the Queen is behind the Bishop.

1. Q-K2, let him take on R3. Is that an idea? Probably not, but you never know. 1...B×R – no, certainly no good because he can close off the Bishop diagonal (...P-B4). Idea abandoned.

1. R-K3 is another possibility, but it's a pity that the diagonal of the Bishop on B1 is blocked. A certain pathological doubt comes over you: you don't know a thing anymore. But I haven't lost heart yet. (Pause) Something must happen
105 with the Rook on R3; you can't just let it be taken. Sacrifice on R6, Pawn takes; 2. B-R7ch, K-R1. I'm all the time trying to get Q-N6 in with check; the plan is clear to me, but I don't see how to
110 realize it. Getting the Queen in play will lead to a quick checkmate. B-N8 is a possibility, threatening mate at KR7 – gets the Bishop out of the way –; I have already roughly looked into this
115 possibility several times.

1. B-R7ch, K-R1; 2. B-N8 threatens mate, R×B. No, nothing. Then 2. Q-N6 but then 2...B×R. The Rook on B1 doesn't do anything; suppose it got
120 into play, it would be simple then.

1. B×P looks so nice, now why doesn't that succeed? 1...P×B – no 1...B×R's better. And then? 2. B-R7ch can not miss, 2...K-R1 is best. Crazy, you'd
125 think you've got something big there.

1. R-N3, King away; then? 2. Q-K2, Bishop goes away. Let's see.

1. R×RP ... (Calculations followed) Anew
130

1. R×RP, P×R; 2. B-R7ch, K-R1;
 3. Q-N6, R-B2; - then I've still got
 nothing.
 135 1. R×RP must be good; 1... P×R;
 2. B×P. What's the threat? 3. B×R but
 that's not important. Any other threat?
 The check on R7 is pointless. The
 Bishop at Q3 is bothering me. Curious.
 Isn't there any forcing move to get rid
 140 of it? In a game I would probably think
 just as long and finally make a dumb
 move anyway.
 1. B-R7ch, K-R1; 2. ? Yes. I've also
 looked several times into the possibility
 145 of opening the King Rook file with
 some sort of sacrifice and then play
 R-B4-KR4. (Long pause)
 1. R-N3 is maybe not so dumb after all,
 1... K-R1; sacrifice on R6, 2... P×B;
 150 3. R-N6. Then I have a nice position at
 the cost of one Bishop. What do I
 threaten? Probably something serious.
 He is almost unable to defend himself.
 It's so irksome. I've a distinct feeling
 that it's somehow or other over. A real
 155 feeling of having fallen short if I don't
 mate him right off the bat. I've also
 thought of bringing the Queen into
 play via KB2, for example, after sac-
 rificing on R6. Then Q-N3 threatens. 160
 (Pause)
 Wait a minute, I do believe that
 1. R×RP is good - can't promise it yet -
 takes; 2. Q-Q2. How does he defend?
 He can't. Ycs, he can, the scoundrel 165
 plays R-B2. He always has a defense.
 No, 1. R×RP isn't the move after all.
 1. B×P. Oh, no, then 1... B×R.
 (Pause)
 1. R-N3. Definitely. In a game maybe 170
 I'd have found it; don't believe that
 it's the move. 172

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INDEX OF SUBJECTS

Apart from referencing the most important terms in the text, this index provides a terminologically based *systematization of the contents of the book* that may be useful to the reader.

Peculiar to the present index are the following procedures:

1. References are to *passages relevant to concepts*, not to terms.
2. *Boldface numbers* refer to pages of definition or otherwise fundamental exposition.
3. The *order of subitems* under a heading is partly alphabetic and partly systematic.
4. For a number of important concepts, such as ANTICIPATION and METHOD, the *special cases* - scattered throughout the text - are *systematically and exhaustively listed*.

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